GEOMETRIC SEGMENTATION OF 3D STRUCTURES

Ron Kimmel

Computer Science Department, Technion – I.I.T., Haifa 32000, Israel

ABSTRACT

Segmentation in volumetric images deals with separating 'objects' from their 'background' in a given 3D data. Usually, one starts with 'edge detectors' that give binary clues on the locations of the objects boundaries. Classical edge detectors that can be adopted from 2D are the Marr–Hildreth, and Haralick or Canny edge detectors. Next, usually one integrates these clues into meaningful contours or surfaces that indicate the boundaries of the objects.

We use our recent variational explanation for the Marr–Hildreth and the Haralick–Canny like edge detectors to extend these classical operators. We combine these operators with a minimal deviation measure that can be tuned to the problem at hand. Finally, an improved 'geometric active surface model' is defined.

1. INTRODUCTION

Variational interpretations of the Marr–Hildreth and the Haralick–Canny like edge detectors, recently presented in [14], allow us to better understand and even improve the process of edge detection and integration. There, it was proposed to use the geodesic active surface model [4] mainly for regularization, and the minimal variance criterion suggested by Chan and Vese [5] for segmentation in case of noisy data. Moreover, it was observed that using alignment of the surface normal with the volumetric image gradient, which is the measure minimized in most advanced edge detectors, is very useful in cases of significant intensity changes near boundaries. This approach allows us to use variational models to address the problem of thin structure segmentation.

It was recently shown that the minimal variance functional is related to segmentation by a threshold and to the Max–Lloyd vector quantization procedure [13]. Histogram based VQ (Vector Quantization) operations are used today as common practice in advanced medical analysis workstations. Adding the geodesic regularization and alignment for better accuracy improves the overall quality of the segmentation, and specifically help us accurately segment fine structures with sub-voxel accuracy.

Recent procedures that deal with thin structures include variational methods [15, 10, 21] introduce various combinations of geometric measures and corresponding level sets minimization techniques. Here we extend our results from [13, 14] to higher dimensions enhanced by our new understanding of the edge detection and VQ procedures.

The most simple edge detectors try to locate points defined by local maxima of the image gradient magnitude. The Marr and Hildreth edges are a bit more sophisticated, and were defined as the zero crossing surfaces of a Laplacian of Gaussian (LoG) applied to the image [18]. The Marr–Hildreth edge detection process can be thought of as a way to locate surfaces in the image space that pass through points where the gradient magnitude is high and whose normal direction aligns with the local edge direction estimated by the image gradient, see [14].

Section 2 introduces some mathematical notations. Section 3, formulates the idea of geometric surface evolution for segmentation, and reviews various types of variational measures (geometric functionals). These functionals describe an integral quantity defined by the surface. Our goal would be to search for a surface that minimizes these integral measures. Next, we compute the first variation of these functionals in Section 4, and comment on how to use it in a dynamic gradient descent surface evolution process.

2. MATHEMATICAL NOTATIONS

Define a 3D gray level image as $I : \Omega \to I\!R^+$ where $\Omega \in I\!R^3$ is the image domain. The image gradient vector field is given by $\nabla I(x, y, z) \equiv \{I_x, I_y, I_z\}$, were we used subscripts to denote the partial derivatives in this case, e.g., $I_x \equiv \partial I(x, y, z)/\partial x$. We search for a surface, $S : [0, 1] \times [0, 1] \to I\!R^3$, given in a parametric form $S(u, v) = \{x(u, v), y(u, v), z(u, v)\}$, whose normal is defined by $\vec{n}(u, v) = \frac{S_u \times S_v}{|S_u \times S_v|}$. This surface interacts with the given 3D image, for example, a surface whose normal aligns with the gradient vector field, where the alignment of the two vectors can be measured by their inner product that we denote by $\langle \vec{n}, \nabla I \rangle$. We also use subscripts to denote full derivatives, such that one surface tangent is given by $S_v = \{x_v, y_v, z_v\}$.

