When Optimal Is Just Not Good Enough: Learning Fast Informative Action Cost Partitionings [*]

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Abstract

Several recent heuristics for domain independent planning adopt some *action cost partitioning* scheme to derive admissible heuristic estimates. Given a state, two methods for obtaining an action cost partitioning have been proposed: optimal cost partitioning, which results in the best possible heuristic estimate for that state, but requires a substantial computational effort, and ad-hoc (uniform) cost partitioning, which is much faster, but is usually less informative. These two methods represent almost opposite points in the tradeoff between heuristic accuracy and heuristic computation time. One compromise that has been proposed between these two is using an optimal cost partitioning for the initial state to evaluate all states. In this paper, we propose a novel method for deriving a fast, informative cost-partitioning scheme, that is based on computing optimal action cost partitionings for a small set of states, and using these to derive heuristic estimates for all states. Our method provides greater control over the accuracy/computation-time tradeoff, which, as our empirical evaluation shows, can result in better performance.

Introduction

Recent years have seen a significant advance in the field of admissible heuristics for domain independent planning. Many of these new heuristics are based upon *action cost partitioning* (simply referred to as cost partitioning throughout the rest of this paper), for example: additive-disjunctive heuristics (Coles et al. 2008), the admissible landmarks heuristic h_{LA} (Karpas and Domshlak 2009), and additive implicit abstractions (Katz and Domshlak 2010b; 2010a). The basic idea underlying cost partitioning is elegantly simple: the cost of each action a is divided between different objects (action representatives or landmarks). The heuristic estimate is computed by summing up estimates obtained from each of these objects, and admissibility is guaranteed by ensuring that each action a does not contribute more than its true cost in the original task to the sum.

Such cost partitioning was originally achieved by accounting for the whole cost of each action in a single object, while ignoring the cost of that action in all the others (Edelkamp 2001; Felner, Korf, and Hanan 2004; Haslum, Bonet, and Geffner 2005). Recently, heuristics that use *arbitrary* cost partitioning have been shown to lead to state-of-the-art performance in cost-optimal planning. Two of these heuristics are the additive implicit abstraction heuristic (Katz and Domshlak 2010b; 2010a), and the admissible landmarks heuristic h_{LA} (Karpas and Domshlak 2009). Both of these heuristics formulate the problem of finding the most informative (that is, an optimal) cost partitioning for a given state as a linear program (LP), which they attempt to solve. Although solving a linear program is polynomial, the resulting heuristic is prohibitively slow. Another option is to use an ad-hoc uniform cost-partitioning scheme, where the cost of each action is divided equally between the relevant objects. While using uniform cost partitioning leads to a much faster heuristic, it is also much less informative.

The optimal and uniform cost-partitioning schemes offer two opposite points of the tradeoff between heuristic accuracy and heuristic computation time. While optimal cost partitioning is informative but slow, uniform cost partitioning is fast but less informative. In order to try and get the best of both worlds, Katz and Domshlak (2010b) proposed computing an optimal cost partitioning for the initial state, and using that cost partitioning to evaluate all other states. While this method offers a compromise between heuristic accuracy and computation-time per state, it does not offer any control over the accuracy/computation-time tradeoff.

In this paper, we present a novel method that provides control over the tradeoff between heuristic accuracy and computation-time per state. Using this method, we can derive heuristic estimates that are almost as informative as those of the optimal cost-partitioning scheme, while keeping computation-time per state at the same order of magnitude as that of the uniform cost-partitioning scheme. Our method consists of selecting a small number of states in a pre-search phase, and computing an optimal cost partitioning for each of these *representative* states. During search, these optimal cost partitionings are used to evaluate every state encountered during search, which is very cheap computationally.

Taking implicit abstraction heuristics (Katz and Domshlak 2010a) as a case study, we empirically show that the suggested approach is competitive with both optimal and any known ad-hoc cost-partitioning schemes.

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Notation

We consider classical planning tasks described by the SAS$^+$ formalism (Bäckström and Nebel 1995) with non-negative action costs. Such a planning task is given by a quintuple $\Pi = \langle V, A, s_0, G, \mathcal{C} \rangle$, where V is a set of *state variables*, each $v \in V$ is associated with a finite domain $dom(v)$; each complete assignment to V is called a *state*, and $S = \times_{v \in V} dom(v)$ is the *state space* of Π. The state s_0 is called an *initial state*. The *goal* G is a partial assignment to V; a state s is a *goal state* iff $G \subseteq s$.

A is a finite set of *actions*. Each action a is a pair $\langle \mathsf{pre}(a), \mathsf{eff}(a) \rangle$ of partial assignments to V called *preconditions* and *effects*, respectively. $\mathcal{C} : A \rightarrow \mathbb{R}^{0+}$ is a real-valued, non-negative *action cost* function.

