FITTING CORRECTLY SHAPED SPLINES TO SHIP LINES GIVEN BY INACCURATE POINTS

E. Kantorowitz * Regina Drabkin and Alexander Krits
Computer Science Dept.
Technion-Israel Institute of Technology, 32000 Haifa Israel
Email: kantor@cs.technion.ac.il

ABSTRACT

Parametric spline curves are sometimes fitted to inaccurate points, e.g. points measured on a small drawing or a sketch. Such errors may corrupt the shape of the spline. A small error in the middle of a nearly flat curve may for example change it from an intended convex to a concave curve. A rule of thumb is introduced for detecting when this may occur. These undesired inflections may be avoided by employing an appropriate number *n* of control points. A method for determining the numbers of control points that produce correctly shaped curves is introduced.

INTRODUCTION

Ship lines can have quite complex shapes as they must meet both hydrostatic, hydrodynamic, aesthetic, and ease of manufacturing requirements. Several methods have been developed for fitting of splines to such complex lines, e.g. (*Söding* 1990) and (*Rabien* 1996). Fitting a spline involves the determination by trial and error of the number and location of the control points. This process can be quite slow, because moving a control point to improve on one shape characteristic may harm another characteristic. In order to shorten the process, this paper suggests starting the process by determining the numbers of control points that produce transverse sectional curves that have the required shape. Using these numbers of control points in the later part of the fairing process ensures to some extend that the transverse sections maintain the required shape. The designer may therefore concentrate on meeting the remaining shape requirements.

The shape of a planar curve is characterized by its convexity/concavity properties. We employ +,- and 0 to denote a convex, concave and straight line segment respectively. The shape shown in Figure 1 is characterized as +-+-0. Further shape characterizations such as fairness (Farin 1992) are not considered by the method proposed in this paper. It is suggested to employ our method to achieve a spline with the required convexity/concavity properties. The resulting splines are by their nature nice curves. Further improvements the fairness of the spline may therefore be achieved by small adjustments at a later stage

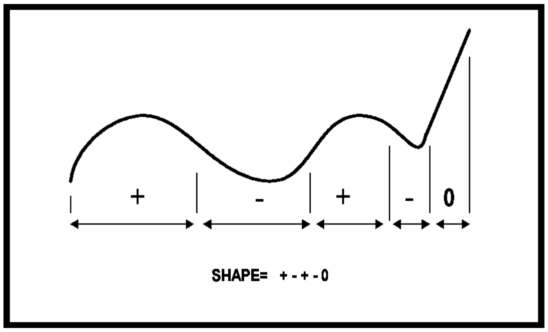

Figure 1. Example of the specification of the shape of a planar curve
.

A special kind of problems occurs when the splines are fitted to inaccurate input points measured on a small scaled drawing or sketch. These errors can cause undesired inflections in splines that have nearly flat parts. To get a feeling on how this works we consider the example in Figure 2:

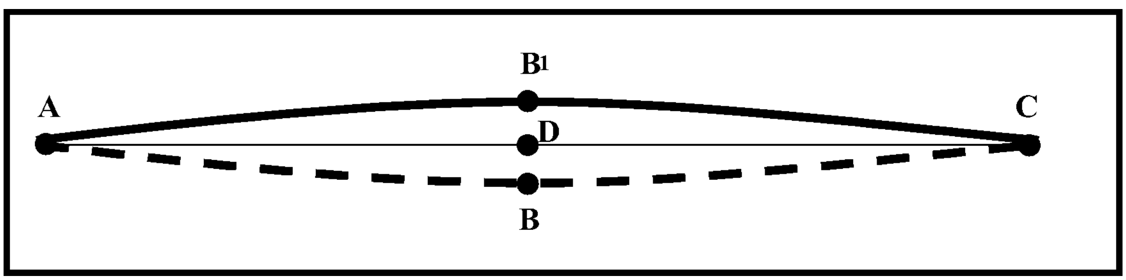

Figure 2. Example of shape corruption. The intended curve was the concave ABC arc. However, because of a measurement error of the size of BB1 that is greater than the distance BD to the chord connecting A and C we get the convex arc AB1C.

Based on the analysis illustrated in Figure 2 this paper suggests a rule of thumb for detecting when such shape corruption may occur:

Rule of thumb: A rough method for estimating when undesired inflections may occur, is to select the flattest part of the curve, and estimate the largest distance between the curve and the cord connecting the first and the last point of this flat part. If the errors in the input points are larger than this distance, the curve may flip to the wrong side of the chord and thus change the shape of the curve.

