Motivation and Goals

- Leading image denoising methods are built upon powerful patch-based (local) image models.
- Non-Local Means (NLM): self-similarity within natural images.
- K-SVD: sparse representation modeling of image patches.
- BM3D: combines a sparsity prior and non-local self-similarity.
- Kernel-regularization: offers a local directional filter.

The Proposed Algorithm

- Given any denoiser, how can we improve its performance?
  
  **I. Strengthen the signal**
  
  **II. Operate the denoiser**
  
  **III. Subtract** the previous estimation from the outcome

**SOS formulation:** $\hat{x}^{k+1} = f(x^k + y) - \hat{x}^k$

- An improvement is obtained since $\text{SNR}(y + x) > \text{SNR}(y)$
- We suggest strengthening the underlying signal, rather than adding the residual back to the noisy image.
- Twisting converges to the noisy image.
- Filtering the previous estimate over and over again.
- Diffusion could lead to over-smoothing, converging to a piece-wise constant image.

Image Denoising - A Matrix Formulation

- In order to study the convergence of the SOS function, we represent the denoiser in its matrix form:
  
  $x^k = f(y) = y$

- The properties of $W$:
  - Kernel-based methods (e.g. Bilateral filter, NLM, Kernel Regression) can be represented as row-stochastic positive definite matrices, which are known to have eigenvalues in the range $[0,1]$.
  - NL-means and other local methods.

- We have showed that it also has eigenvalues in the range $[0,1]$ and (other interesting properties)

Convergence Study

- The SOS recursive function converges if $\|W - W^\dagger\|_2 < 1$
- Holds both for kernel-based (Bilateral filter, NLM, Kernel Regression), and sparsity-based methods (K-SVD).

For most denoising algorithms the SOS boosting is "guaranteed" to converge to $x^\dagger = (I - (I - W)^{-1})Wy$

Graph-Laplacian

- The normalized graph Laplacian can be defined as $L = I - W$.
- Encapsulates the structure of underlying signal.
- Most of the image content is represented by the eigenvectors that correspond to the large eigenvalues.
- Most of the noise is represented by eigenvectors that correspond to the large eigenvalues.
- What can we do with $L$?

- Regularize the inverse problem by encouraging similar pixels to remain similar in the final estimate.
- This can be done by minimizing $\text{Bougleux et al. [69]}$
  
  $\min \|x - y\|^2 + \lambda \|Wx\|^2$

Verifying the Convergence

- We apply the K-SVD on the noisy House image ($\sigma = 25$), with a fixed $W$

- Original: $32.34$dB
- SOS: $32.74$dB

Graph-Based Analysis

- The filter-matrix $W$ is an adaptive filter, where the $i^{th}$ denoised pixel is obtained by $\hat{x}_i = \sum_j W_{ij} y_j$
- $W_{ij}$ measures the similarity between the $i^{th}$ and $j^{th}$ pixels.
- A large value implies large similarity (and vice versa).
- For kernel-based methods (e.g. NLM, Bilateral, LARK): $W_{ij}$ is a function of the Euclidean distance between pixels
- For sparsity-based methods (K-SVD): $W_{ij}$ is a function of the dictionary atoms that were chosen to represent the patch (measures the affinity between pixels, through a dictionary).

Table: Average PSNR Improvement

<table>
<thead>
<tr>
<th>$\sigma$</th>
<th>K-SVD</th>
<th>NLM</th>
<th>BM3D</th>
<th>EPLL</th>
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<td>0.44</td>
<td>0.01</td>
<td>0.13</td>
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<td>0.25</td>
<td>0.04</td>
<td>0.25</td>
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<td>0.26</td>
<td>0.41</td>
<td>0.03</td>
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<tr>
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<tr>
<td>1.00</td>
<td>0.81</td>
<td>0.36</td>
<td>0.14</td>
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</tr>
</tbody>
</table>

Results

- We successfully boost several state-of-the-art denoising algorithms: K-SVD, NLM, BM3D, and EPLL.
- Without any modifications, simply by applying the original software as a "black-box".

Relation to Game Theory

- Inspired by the "Consensus and Sharing" problem:
  
  - There are several agents, each of them aims to minimize its individual cost (i.e., representing the noisy patch sparsely).
  
  - These agents affect a shared objective term, describing the noisy signal (i.e., representing the globally denoised image).
  
  - We push the overlapping patches towards an agreement.

- Imitating the "consensus and sharing" concept, we suggest "sharing the disagreement".
- We define the local disagreements by

- The disagreements are naturally exist since each noisy patch is denoised independently.

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References

- Bougleux S., Chambon P., "A novel measure of similarity between denoising algorithms", in Proc. 9th Conf. Computer Vision and Imaging, 2013.