Lecture 16 – Dataflow Analysis

THEORY OF COMPILATION

Eran Yahav

www.cs.technion.ac.il/~yahave/tocs2011/compilers-lec16.pptx

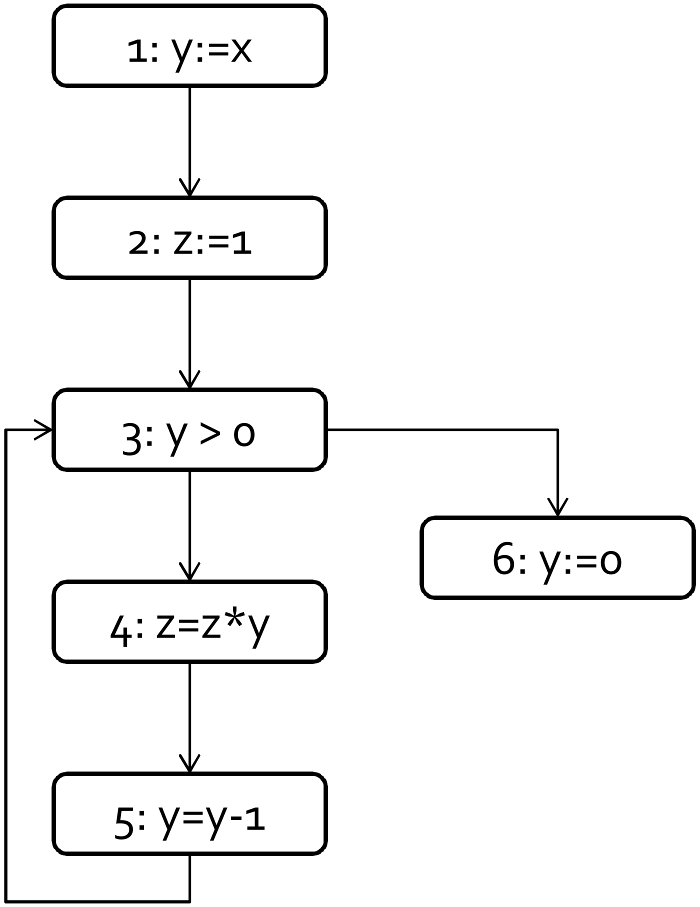
Reference: Dragon 9, 12

Last time… Dataflow Analysis

- Information flows along (potential) execution paths
- Conservative approximation of all possible program executions
- Can be viewed as a sequence of transformations on program state
    - Every statement (block) is associated with two abstract states: input state, output state
    - Input/output state represents all possible states that can occur at the program point
    - Representation is finite
    - Different problems typically use different representations

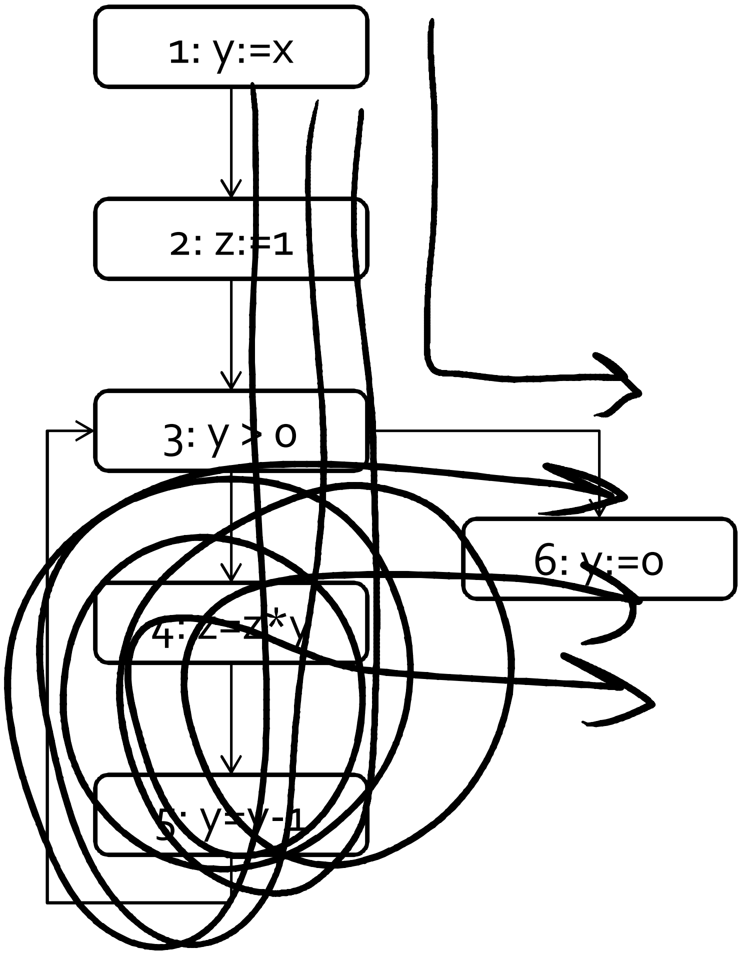
Control-Flow Graph

```
1: y := x;
2: z := 1;
3: while y > 0 {
4:    z := z * y;
5:    y := y – 1
 }
6: y := 0
```



