

The Blackboard Bold Font

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Blackboard bold also called *double struck* is a typeface in which certain lines of the symbol (usually vertical, or near-vertical lines) are doubled. It is primarily used for upper case letters, which usually describe sets of numbers.

The name originated from the attempt to distinguish bold letters on blackboards by double striking them.

In this short document I try to demonstrate the common use of the blackboard bold font in mathematics and what you have to do in $\text{\LaTeX} 2_{\epsilon}$ in order to generate these symbols.

1 Accessing the blackboard font

The blackboard font is delivered with \AMSTeX -LaTeX distribution, which is part of the standard $\text{\LaTeX} 2_{\epsilon}$ distribution.

The following is one the shortest possible document that uses the blackboard bold font.

```
\documentclass{article}
\usepackage{amssymb}

\begin{document}
  $\mathbb{A},\mathbb{B}, \mathbb{C}, \ldots$
\end{document}
```

The output is then

$\mathbb{A}, \mathbb{B}, \mathbb{C}, \dots$

It would have been sufficient to write `\usepackage{amsfonts}`. However, the `amssymb` package includes the `amsfonts` package.

2 The Symbols

Only upper case letters exist in the blackboard bold font.

A B C D E F G
H I J K L M N
O P Q R S T U
V W X Y Z

3 Using the font

Excessive change of typefaces, unrestrained creativity in inventing new notation, will deter the reader. It is even worse to overload existing and familiar notation with idiosyncratic meaning. Here is a list of the common uses of the blackboard bold font in mathematics. If possible, try not to deviate from this list.

1. Number theory often uses the set \mathbb{N} of natural numbers (which may or may not include 0), and the *ring* \mathbb{Z} of integers. It is also common to use \mathbb{P} to denote the set of primes. We can write

$$\mathbb{P} \subset \mathbb{N} \subset \mathbb{Z}.$$

2. *Fields* in common use include \mathbb{Q} (the rational numbers), \mathbb{R} (the real numbers), and \mathbb{C} (the complex plane), which obey the relation

$$\mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}.$$

All the above are infinite fields, also called fields of 0 characteristic.

3. An unspecified field is usually denoted by \mathbb{K} . The notation \mathbb{F} is usually reserved to finite fields. A finite field of order n is usually denoted as \mathbb{F}_n . However, since for all $p \in \mathbb{P}$

$$\mathbb{Z} \bmod p = \mathbb{F}_p$$

people often use the notation \mathbb{Z}_p for the finite field of prime order p . A finite field is called a Galois field and is completely defined by its order, which must be a prime power. Therefore people often write $\text{GF}(p^k)$ (where $p \in \mathbb{P}$ is the field characteristic and $k \in \mathbb{N}$, $k \geq 1$) instead of \mathbb{F}_{p^k} .

4. The field obtained by an algebraic closure of the rational numbers is written as either $\overline{\mathbb{Q}}$ or \mathbb{A} . We have

$$\overline{\mathbb{Q}} = \mathbb{A} \subset \mathbb{C}.$$

5. There are several extensions of the complex field, none of which is a field. The quaternions \mathbb{H} are a four dimensional set in which multiplication is non-commutative. The octonions \mathbb{O} are a nonassociative extension of

the quaternions. The sedenions \mathbb{S} are a 16-dimensional extension of the octonions. Thus,

$$\mathbb{C} \subset \mathbb{H} \subset \mathbb{O} \subset \mathbb{S}.$$

6. The unit disk in the complex plane is denoted by \mathbb{D} .
7. The symbol \mathbb{B} is used for denoting a ball, \mathbb{S} is the sphere, while \mathbb{T} often denotes a torus. A superscript is used to denote the dimension. Thus, \mathbb{B}^n is an n -dimensional ball.

I like to define meaningful macro names for all mathematical symbols I use. Table 1 lists the macros provided by the `bbbold` package for this purpose.

<code>\FiniteField</code>	\mathbb{F}	<code>\Algebraics</code>	\mathbb{A}
<code>\Field</code>	\mathbb{K}	<code>\ComplexPlane</code>	\mathbb{C}
<code>\Booleans</code>	\mathbb{B}	<code>\Quaternions</code>	\mathbb{H}
<code>\Primes</code>	\mathbb{P}	<code>\Octonions</code>	\mathbb{O}
<code>\Naturals</code>	\mathbb{N}	<code>\Sedenions</code>	\mathbb{S}
<code>\Integers</code>	\mathbb{Z}	<code>\UnitDisk</code>	\mathbb{D}
<code>\Rationals</code>	\mathbb{Q}	<code>\Ball</code>	\mathbb{B}
<code>\Reals</code>	\mathbb{R}	<code>\Sphere</code>	\mathbb{S}
<code>\Algebraics</code>	\mathbb{A}	<code>\Torus</code>	\mathbb{T}

Table 1: Macros provided by the `bbbold` package.

4 Alternatives

There is an ever increasing need for mathematical notation. If you need a variation on capital letters, you may want to consider the following alternatives.

Calligraphic letters \LaTeX offers a calligraphic style for upper case letters. This is obtained by the `\mathcal` command, as in `\mathcal{ABC}` which produces \mathcal{ABC} .

Boldface letters To obtain characters mathematics, use the `\mathbf` command, which works for both upper and lower case letters. e.g., the input `\mathbf{a B}` will produce \mathbf{aB} .

Boldface italics letters Command `\boldsymbol`, made available by the `amssymb` package, makes it possible to use the boldface version of a symbol, while keeping its slanted appearance. Thus, typing

$$\code{\boldsymbol{A} \neq A \neq \mathbf{A}}$$

you will see that

$$\mathbf{A} \neq A \neq \mathbf{A}.$$

Bold Greek If you, like others, do not think that the difference between the above three version of “A” is sufficient to let them denote different entities, you can still use `\boldsymbol` to generate a visually distinctive version of Greek letters. Consider the difference between $\boldsymbol{\Gamma}$ and $\mathbf{\Gamma}$, between $\boldsymbol{\Delta}$ and $\mathbf{\Delta}$, etc. It is even possible to generate bold face version of lower case Greek letters, such as $\boldsymbol{\alpha} \neq \alpha$ and $\boldsymbol{\beta} \neq \beta$.

Sans Serif letters I like using these for proper words. Thus,

```
$t_\mathsf{QuickSort}$
```

will produce

t_{QuickSort}.

Fraktur letters One of the nicest and relatively unknown options you have is to invoke the `\mathfrak` command to generate Fraktur typeface, available for both upper case letters

$\mathfrak{ABCDEFGHIJKLMNOPQRSTUVWXYZ}$,

(produced by `\mathfrak{ABCDEFGHIJKLMNOPQRSTUVWXYZ}`) and lower case letters

$\mathfrak{abcdefghijklmnopqrstuvwxyz}$

(produced by `\mathfrak{abcdefghijklmnopqrstuvwxyz}`).