These unwanted shape corruption indicated by the rule of thumb may be avoided by designers employing B splines whose shape is controlled by moving of control points. These designer may exploit the theorem (Kantorowitz 1993) stating that the shape of uniform quadratic and cubic spline curves that do not intersect themselves is the same as the shape of their control polygon (the polygon connecting the control points). This is illustrated in Figure 3. The designer has in Figure 3 placed the control points such that a convex control polygon is obtained. The theorem guarantees that the corresponding spline curve is also convex. This method is especially useful in the flat parts, because it is quite easy to see if a polygon is convex or concave.

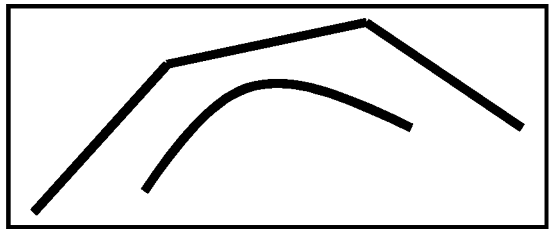

Figure 2. A convex control polygon and the corresponding convex spline

A number of experiments were made to evaluate methods for overcoming the problems indicated by this rule of thumb. All the experiments reported in this paper employed parametric cubic B splines using arc length parameterization. Splines with these characteristics are often employed in industry with satisfactory results. The splines were computed such that the best least squares fit to the erroneous input points is obtained using methods described in (Linnik 1961) and (Lau 1994).

In all the experiments described in the following sections we consider a computed spline to be satisfactory if it meets the criteria:

1. The spline has the required shape, i.e. has the required concavity/convexity properties.
2. The standard deviation between the input points and the corresponding spline points is less than δ=0.001 of the maximal ordinate of the curve. The standard deviation is computed as

$$s = \sqrt{\frac{\sum_{i=1}^{n} d_i}{n-1}} \qquad (1)$$

where n is the number of inaccurate input points and di is the difference between the y ordinate of the i^{th} input point and the ordinate of the corresponding spline point.

The rule of thumb tells us when there is a possibility of undesired inflections. The more important problem is how to avoid such unintended inflections. Generally the shape of a spline may be controlled by its degree, the number and locations of the control points and the method of parameterization. In this paper we investigate what the selection of a proper number of control points can do. The degree of the spline and the parameterization method are not varied. Using a large number of control points produces a highly flexible spline that enables undesired inflections. Using a small number of control points may produce a spline that is not sufficiently flexible to fit to the shape of the curve. It is therefore necessary to look for a useful number of control points that is sufficiently large to enable fitting to the shape of the curve and at the same time small enough to hinder undesired inflections. An experimental method for the detection of such useful numbers of control points is presented in this paper.

DETERMINATION OF THE USEFUL NUMBER OF CONTROL POINTS

The experiments were made with the curves of a German destroyer C4, whose shape was specified in (Kracht 1966) by offsets at the transverse sections:

x_i=-1,-0.95, -0.9,-0.8,-0.7,…0.8, 0.9 0.95, 1.0. (2)

In this study we fitted B-splines $y(z)$ to the given transverse sectional. A novel method is introduced to determine the useful number of control points.. We illustrate the method by applying it for section x=-0.6 shown in Figure 4. The determination of the useful numbers of control points is illustrated by Table 1

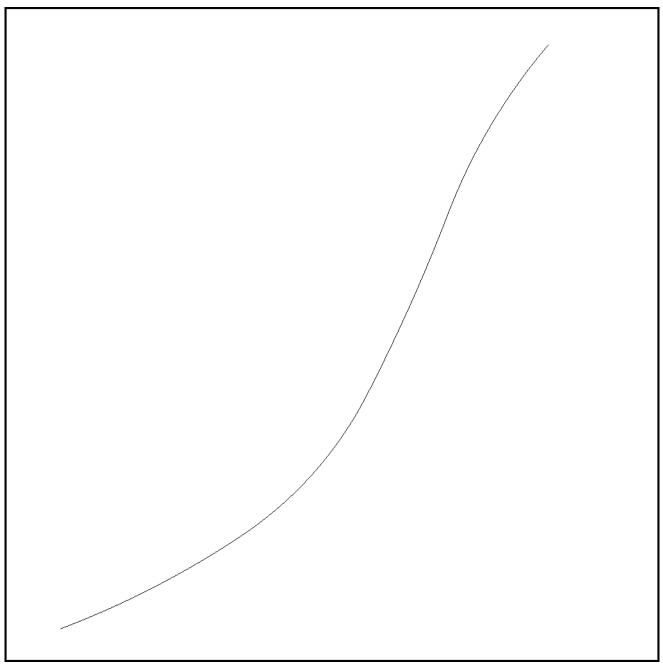

Figure 4. Section x=-0.6 has neither highly curved nor very flat parts. The shown plot is made with 6 control points

Number control points	Max deviation between spline and given points	Shape	Useful
4	0.002	+-	Yes
5	0.001	+-	Yes
6	0.000	+-	Yes
7	0.000	+-	Yes
8	0.000	+-	Yes
9	0.000	+-	Yes
10	0.000	+-	Yes
11	0.000	+-	Yes
12	0.001	+-+-	No
13	0.011	+-+-	No

Table 1. The useful number of control points for the section x=0.6 are 4 to 11. At 12 control points the shape is corrupted and at 13 control points the deviations from the given input data are unacceptably high.