Define, H to be the mean curvature of the surface S, and the curvature vector $H\vec{n}$. We also define Ω_S to be the domain inside the surface S.

Here, we also define $da = S_u \times S_v dudv$ to be an area element, and Ω_S the region inside the surface $\partial\Omega_S = S$.

3. GEOMETRIC INTEGRAL MEASURES FOR ACTIVE SURFACES

The evolution of dynamic edge integration processes and active contours started with the classical snakes [12], followed by non-variational active contours [17, 2], and more recent geodesic active contours [3]. Here, we restrict our discussion to geometric functionals, that depend on geometry and the properties of the image. From these functionals we extract the first variation, and use it as a gradient descent process, that we refer to as *geometric active surface*.

Alignment Term: Consider the geometric functional

$$E_A(S) = \iint \langle \nabla I(x(u,v), y(u,v), z(u,v)), \vec{n}(u,v)\rangle da,$$

or in its 'robust' form

$$E_{AR}(S) = \iint |\langle \nabla I(x(u,v), y(u,v), z(u,v)), \vec{n}(u,v)\rangle| da,$$

where the absolute value of the inner product between the image gradient and the surface normal is our alignment measure. The motivation is the fact that in many cases, the gradient direction is a good estimator for the orientation of the edge surface. The inner product gets high values if the surface normal aligns with the image gradient direction. This measure also uses the gradient magnitude as an edge indicator. Therefore, our goal would be to find surfaces that maximize this geometric functional.

Weighted Region: In some cases we have a quantity we would like to maximize by integration inside the region Ω_S, defined by the surface S. In its most general setting, this weighted volume measure is

$$E_W(S) = \iint_{\Omega_S} f(x,y,z) dxdydz,$$

where $f(x,y,z)$ is any scalar function. A simple example is $f(x,y,z) = 1$, for which the functional $E(S)$ measures the volume inside the surface S, that is, the volume of the region Ω_S.

Minimal Variance: In [5], Chan and Vese proposed a minimal variance criterion, given by

$$\begin{aligned} E_{MV}(S, c_1, c_2) &= \frac{1}{2}\iiint_{\Omega_S} (I(x,y,z) - c_1)^2 dxdydz \\ &+ \frac{1}{2}\iiint_{\Omega \setminus \Omega_S} (I(x,y,z) - c_2)^2 dxdydz. \end{aligned}$$

In the optimal case, the two constants, c_1 and c_2, get the mean intensities in the interior (inside) and the exterior (outside) the surface S, respectively. The optimal surface would best separate the interior and exterior with respect to their relative average values. In the optimization process we look for the best separating surface, as well as for the optimal expected values c_1 and c_2. Such optimization problems are often encountered in color quantization and classification problems.

In order to control the smoothness of their active contour, Chan and Vese also included the area $\iint da$ as a regularization term. Here we propose to use the more general weighted area, $\iint g(S(u,v))da$, also known as the geodesic active surface functional [3], as a data sensitive regularization term. It can be shown to yield better results in most cases and simplifies to the regularization used by Chan and Vese for the selection of $g = 1$.

One could consider other region based measures like the robust measure

$$\begin{aligned} E_{RMV}(S) &= \iiint_{\Omega_S} |I(x,y,z) - c_1| dxdydz \\ &+ \iiint_{\Omega \setminus \Omega_S} |I(x,y,z) - c_2| dxdydz. \end{aligned}$$

Geodesic Active Surface: The geodesic active surface [4] model is defined by the functional

$$E_{GAS}(S) = \iint_{\partial\Omega_S} g(S(u,v)) da.$$

It is an integration of an inverse edge indicator function, like $g(x,y,z) = 1/(1 + |\nabla I|^2)$, along the surface. The search, in this case, would be for a surface along which the inverse edge indicator gets the smallest possible values. That is, we would like to find the surface S that minimizes this functional. The geodesic active contour was shown in [14] to serve as a good regularization for other dominant terms like the minimal variance in noisy images, or the alignment term in cases we have good orientation estimation of the edge. A well studied example is $g(x,y,z) = 1$, for which the functional measures the total area of the surface.