An action a is applicable in a state s iff $\mathsf{pre}(a) \subseteq s$. Applying a changes the value of each state variable v to $\mathsf{eff}(a)[v]$ if $\mathsf{eff}(a)[v]$ is specified. The resulting state is denoted by $s[\![a]\!]$; by $s[\![\langle a_1, \ldots, a_k \rangle]\!]$ we denote the state obtained from sequential application of the (respectively applicable) actions $a_1, \ldots, a_k$ starting at state s. Such an action sequence is an *s-plan* if $G \subseteq s[\![\langle a_1, \ldots, a_k \rangle]\!]$, and it is a *cost-optimal* (or, in what follows, *optimal*) s-plan if the sum of its action costs is minimal among all s-plans. The purpose of (optimal) planning is finding an (optimal) s_0-plan.

For a pair of states $s_1, s_2 \in S$, by $\mathcal{C}(s_1, s_2)$ we refer to the cost of a cheapest action sequence taking us from s_1 to s_2 in the state model of Π; $h^*(s) = \min_{s' \supseteq G} \mathcal{C}(s, s')$ is the notation for the cost of optimal s-plans for Π.

Background

In this paper, we are concerned with solving planning tasks optimally, using A^* with a cost-partitioning based heuristic. Furthermore, we are only interested in heuristics that have a feasible method for finding an optimal cost partitioning for a specific state. We are familiar with two such heuristics: the admissible landmarks heuristic h_{LA}, and the additive implicit abstraction heuristic.

The h_{LA} heuristic (Karpas and Domshlak 2009) works by partitioning the cost of each action between the landmarks it achieves (a landmark is a fact which must be true at some point along *every* plan). A set of linear constraints ensures admissibility, and thus finding an optimal cost partitioning can be formulated as a linear program (LP). However, the h_{LA} heuristic is defined using the set of landmarks that still need to be achieved, which is determined by the path taken to reach the state, and not by the state itself. Therefore, h_{LA} is not a proper, state-dependent heuristic, but rather a path-dependent heuristic. More accurately, it is multi-path dependent, and can thus give several different heuristic estimates for the same state, depending upon the path or paths found leading to that state.

The (additive) implicit abstraction heuristic (Katz and Domshlak 2010a; 2010b) belongs to the family of additive ensembles of admissible heuristics. An additive ensemble is composed of several different component heuristics, and the cost of each action is divided between its representatives in the component heuristics, while ensuring that the cumulative counting of the cost of the action does not exceed its true cost in the original task. This admissibility condition subsumes earlier admissibility criteria for additive pattern database heuristics by Edelkamp (2001) and for general admissible heuristics by Haslum et al. (2007).

Of course, different cost partitions lead to additive heuristics of different quality. For abstraction based heuristics, a procedure for computing an optimal cost partitioning for a given state exists (Katz and Domshlak 2010b). This procedure works by formulating the conditions for admissibility as linear constraints, and solving an LP that maximizes the heuristic estimate. This procedure gives the most informative estimate possible (for the given state), but is very slow, and does not work well in the standard time-bounded setting.

Another option is to use an ad-hoc uniform cost-partitioning scheme. This results in a heuristic which is much faster, but also less informative. Katz and Domshlak (2010b) suggested a method that provides a compromise between the optimal and uniform cost-partitioning schemes. Their compromise involves computing optimal cost partitionings for only a subset of evaluated nodes. They only implemented this for the trivial case, where this subset of evaluated nodes contains only the initial state. However, even this sometimes leads to an improvement over using the uniform cost-partitioning scheme.

In this paper, we suggest a concrete method for obtaining a set of states for which optimal cost partitionings can be computed. We also suggest two different ways of using these cost partitionings to derive a heuristic. The setup we consider is as follows: we are given a single planning task Π which we want to solve. Our method takes a parameter k, that controls the tradeoff between heuristic accuracy and heuristic computation time. The method we present below then selects k states and computes an optimal cost partitioning for each of them. These states are used to create a heuristic, which is then used by the A^* search algorithm. Note that although our method works before search, it works for a single specific planning task, and can thus be considered to work in a pre-search phase. Specifically, the time our method uses is counted as part of the time-limit per task.

Since the h_{LA} heuristic is multi-path dependent (and not state-dependent), applying our method to the h_{LA} heuristic is a bit complicated. Therefore in this paper we apply our method only to implicit-abstraction heuristics, and leave application to h_{LA} as future work.

Algorithm

Given some cost-partitioning based heuristic family h, let h_c denote the heuristic induced by cost partitioning c. Denote by c_s some optimal cost partitioning for heuristic h at state s (that is, $c_s = \mathrm{argmax}_{c \in \mathcal{P}} h_c(s)$ where $\mathcal{P}$ denotes the set of all admissible cost partitionings). By definition, the best estimate we can get for state s using heuristic h is $h_{c_s}(s)$. Note that computing $h_{c_s}(s)$ consists of two parts: computing c_s, which is usually very slow, and then computing the heuristic estimate h_c given $c = c_s$, which is much faster.

The approach proposed in this paper is based on the assumption that if two states s and s' are close to each other (in the transition system of the planning task), then an optimal cost partitioning for s will also give a good (informative) heuristic estimate for s' (that is, the loss of accuracy $h_{c_s}(s) - h_{c_{s'}}(s)$ is small when s and s' are close). Although we cannot prove this assumption, our empirical evaluation supports it.