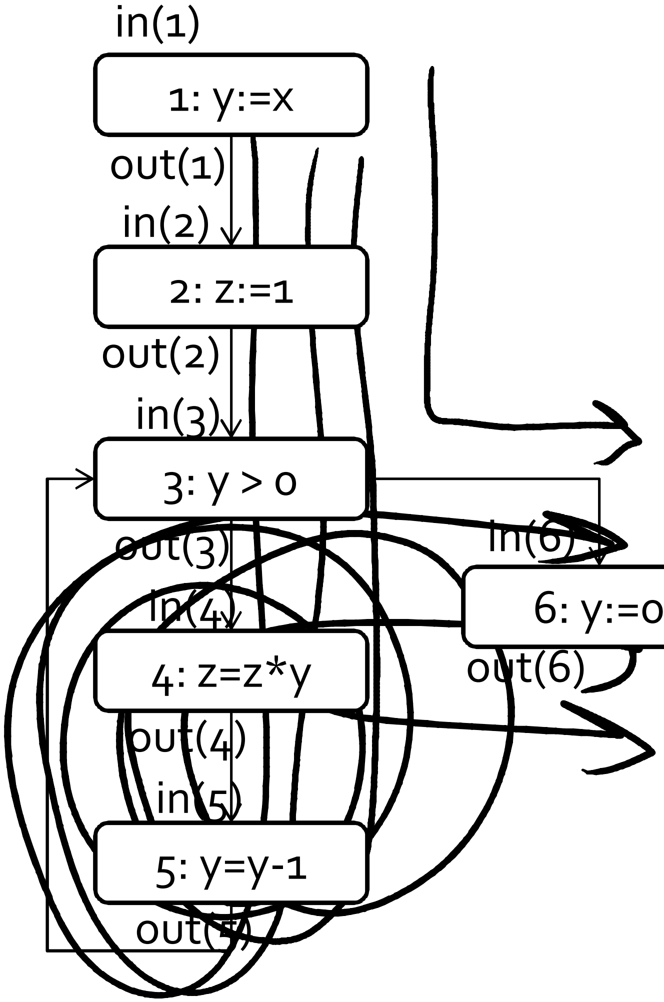
Executions

```
1: y := x;
2: z := 1;
3: while y > 0 {
4:    z := z * y;
5:    y := y – 1
}
6: y := 0
```

