

Exercise 1 - Due 17.11.2005

Purpose

Become familiar with parameters of PCPs and their basic transformations (this will be used extensively later on).

Questions

Assume $L \subseteq \{0, 1\}^*$ is in the class

$\text{PCP}[\text{time} = t(n), \text{length} = l(n), \text{randomness} = r(n), \text{query} = q, \text{completeness} = 1, \text{soundness} = s > 0]$,

where q, s are constants (independent of n). Prove the following statements and replace the "...” with the (best possible) missing parameters.

1. If $t(n) = n^{O(1)}, l(n) = n^{O(1)}, r(n) = O(\log n), q = 2$, then L is decidable in polynomial time (i.e. $L \in \text{P}$).
2. There exists constant $s' > 0$ such that

$$L \in \text{PCP}[\dots, \text{query} = 3, \text{completeness} = 1, \text{soundness} = s'].$$

What is s' ?

3. There exists constant $s' > 0$ and finite alphabet $\Sigma \supseteq \{0, 1\}$ such that

$$L \in \text{PCP}[\dots, \text{query} = 2, \text{completeness} = 1, \text{soundness} = s'].$$

(Each query is answered by a symbol of Σ). What is $|\Sigma|$? What is s' ?

4. For all $s' < 1$, there exists constant q' such that

$$L \in \text{PCP}[\dots, \text{query} = q', \text{completeness} = 1, \text{soundness} = s'].$$

What is q' ?