General Framework

Assume we have a set of k states, $R = \{s_1, \ldots, s_k\}$. Assume we also have a set of associated optimal cost partitionings $C = \{c_1, \ldots, c_k\}$, such that c_i is an optimal cost partitioning for state s_i.

We can define a heuristic which combines these as follows:

$$h_{all}(s) = \max_{c \in C} h_c(s)$$

Since computing h given a cost partitioning is fast, h_{all} is still rather fast — it requires evaluating h (with a given cost partitioning) k times.

Another option is to only look at at the state in R that is closest to s. Based on our assumption, using the closest state's cost partitioning should give the best heuristic estimate possible by using any cost partitioning from C. However, finding the closest state to s is as hard as optimal planning, so we use a distance metric d instead of the true distance. This leads to the following heuristic:

$$h_{nn}(s) = h_{c_{N(s)}}(s)$$

where $N(s) := \text{argmin}_{s' \in R}\, d(s, s')$ is the state in R which is closest to s. This requires using h (with a given cost partitioning) only once, but computing the distance metric k times. Note that a good distance metric might be as expensive to compute as h with a given cost partitioning.

These heuristics lead to the following method for fast, informative cost partitioning:

1. Choose a set R of k states. Assume $R = \{s_1, \ldots, s_k\}$.
2. For each $s_i \in R$ compute an optimal cost partitioning c_i. Let $C = \{c_1, \ldots, c_k\}$.
3. Use either h_{all} or h_{nn} (with an appropriate distance metric) as a heuristic.

This leads to the obvious question of — how to choose the set of states R?

In the following we assume we are using h_{all}, although the arguments for h_{nn} are similar. Given a budget for k optimal cost partitionings, our objective is to choose R, so as to minimize the number of states expanded by A^*.

From the well-known analysis of A^* (Pearl 1984), this can be achieved by making h_{all} as informative as possible (ignoring pathological cases with tie-breaking). In other words, we want to maximize the informativeness of h_{all} over the entire state space. However, it is not feasible to work with the entire state space, so we limit ourselves to a *sample* of the state space.

Given a sample of the state space (denoted by S'), we must still choose k states out of S'. Here, we use our assumption that if two states are close to each other, then an optimal cost partitioning for one will also give a good heuristic estimate for the other. Therefore, we want to choose R so as to minimize the distance from each state in our state space sample to a state in R. Although this objective can be formulated mathematically in several ways, we choose to minimize the sum of distances from each state $s \in S'$ to the *closest* state to it in R (that is, we want to minimize $\sum_{s \in S'} \min_{s' \in R} \mathcal{C}(s, s')$). It is also possible to consider minimizing the maximum distance for each state to the closest state, but we consider this a bad alternative, since it only takes into account one pair of states.

Since we don't know the true cost $\mathcal{C}(s, s')$, we can approximate it by some easier to compute distance metric d, and try to minimize $\sum_{s \in S'} \min_{s' \in R} d(s, s')$. Although it might seem like it is difficult to find such a set of states R, this is basically a clustering problem, with the objects to cluster being states (from the sample), and the distance between them defined by d.

After we cluster our state space sample, we need to choose a representative from each cluster, and compute an optimal cost partitioning for that representative. The other states in the cluster should be similar to the representative, and so (hopefully), will get a high heuristic estimate using the representative's optimal cost partitioning. Note that this is similar to the ISAC instance specific algorithm configuration method (Kadioglu et al. 2010), where problem instances are objects, and they are clustered according to some user-defined similarity measure, and then a "best configuration" is found for each cluster.

The approach described above is rather general. In the following, we give a more detailed description of each of the steps we use in our approach.

State Space Sampling

The state space sampling procedure we use here is the one suggested by Haslum et al. (2007). An initial goal depth estimate is first obtained. We estimate the goal depth by using twice the value of the FF heuristic's (Hoffmann and Nebel 2001) estimate of the initial state. Then a series of random walks is performed, where the last state in each such walk is added to the sample.

Each random walk has a random depth limit, which is drawn from a binomial distribution that is centered around the estimated goal depth. Each walk is uniformly random (that is, one of the successors of the current state is chosen with equal probability), and walks that reach dead ends are discarded. The number of random walks we perform is 1000 times the number of clusters we want (that is, $1000k$).

We remark that other state space sampling methods exist, such as the probing method proposed by Domshlak, Karpas, and Markovitch (2010). However, the probing method does not provide an independent sample, since a state cannot be in the sample without one of its predecessors.

Clustering Algorithm

While clustering algorithms abound, we have a few requirements that limit our selection. We require the clustering algorithm to allow manual control of the number of clusters generated. We also need the algorithm to return a representative for each cluster, which will later be used as one of the states for which an optimal cost partitioning is computed.

The k-means algorithm (Hartigan and Wong 1979) seems to fit the above requirements, but it has one problem. The k-means algorithm maintains a centroid for each cluster, and assigns instances (in our case, states from the state space sample) to the closest centroid. However, these centroids are not necessarily instances (states), but rather the arithmetic mean of several of those. In our case, they would be the mean of several states, which is not well defined (consider the meaning of $0.5 \cdot on(A, B) + 0.5 \cdot on(A, C)$).