4. CALCULUS OF VARIATIONS FOR GEOMETRIC MEASURES

Given a surface integral of the general form,

$$E(S) = \iint_S L(S_u, S_v, S) dudv,$$

we compute the first variation $\frac{\delta E(S)}{\delta S}$. The extremals of the functional $E(S)$ can be identified by the Euler–Lagrange equation $\delta E(S)/\delta S = 0$. A dynamic process known as gradient descent, that takes an arbitrary surface towards a maximum of $E(S)$, is given by the surface evolution equation $\frac{\partial S}{\partial t} = \frac{\delta E(S)}{\delta S}$, where we added a virtual 'time' parameter t to our surface to allow its evolution into a family of surfaces $S(u, v, t)$. Our hope is that this evolution process would take an almost arbitrary initial surface into a desired configuration, which gives a significant extremum of our functional. Here, we restrict ourselves to closed surfaces.

Lemma 1 *Given the vector field $\vec{V}(x, y, z)$, we have the alignment measure, $E_A(S) = \oint_S \langle \vec{V}, \vec{n} \rangle da$, for which the first variation is given by $\frac{\delta E(S)}{\delta S} = \mathrm{div}(\vec{V})\vec{n}$.*

An important example is $\vec{V} = \nabla I$, for which we have as first variation $\frac{\delta E_A(S)}{\delta S} = \Delta I \vec{n}$, where $\Delta I = I_{xx} + I_{yy} + I_{zz}$ is the image Laplacian. The Euler–Lagrange equation $\delta E_A / \delta S = 0$ gives a variational explanation for a generalization of the 2D Marr–Hildreth edge detector that is defined by the zero crossings of the Laplacian, as reported in [14].

Lemma 2 *The robust alignment term given by $E_{AR}(S) = \oint_S |\langle \vec{V}, \vec{n} \rangle| da$, yields the first variation $\mathrm{sign}(\langle \vec{V}, \vec{n}(s) \rangle)\mathrm{div}(\vec{V})\vec{n}$.*

An important example is $\vec{V} = \nabla I$, for which we have $\frac{\delta E_{AR}(S)}{\delta S} = \mathrm{sign}(\langle \nabla I, \vec{n}(s) \rangle)\Delta I \vec{n}$.

Lemma 3 *The weighted region functional $E_W(S) = \iiint_{\Omega_S} f(x, y, z) dxdydz$, yields the first variation $\frac{\delta E_W(S)}{\delta S} = -f(x, y, z)\vec{n}$.*

In 2D, this term is sometimes called the weighted area [23] term, and for f constant, its resulting variation is known as the 'balloon' [8] force. If we set $f = 1$, the gradient descent surface evolution process is the constant flow. It generates offset surfaces via $S_t = \vec{n}$, and its efficient implementation is closely related to Euclidean distance maps [9, 6] and fast marching methods [20].

Lemma 4 *The geodesic active surface model is $E_{GAS}(S) = \oint_S g(S(s)) da$, for which the first variation is given by $\frac{\delta E_{GAS}(S)}{\delta S} = -(Hg - \langle \nabla g, \vec{n} \rangle)\vec{n}$.*

We will use this term mainly for regularization. If we set $g = 1$, the gradient descent surface evolution result given by $S_t = -\delta E_{GAS}(S)/\delta S$ is the well known curvature flow, $S_t = H\vec{n}$.

