Convolutional Neural Networks
Analyzed via Convolutional Sparse Coding*

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Convolutional Neural Networks (CNN)

Sparse representations

\[ \mathbf{X} = \mathbf{D} \Gamma \]
Convolutional Sparse Coding (CSC)

Convolutional Neural Networks (CNN)

Sparse representations

\[ X = D \Gamma \]
Sparse representations

Convolutional Sparse Coding (CSC)

Convolutional Neural Networks (CNN)
The representations are locally sparse

\[
\begin{align*}
X &= D_1 \Gamma_1 \\
\Gamma_1 &= D_2 \Gamma_2 \\
\vdots \\
\Gamma_K &= D_K \Gamma_K
\end{align*}
\]
Sparse representations

Convolutional Sparse Coding (CSC)

Multi-Layer Convolutional Sparse Coding (ML-CSC)

Forward pass

Convolutional Neural Networks (CNN)
The forward pass is a sparse-coding algorithm, serving the ML-CSC.
Forward pass

Multi-Layer Convolutional Sparse Coding (ML-CSC)

Sparse representations

Convolutional Sparse Coding (CSC)

Convolutional Neural Networks (CNN)

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Extension of the classical sparse theory to a multi-layer setting

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Sparse representations

Forward pass
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Sparse representations

Forward pass
The feature maps CNN aims to recover are unique

The problem CNN aims to solve is stable

The forward pass is stable

The above are guaranteed assuming that the sparse representations are locally sparse and the noise is locally bounded

We also propose a better pursuit that is shown to be theoretically superior to the conventional forward pass