Therefore we use k-medoids (Kaufman and Rousseeuw 1987), which performs clustering around representatives, such that the representative for each cluster is the most "central" member of that cluster (the median), and is always one of the given instances. We used an efficient implementation of k-medoids, *ultra k-medoids* (Breitkreutz and Casey 2008), for the weka toolkit (Hall et al. 2009).

Distance Metric and Features

Any type of clustering algorithm requires a distance metric between instances (which could be defined by features for each instance). In this paper, we use the Euclidean distance metric (l_2), over two types of features. First, we use the values of state variables as features (as suggested by Domshlak, Karpas, and Markovitch 2010). These features can be used regardless of the exact heuristic we use.

The second type of features, which we call the *ensemble member value* features, is derived from the component heuristics that define the additive implicit abstraction heuristic. The feature values are the heuristic values of each individual component heuristic (ensemble member) under the *uniform* cost partitioning. That is, the number of features is equal to the number of component heuristics, and for state s, the value of feature f_i is simply the heuristic estimate induced by the i^{th} component heuristic, $h_i(s)$, under the uniform cost partitioning. Note that any other cost partitioning can be used to generate features — the uniform cost partitioning is simply the natural choice since it is easy to compute, and does not require any additional information.

Once we have our features, we need to define a distance metric based upon them. The ensemble member value features are numeric, and thus require no further definitions. However, the state variable value features are discrete, but not numeric, and thus we define the distance as 0 for equal values, and 1 for different values of the same feature.

In our empirical evaluation we use both types of features (state variable value features and ensemble member value features). We also tried using both types of features together, but this does not make a noticeable difference, and so for clarity of presentation, we do not give these results.

Empirical Evaluation

The approach presented above is based on the assumption that if state s is close to s', then an optimal cost partitioning for s will result in a good heuristic value for s'. First, we attempt to validate this assumption empirically.

Validating the Basic Assumption

In order to validate our assumption emprically, we must first formulate it in statistical terms: Let s, s' be two states, such that the minimal distance from s to s' is d. Denote the relative loss of accuracy from using an optimal cost partitioning of s' to evaluate s by $\Delta_{s,s'} := (h_{c_s}(s) - h_{c_{s'}}(s))/h_{c_s}(s)$. Then $\Delta_{s,s'}$ and $\Delta_{s',s}$ are positively correlated with d.

Domain	Inverted Forks		Forks	
	T	S	T	S
airport-ipc4	9	5	2	0
blocks-ipc2	19	16	19	16
depots-ipc3	0	0	3	3
driverlog-ipc3	6	6	0	0
freecell-ipc3	5	5	3	3
grid-ipc1	1	0	0	0
gripper-ipc1	7	7	7	7
logistics-ipc1	2	2	2	2
logistics-ipc2	10	10	10	10
miconic-strips-ipc2	43	43	43	39
mprime-ipc1	16	13	13	9
mystery-ipc1	18	12	12	9
openstacks-ipc5	0	0	0	0
pathways-ipc5	2	1	2	1
pw-notank-ipc4	15	13	5	4
pw-tank-ipc4	0	0	6	4
psr-small-ipc4	33	30	33	28
rovers-ipc5	7	7	7	7
satellite-ipc4	4	4	4	4
schedule-ipc2	41	25	28	18
tpp-ipc5	4	3	5	5
zenotravel-ipc3	7	7	7	6
TOTAL	249	209	211	175

Table 1: Results of Basic Assumption Statistical Test
For each domain, we list the total number of tasks with at least 30 points in their sample (T) and the number of tasks for which the correlation was statistically-significant with $p < 0.05$ (S).

In our empirical evaluation, we collect pairs of states, for which we know the minimal distance. For each pair $\langle s, s' \rangle$, we compute $\Delta_{s,s'}$ and $\Delta_{s',s}$. We do this for each planning task in our benchmarks, by choosing 10 states using random walk (in the same manner as our state space sampling). From each of these states, we perform breadth-first expansion (eliminating duplicates), up to a depth of $h_{FF}(s_0)$. For each such expansion (based in state s) and for each layer $1 \leq l \leq h_{FF}(s_0)$, we randomly and uniformly select a state s_l from layer l, and add Δ_{s,s_l} and $\Delta_{s_l,s}$ with minimal distance l to our sample. While we do not claim that this is an unbiased sample, we believe it can provide a good indication whether our assumption is valid.

Having collected such a sample for each planning task, we can use a statistical test to validate our assumption — that there is a positive correlation between distance and relative loss of accuracy. We use the Kendall rank correlation test (Kendall 1938), which is a non-parametric test, that does not assume a linear correlation between the variables. The specific variant of the Kendall test we use is the τ-b variant, which makes adjustments for ties. We discard planning tasks for which we do not have at least 30 pairs in our sample, and perform the Kendall test for each of the remaining planning tasks. The results of this evaluation are detailed in Table 1. Overall, we found a statistically significant correlation ($p < 0.05$) in over 80% of the tasks, for both forks and inverted forks.

Evaluating Our Approach

Even if we accept the basic assumption, there are several approximations involved in using our approach: using the state space sample is an approximation (of the unfeasible option) of using the entire search space, and clustering is an approximation of some unknown target function. Therefore we also performed an empirical evaluation of the performance of our approach on planning tasks from well known benchmarks.

