Using Homogeneous Weights for Approximating the Partial Cover Problem

Reuven Bar-Yehuda

Computer Science Department
Technion - IIT, Haifa 32000 Israel

e-mail: reuven@cs.technion.ac.il
http://www.cs.technion.ac.il/~reuven

The t-VC Problem (Special Case)

Given: $G = (V, E)$, $w : V \to R^+$, $0 \le t \le |E|$
Find:

$$\begin{array}{ll} \min & w(C) \\ s.t. & C \subseteq V \\ & |E(C)| \ge t \end{array}$$

r-Approximation

C is an r-approximation if
C is feasible and $w(C) \le r \cdot w^*$

An algorithm is an r-approximation if
for any G, w, t it returns an r-approximation.

t-VC

Bshouty and Burroughs 98 2-approximation.

Hochbaum 98 $O(|E|\,|V|\log(\frac{|V|^2}{|E|})\log|V|)$ time.

Here: $O(|V|^2)$ time.

t-VC with edge lengths

Hochbaum 98 3-approximation.

Here: 2-approximation ($O(|V|^2)$ time).

t-SC

Slavík 97 ($\ln \Delta_V$)-approximation for $t = p \cdot |E|$.

Burroughs 98 ($\ln \Delta_V$)-approximation.

t-SC with edge lengths

Here: Δ_E-approximation $O(|V|^2 + |H|)$ time.

Homogeneous Weight Function

Given: $G = (V, E)$, $\delta : V \to R^+$, $0 \leq t \leq |E|$

$$\text{s.t. } \forall_{v \in V} \delta(v) = \min(t, d(v))$$

Claim: C is minimal t-VC $\Longrightarrow \delta(C) \geq t$

Proof: If $\exists_{v \in C} \delta(v) = t \qquad \surd$

$$\text{else } \delta(C) = \sum_{v \in C} \delta(v) = \sum_{v \in C} d(v) \geq |E(C)|$$

$\square$

Claim: C is minimal t-VC $\Longrightarrow \delta(C) \leq 2t$

Proof: Let $v \in C$,

$$\left.\begin{array}{l} \delta(\{v\}) \leq t \\ \delta(C \setminus \{v\}) \leq t \end{array}\right\} \quad \Rightarrow \quad \delta(C \setminus \{v\} \cup \{v\}) \leq 2t$$

$\square$

Corollary:

C is minimal t-VC $\Longrightarrow$ C is 2-approximation.

Weight Decomposition

Local-Ratio Theorem:

$$\left.\begin{array}{l} C \text{ is an } r\text{-approximation w.r.t. } \delta \\ C \text{ is an } r\text{-approximation w.r.t. } w-\delta \end{array}\right\}$$

$\Longrightarrow$

C is an r-approximation w.r.t. w

Proof:

$$\left.\begin{array}{l} \delta(C) \leq r \cdot \delta^* \\ (w-\delta)(C) \leq r \cdot (w-\delta)^* \end{array}\right\}$$

$\Longrightarrow$

$$\begin{aligned} w(C) &\leq r \cdot (\delta^* + (w-\delta)^*) \\ &\leq r \cdot w^* \end{aligned}$$

$\square$

Algorithm Cover(V, E, w, t)

If $t \leq 0$ return ϕ

If $|E| < t$ return "no solution"

If $\exists_{v \in C} d(v) = 0$ return **Cover**$(V \setminus v,\ E,\ w, t)$

If $\exists_{v \in C} w(v) = 0$ return

$\{v\} \cup$ **Cover**$(V \setminus \{v\},\ E \setminus E(v),\ w, t - |E(v)|)$

$\epsilon = \min_{v \in V} \frac{w(v)}{\min(t, d(v))}$

for each $v \in V$

$\delta(v) = \epsilon \cdot \min(t, d(v))$

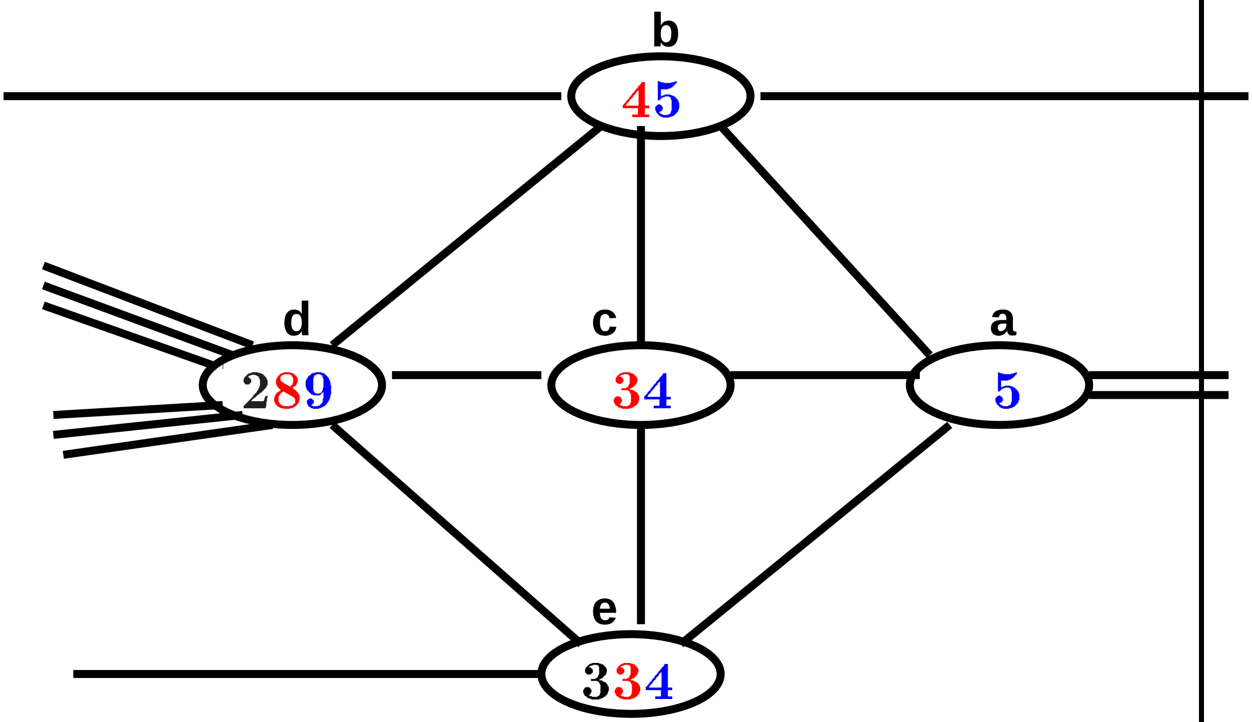
$C=$ **Cover**$(V, E, w - \delta, t)$

while $\exists_{v \in C} C \setminus \{v\}$ is a t-VC

$C = C \setminus \{v\}$

return C

Example



i	ε	$w(?) = 0$	t_i
0			13
1	1	$\{a\}$	$t_0 - 5 = 8$
2	10	$\{b, c\}$	$t_1 - 5 = 2$
3	100	$\{d\}$	$t_2 - 5 = -5$