The section x=-0.6 has many useful numbers of control points because it has neither very flat nor very curved parts. The analysis shown for section x=-6 is done for all transverse sections. Ideally we would like to use the same number of control points for all the transverse section. It was, however not possible to find a number of control points that was useful for all sections. We employed therefore 5 control points for the curves in the fore ship. Going higher caused unwanted inflections in the curves around in x=-0.3 which have quite flat parts (see Figure 6). Some of the sections in the aft body have a complex shape, and 8 control points were therefore employed. The high number of control points caused unwanted inflections which were suppressed by adding more input points. These points were obtained by interpolation in the offsets given in (Kracht 1966). The most severe problems arose in the transverse sections of the stern, where each section is composed of two very flat parts connected by a highly curved part, as illustrated by Figure 5.

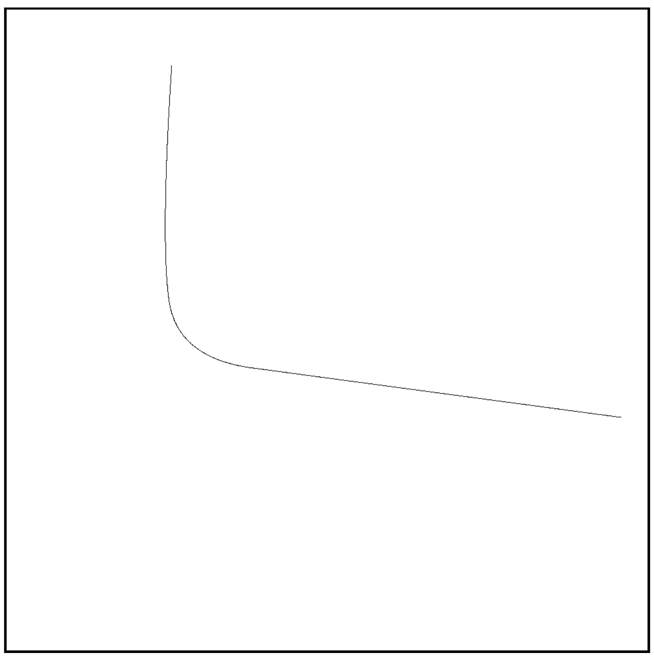

Figure 5: A transverse section at the stern is composed of two very flat parts connected by a highly curved part. The figure shows the case of section x=-0.9

For these stern sections 8 control points caused undesired inflections. Adding input points helped, but a considerable number of points is needed. Getting more input points involves work and seem not to be a good solution. We looked there for another method that employs only the given input points. There are two main problems in the stern sections. The first is that the employed cubic splines have continuous second derivatives. However, at the point where the very flat part is connected to the highly curved part, the second derivative is discontinuous (the second derivative of the near straight line part is close to zero, while the second derivative of the connected highly curved part is quite different from zero).

A solution to this problem is to employ a spline whose second derivative is discontinuous. This can be achieved by employing double knots in the cubic spline. The second reason for the difficulties is that we have employed uniform splines, i.e. splines where the knots of the spline are uniformly distributed over the length of the curve. This means that there are many control points in the quite long flat parts. These many control points means that many inflections are possible in the flat part. The possibility for inflection can be completely avoided by employing a single quadratic spline (3 control points only) for a plat part of the section. Using instead a single cubic spline (4 control points) enable a better fit to the shape of the section, but enable a single inflection point in the worse case. Each one of the sections at the stern was therefore computed by three different cubic splines fitted respectively to the lower flat part, the highly curved middle part and to the nearly flat upper part.. The three splines were connected at the two points of connection with a common first derivative only. Such three cubic splines were enough for a satisfactory solution in the stern region (see the curves x=–0.9 and x=-1.0 in Figure 6).