We implemented our approach on top of the Fast Downward planning system (Helmert 2006), and conducted an empirical evaluation on the same set of benchmark problems used by Katz and Domshlak (2010b). Since preliminary results showed that h_{all} does better than h_{nn}, we only evaluate h_{all} emprically.

We used two different types of implicit abstraction heuristics: forks and inverted forks. For both of these, we compare our approach to the three baseline approaches proposed by Katz and Domshlak (2010b): optimal cost partitioning at each state (*opt*), uniform cost partitioning (*uni*), and using an optimal cost partitioning for the initial state to evaluate all states (*ini*). We compare these to h_{all} with various values for k, where the representative states are chosen using clustering with the different features discussed above: state features (*clstr-s*) or ensemble features (*clstr-e*). In addition, we compare to a variant which chooses k representative states from the sample at random (*rand*). All of the experiments were run on Intel E8400 machines with the standard time limit of 30 minutes and a memory limit of 1.5 GB.

There are a few differences between fork-based and inverted fork-based implicit abstraction heuristics, which are worth noting. First, using uniform cost partitioning seems to be much more effective for forks than for inverted forks. As our empirical evaluation demonstrates, with fork-based heuristics, *uni* is slightly more informative than *ini*, while with inverted forks, *ini* is much more informative than *uni*.

Another important difference is that finding an optimal cost partitioning for a given state with an inverted fork-based heuristic is usually much faster than finding the optimal cost partitioning for a fork-based heuristic. This is because the LP formulation for inverted forks is much smaller, and so the LP solver can find an optimal solution much faster. In fact, for some planning tasks, with fork-based heuristics, the LP Solver we use (MOSEK 2009) fails to find an optimal cost partitioning within our default timeout of 5 minutes, and so any method that tries to find an optimal cost partitioning fails on these tasks. Therefore, for the fork-based heuristic, we provide evaluation over two separate sets of tasks: all tasks (including those where the LP solver fails to find an optimal cost partitioning), and *non-hopeless* tasks, which we define as the planning tasks for which an optimal cost partitioning for the initial state was found by the LP solver. For inverted forks, almost all tasks that can be solved are non-hopeless, and so we only report results for the set of all tasks.

In our evaluation we examine the tradeoff between heuristic accuracy and heuristic computation time. We also look at the bottom line — the number of planning tasks solved.

Informativeness

We evaluate heuristic accuracy (informativeness) on the set of tasks solved by all methods (236 tasks for inverted forks and 194 tasks for forks). Note that these are always non-hopeless tasks, since they are solved by *opt*. For a given task, the method which expanded the least number of states is assigned a score of 1, and the other methods are assigned a score in the range $(0, 1]$, relative to the best method (so if a method expanded twice as many states as the best method, it is assigned a score of 0.5). The total score for a method is the sum of scores over all tasks. The same scoring method was used by Domshlak, Katz, and Lefler (2010) to evaluate informativeness, and it is based on the metric score for evaluating plan quality introduced in IPC-2008.

Figure 1 shows the informativeness of each method, as measured by the score defined above. Looking at the graph, we can see that the informativeness of our approach increases with k, and for $k = 3$ it already passes the best of uni and ini. In addition, we can see that using clustering results in more informative heuristic estimates than random selection of representatives. The only exception is the case of $k = 2$ with forks, for which we can only speculate that clustering the state space sample into 2 parts leaves a lot of states in the "middle" of the sample with a large distance to the closest representative.

Computation Time

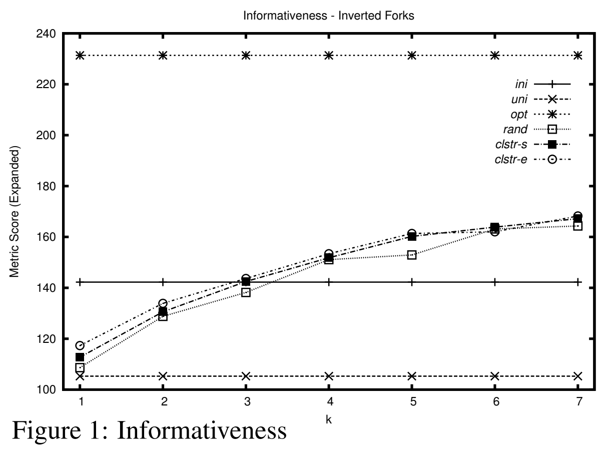
Figure 2 shows the average time per state for each method. This is the total run-time divided by the number of evaluated states, averaged over all tasks solved by all methods. The graph shows that the runtime of our approach increases with k, but is still much lower than the time per state of using optimal cost partitioning at each state (note that the y-axis is in logscale). In addition, the overhead of our approach (sampling the state space, clustering) is evident, since with $k = 1$, it still takes more time per state than using an optimal cost partitioning for the initial state.