Lemma 5 *The Chan–Vese minimal variance criterion [5] is given by*

$$E_{MV}(S, c_1, c_2) = \frac{1}{2}\iiint_{\Omega_S} (I - c_1)^2 dxdydz + \frac{1}{2}\iiint_{\Omega \setminus \Omega_S} (I - c_2)^2 dxdydz,$$

for which the first variation is

$$\frac{\delta E_{MV}}{\delta S} = (c_2 - c_1)\left(I - \frac{c_1 + c_2}{2}\right)\vec{n}$$

$$\frac{\delta E_{MV}}{\delta c_1} = \iiint_{\Omega_S} I dxdydz - c_1 \iiint_{\Omega_S} dxdydz$$

$$\frac{\delta E_{MV}}{\delta c_2} = \iiint_{\Omega \setminus \Omega_S} I dxdydz - c_2 \iiint_{\Omega \setminus \Omega_S} dxdydz.$$

One could recognize the variational interpretation of segmentation by the threshold $(c_1 + c_2)/2$ given by the Euler–Lagrange equation $\delta E_{MV}/\delta S = 0$.

Finally, we treat the robust minimal deviation measure E_{RMV}.

Lemma 6 *The robust minimal total deviation criterion is given by*

$$E_{RMV}(S, c_1, c_2) = \iiint_{\Omega_S} |I - c_1| dxdydz + \iiint_{\Omega \setminus \Omega_S} |I - c_2| dxdydz,$$

for which the first variation is

$$\frac{\delta E_{RMV}}{\delta S} = (|I - c_1| - |I - c_2|)\vec{n}$$

$$\frac{\delta E_{RMV}}{\delta c_1} = \iiint_{\Omega_S} \mathrm{sign}(I - c_1) dxdydz$$

$$\frac{\delta E_{RMV}}{\delta c_2} = \iiint_{\Omega \setminus \Omega_S} \mathrm{sign}(I - c_2) dxdydz$$

The computation of c_1 and c_2 can be efficiently implemented via the intensity histograms in the interior or the exterior of the surface. One approximation is the median of the pixels inside or outside the contour.

The robust minimal deviation term should be preferred when the penalty for isolated pixels with wrong affiliation is insignificant. The minimal variance measure penalizes large deviations in a quadratic fashion and would tend to over-segment such events or require large regularization that could over-smooth the desired boundaries.

We embed a closed surface in a higher dimensional $\phi(x, y, z)$ function, which implicitly represents the surface S as a zero set, i.e., $S = \{\{x, y, z\} : \phi(x, y, z) = 0\}$. This way, the well known Osher–Sethian [19] level-set method can be employed to implement the surface propagation toward its optimal location. Efficient solutions for the resulting evolution equations can use either AOS [16, 22] or ADI methods, coupled with a narrow band approach [7, 1], as first introduced in [11] for the geodesic active contour.

The following table summarizes the functionals, the resulting first variation for each functional, and the level set formulations for these terms.

5. CONCLUSIONS

We presented a variational geometric framework for volumetric image analysis. It is based on variational explanations of basic edge detection and threshold operators that

Measure	$E(S)$	$\delta E/\delta S$	level set form
Weighted Area	$\iiint_{\Omega_S} f(x,y,z)dxdydz$	$-f(x,y,z)\vec{n}$	$-f(x,y,z)\|\nabla\phi\|$
Minimal Variance	$\iiint_{\Omega_S}(I-c_1)^2 dxdydz+$ $\iiint_{\Omega\setminus\Omega_S}(I-c_2)^2 dxdydz$	$(c_2-c_1)\times$ $(I-(c_1+c_2)/2)\vec{n}$	$(c_2-c_1)\times$ $(I-(c_1+c_2)/2)\|\nabla\phi\|$
GAS	$\iint_S g(S(s))da$	$(\langle\nabla g,\vec{n}\rangle - Hg)\vec{n}$	$-\text{div}\left(g\frac{\nabla\phi}{\|\nabla\phi\|}\right)\|\nabla\phi\|$
Alignment	$\iint \|\langle\nabla I,\vec{n}\rangle\| da$	$\text{sign}(\langle\nabla I,\vec{n}\rangle)\Delta I\vec{n}$	$\text{sign}(\langle\nabla I,\nabla\phi\rangle)\Delta I\|\nabla\phi\|$

allow us to combine these measures into a global geometric functional from which we can derive an efficient and accurate segmentation tool. At the conference we will present several examples and a medical image analysis system for 3D volumetric imaging based on the above derivations.

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