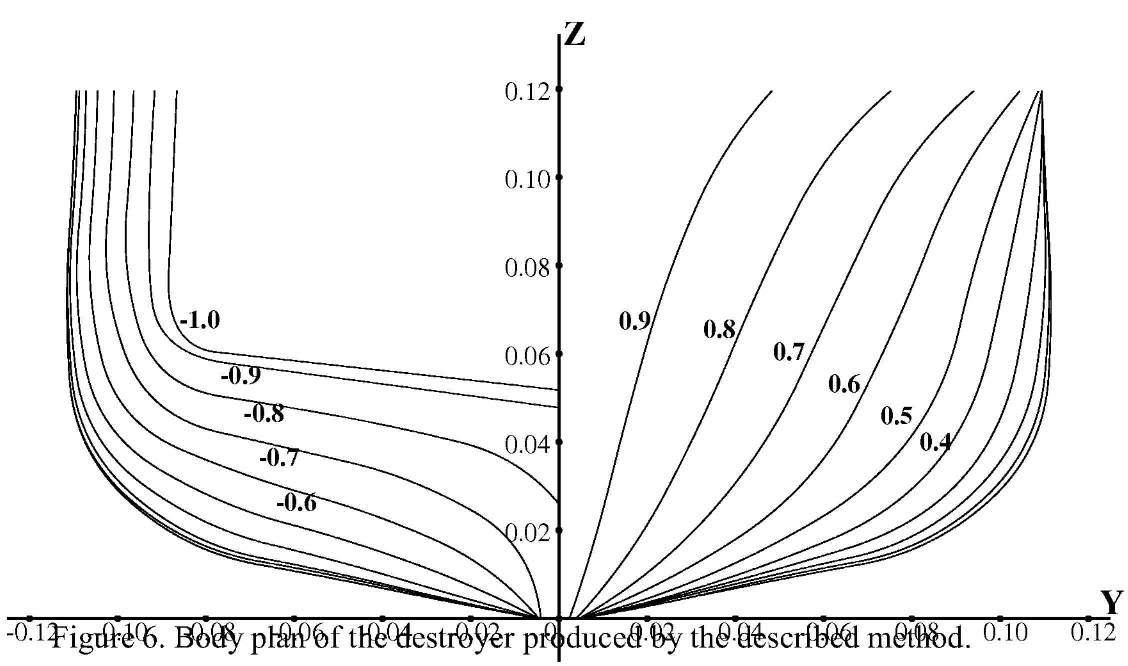

Figure 6. Body plan of the destroyer produced by the described method.

DISCUSSION AND CONCLUSIONS

The paper introduces a rule of thumb for detecting risks for unwanted inflections in the splines. It is, however, not necessary to employ the rule of thumb if we have another way of reducing the risks for unwanted inflections. Such a way is the program described in this paper, which determine for each section the numbers of control points that produce the required shape without unwanted inflections. The program was tested on the body of the a destroyer with a quite difficult shape composed of both flat and curved parts. The program detected useful numbers of control points in all the transverse sectional curves with the exception of the curves in the stern. This is an indication that it is difficult to fit uniform splines to the shapes in the sten. Each one of these curves is composed of three parts, i.e. two very flat parts connected to highly curved part, with a discontinuous second derivative at the two points of connection. Each one of these sections was therefore fitted by three connected cubic spline sections, such that a discontinuous second derivative is obtained at points where the flat and highly curve sections meet.

It is proposed that the fairing process be started with the program that determines the numbers of control points that can produce the required shape with a little risk for unwanted inflections. Then we select one of these useful numbers of control points for the following part of the fairing process. This ensures to some extent that the shape of the transverse section is correct during the remainder of the process. This enables the designer to concentrate on meeting other requirements to the shape. Starting with a useful number of control is therefore expected to reduce the total time it takes to achieve the final results.

REFERENCES

Söding H, (1990) *Computer Handling of Ship Hull Shapes and Other Surfaces*, Ship Technology Research, 37 ,pp 85-91.

[2] Rabien. U. (1996), *Ship Geometry Modeling*, Ship Technology Research, 43 , pp 115-123.

[3] Farin G. (1992), *Curves and Surfaces for Computer Aided Geometric Design*, A Practical Guide, Academic Press.

[4] Kantorowitz E., Schechner Y. (1993), *Managing the Shape of Planar Splines by their Control Polygons,* Computer Aided Design, 25:355-364.

[5] Kracht A., Grim O. (1966), *Widerstand Propulsion, Bewegung und Beanspruchung Schneller Verdrangungsfahrzeuge in Glatten Wasser und in Regelmassingem Seegang*, Institut fur Schiffbau der Universitat Hamburg, Technical Report IFS 167, 1966.

[6] Drabkin R. (1997), *Controlling the Shape of B splines Curves and Surfaces,*M. Sc. Thesis, Applied Mathematics Dept. Technion-Israel Institute of Technology, 1997.

[7] Linnik Yu V. (1961), *Method of Least Squares and Principles of the Teory of Observations,* Pergamon Press 1961.

[8] Lau H. T. (1994), A Numerical Library in C for Scientists and Engineers, Academic Press, 1994.