Tradeoff

While figures 1 and 2 each look at one aspect of the tradeoff between heuristic accuracy and heuristic computation time, Figure 3 gives an overall view of this tradeoff. Each point in the plot gives heuristic computation time (as measured by the average time per evaluated state) as the x-coordinate, and heuristic accuracy (as measured by the informativeness score defined above) as the y-coordinate. Both accuracy and computation time are averaged over the tasks solved by all methods. Each of the baseline approaches is a single point in this plot. Our approach gives us a curve (for each method of choosing representatives), which illustrates that our approach gives us more control over this tradeoff. Note that a point in this plot dominates the area below and to the right of it. As the plots show, with fork-based heuristics, uni dominates ini. In contrast, our methods are non-dominated with $k \geq 3$ for both forks and inverted forks.

Number of Solved Tasks

Although looking at the tradeoff between heuristic accuracy and computation time is very interesting, the bottom line (at least as measured in the International Planning Competition) is the number of tasks solved. Table 2 gives the number of solved tasks for each configuration. As explained above, our methods, which compute an optimal cost partitioning, have an inherent disadvantage in comparison to *uni* with fork-based heuristics. Therefore, for fork-based heuristics we



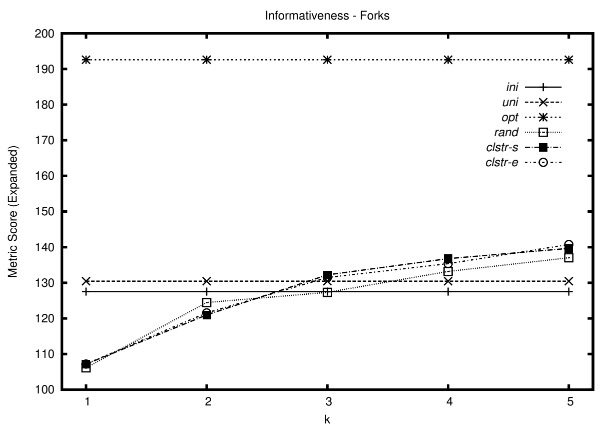
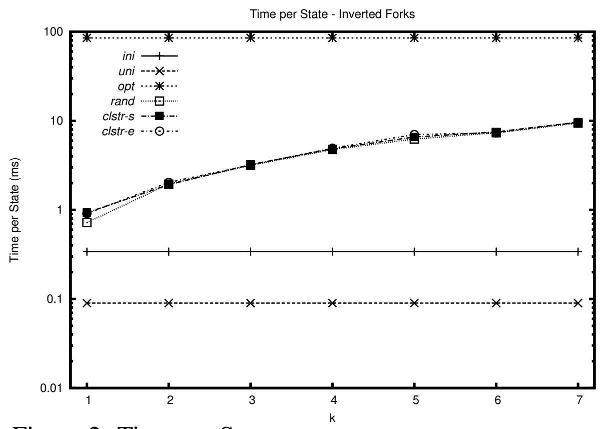


Figure 1: Informativeness
The value for each method is the total metric score of the number of expanded states (over tasks solved by all methods). The metric score of method i for a task is e^*/e_i, where e_i is the number of states expanded by method i, and $e^* = min_i e_i$. The total score is the sum of the metric scores across all tasks solved by all methods.



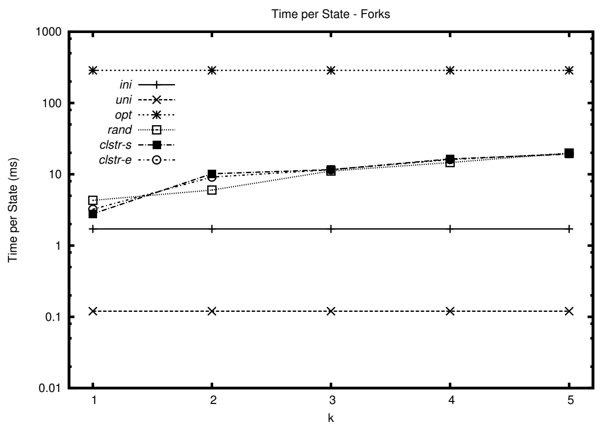


Figure 2: Time per State
The value for each heuristic is the average (over tasks solved by all configurations) time per state using that heuristic. Note that the y-axis is in log-scale.

give results over the non-hopeless tasks in addition to results over all tasks. We can justify this by arguing that if, when using our approach, the LP solver fails to find a solution, we can use *uni* as a fall-back method. The same fall-back technique was used by Katz and Domshlak (2010b) for *ini*, when computing the optimal cost partitioning for the initial state failed, with very good results. However, our purpose here is to study our cost-partitioning scheme in a clean setting, and so we did not evaluate this fall-back option empirically.

The results in Table 2 show that with inverted forks, using $k = 5$ and clustering by state features yields a slight improvement over the best baseline (*ini*). With forks, *uni* dominates every other method when considering all tasks. However, looking only at the non-hopeless tasks (those for which an optimal cost partitioning for the initial state was found within 5 minutes), using $k = 3$ and selecting representative states at random emerges as the winner. Although understanding why random selection outperforms clustering with forks is future work, we believe this means there is potential for improvement by using better features and/or clustering algorithms.

Looking at the number of tasks solved given an arbitrary (although commonly accepted) timeout of 30 minutes does not tell the complete tale. Figure 4 plots the number of tasks solved under different timeouts for the three baseline methods, as well as for the best of our methods. The plot shows that for both forks and inverted forks, both *uni* and *ini* reach a plateau fairly quickly. On the other hand, our methods seem to still be in an increasing region when the 30 minute timeout expires. This is a good indication that given more time, the gap between our method and the best basline methods will keep growing. Note that *opt* is also still increasing after 30 minutes, since the main bottleneck there is time rather than memory. However, the timeout until *opt* pays off is very large, and is infeasible in practice.

Table 2 and Figure 4 both look at results where a single value for k is used across all planning tasks, from different domains. However, the k parameter is used to control the tradeoff between heuristic accuracy and heuristic computation time, and there is no reason to assume that a single value of k is suitable for all domains. Table 3 gives the number of tasks solved for each domain, when using the best k for each method, for that domain. The table shows that with inverted forks, there are 6 domains where using our approach

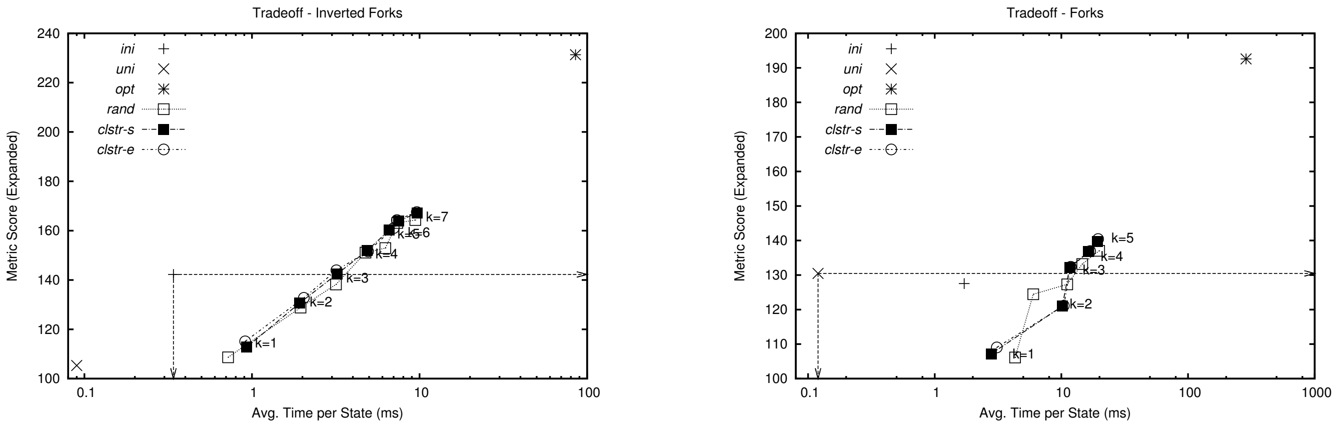


Figure 3: Tradeoff between Time and Accuracy
Each point in the graph corresponds to one method. The x- and y-axes capture the average computation-time per state and the total metric score for expanded states, respectively. Both are over all tasks solved by all methods. The region dominated by *ini* for inverted forks and by *uni* for forks is marked by dotted arrows. Note that the x-axis is in log-scale.

Inverted Forks

k	*ini*	*uni*	*opt*	*rand*	*clstr-s*	*clstr-e*
1				341	346	345
2				356	356	354
3				354	354	356
4	358	329	238	354	354	355
5				353	**359**	358
6				351	357	355
7				353	353	353

Forks
Non-Hopeless Tasks

k	*ini*	*uni*	*opt*	*rand*	*clstr-s*	*clstr-e*
1				310	311	313
2				316	312	313
3	315	311	196	**318**	314	314
4				316	313	312
5				313	312	313

All Tasks

k	*ini*	*uni*	*opt*	*rand*	*clstr-s*	*clstr-e*
1				312	316	317
2				318	315	315
3	315	**362**	196	319	316	315
4				317	315	313
5				314	313	314

Table 2: Total Number of Tasks Solved Using Each Method

results in more tasks solved, and with forks there are 3 domains where our approach helps, and only 1 where it hurts. Note that the best value of k for a domain can be obtained by using automated parameter tuning (Hutter et al. 2009; Ansótegui, Sellmann, and Tierney 2009), so there is no need to try all possible values for k across all tasks.

Discussion

We presented a method for performing fast, informative cost partitioning. Our method is parameterized by k, the number of optimal cost partitionings that are computed. This parameter gives us much more control over the tradeoff between heuristic accuracy and heuristic computation time than previous methods.

The representative states, for which an optimal cost partitioning is computed, are chosen by selecting k states out of a sample of the state space, using either a clustering algorithm, or simply at random. The state space sampling method, as well as the clustering algorithm and the features or distance metric it uses are all components of our method.

We evaluated our approach empirically, and showed that it outperforms any of the previous methods of using implicit abstraction heuristics when an optimal cost partitioning can be feasibly computed. The fact that random selection of representatives sometimes leads to better results than clustering implies that finding a principled way of choosing representatives is of definite interest, and that there is potential for improvement. In addition, coming up with a better state space sampling technique, which does not depend on a heuristic estimate of the goal depth, might also prove beneficial.

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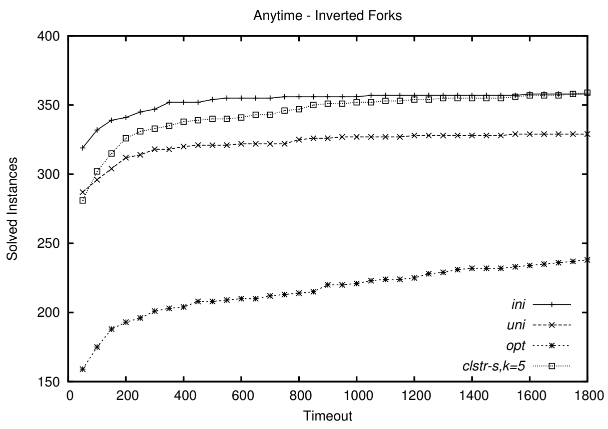

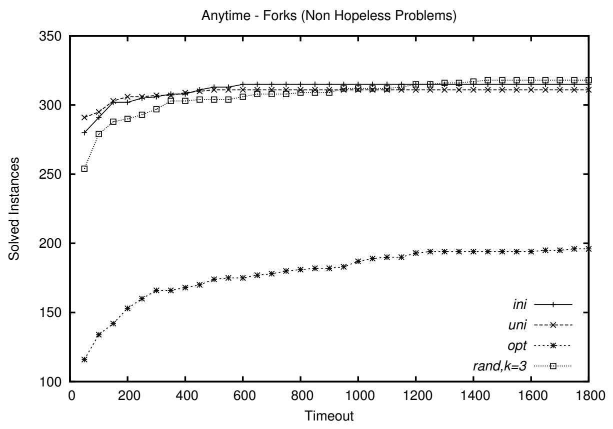

Figure 4: Anytime Results
Number of solved instances under different timeouts. The x-axis captures the timeout in seconds, and the y-axis captures the number of tasks solved under that timeout.

Inverted Forks

Domain	*ini*	*uni*	*opt*	*rand*	*clstr-s*	*clstr-e*
airport-ipc4	21	20	15	21	**22**	**22**
blocks-ipc2	21	18	17	**22**	**22**	**22**
depots-ipc3	7	4	2	7	7	7
driverlog-ipc3	14	12	8	14	14	14
freecell-ipc3	4	4	1	**5**	**5**	**5**
grid-ipc1	2	1	0	2	2	2
gripper-ipc1	7	7	3	7	7	7
logistics-ipc1	4	4	2	4	4	4
logistics-ipc2	20	16	16	19	20	20
miconic-strips-ipc2	53	50	30	53	53	53
mprime-ipc1	19	19	13	19	19	19
mystery-ipc1	13	13	10	13	13	13
openstacks-ipc5	7	7	5	7	7	7
pathways-ipc5	4	4	4	4	4	4
pw-notank-ipc4	17	15	6	**18**	**18**	**18**
pw-tank-ipc4	11	9	2	11	**12**	**12**
psr-small-ipc4	48	49	46	49	49	49
rovers-ipc5	7	7	4	7	7	7
satellite-ipc4	7	6	5	7	7	7
schedule-ipc2	49	40	34	53	**54**	53
tpp-ipc5	6	6	5	6	6	6
trucks-ipc5	7	7	2	7	7	7
zenotravel-ipc3	10	11	8	10	10	11
TOTAL	358	329	238	365	**369**	**369**

Forks (Non-Hopeless Tasks)

Domain	*ini*	*uni*	*opt*	*rand*	*clstr-s*	*clstr-e*
airport-ipc4	9	9	6	9	9	9
blocks-ipc2	21	21	11	21	21	21
depots-ipc3	4	4	1	4	4	4
driverlog-ipc3	11	11	6	11	11	11
freecell-ipc3	3	3	1	3	3	3
grid-ipc1	0	0	0	0	0	0
gripper-ipc1	7	7	3	7	7	7
logistics-ipc1	5	5	3	5	5	5
logistics-ipc2	23	22	18	**25**	**25**	**25**
miconic-strips-ipc2	53	51	35	**54**	**54**	**54**
mprime-ipc1	16	16	9	16	16	16
mystery-ipc1	10	10	7	10	10	10
openstacks-ipc5	7	7	5	7	7	7
pathways-ipc5	4	4	4	4	4	4
pw-notank-ipc4	8	7	1	8	8	8
pw-tank-ipc4	6	6	1	6	6	6
psr-small-ipc4	49	49	41	49	49	49
rovers-ipc5	7	6	4	7	7	7
satellite-ipc4	6	6	4	6	6	6
schedule-ipc2	41	44	21	**45**	43	43
tpp-ipc5	6	6	5	6	6	6
trucks-ipc5	6	6	2	6	6	6
zenotravel-ipc3	**13**	11	8	12	11	11
TOTAL	315	311	196	**321**	318	318

Table 3: Number of Tasks Solved by Using the Best k per Domain for Each Method
The best result in each domain is highlighted, unless there was a tie between the best baseline method and the best of our methods.

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