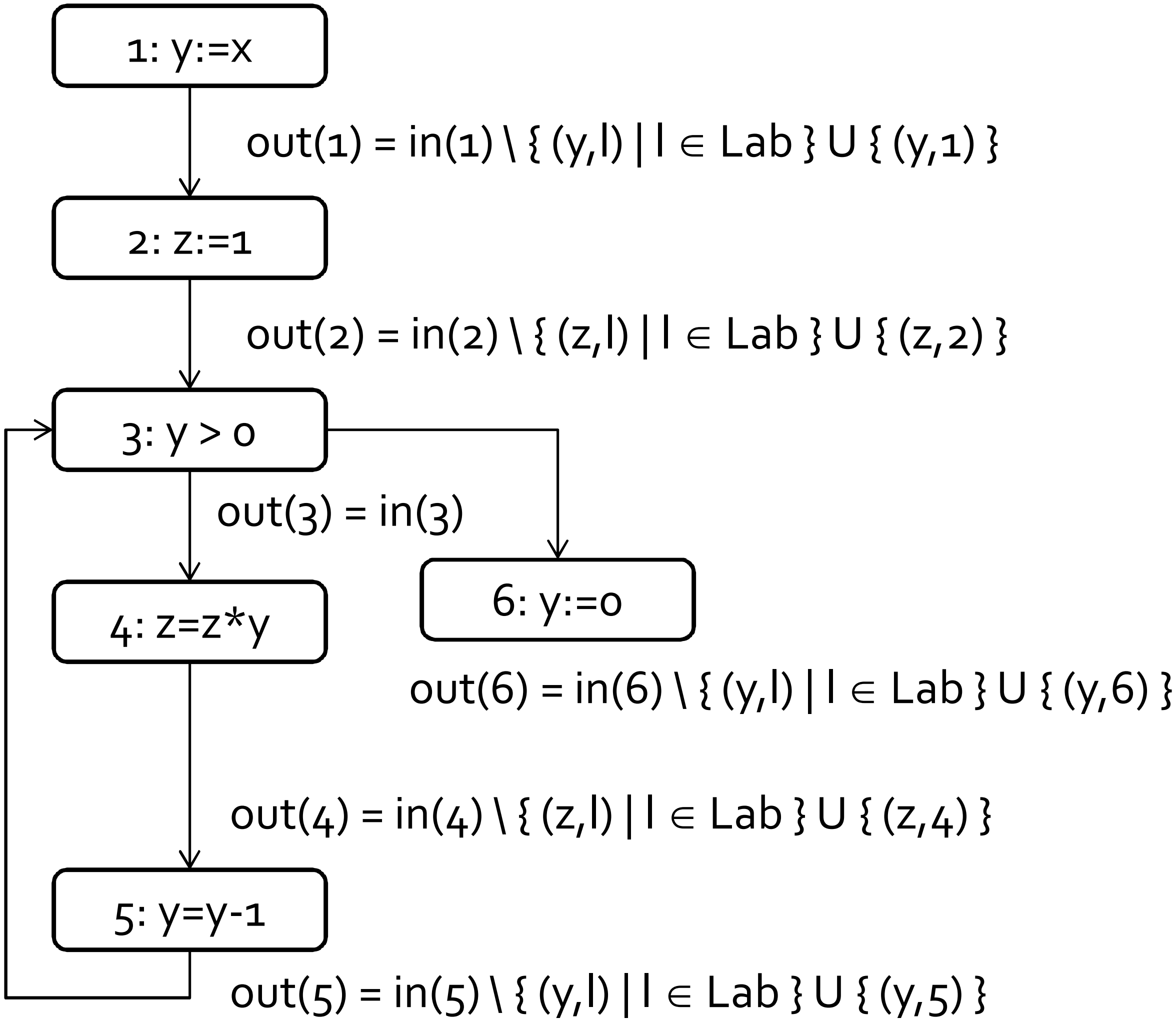


Input/output Sets

1: y := x;
2: z := 1;
3: while y > 0 {
4:     z := z * y;
5:     y := y – 1
}
6: y := 0



Transfer Functions



1: y:=x

out(1) = in(1) \ { (y,l) | l ∈ Lab } U { (y,1) }

2: z:=1

out(2) = in(2) \ { (z,l) | l ∈ Lab } U { (z,2) }

3: y > 0

out(3) = in(3)

4: z=z*y

6: y:=0

out(6) = in(6) \ { (y,l) | l ∈ Lab } U { (y,6) }

out(4) = in(4) \ { (z,l) | l ∈ Lab } U { (z,4) }

5: y=y-1

out(5) = in(5) \ { (y,l) | l ∈ Lab } U { (y,5) }

in(1) = { (x,?), (y,?), (z,?) }
in(2) = out(1)
in(3) = out(2) U out(5)
in(4) = out(3)
in(5) = out(4)
in(6) = out(3)

Kill/Gen formulation for Reaching Definitions

Block	**out (lab)**
$[x := a]^{lab}$	$in(lab) \setminus \{ (x,l) \mid l \in Lab \} \cup \{ (x,lab) \}$
$[skip]^{lab}$	$in(lab)$
$[b]^{lab}$	$in(lab)$

Block	**kill**	**gen**
$[x := a]^{lab}$	$\{ (x,l) \mid l \in Lab \}$	$\{ (x,lab) \}$
$[skip]^{lab}$	$\emptyset$	$\emptyset$
$[b]^{lab}$	$\emptyset$	$\emptyset$

For each program point, which assignments may have been made and not overwritten, when program execution reaches this point along some path.

Solving Gen/Kill Equations

```
OUT[ENTRY] = $\emptyset$;
for (each basic block B other than ENTRY) OUT[B] = $\emptyset$;
while (changes to any OUT occur) {
   for (each basic block B other than ENTRY) {
      OUT[B]= (IN[B] $\setminus$ killB) $\cup$ genB
      IN[B] = $\cup$p$\in$pred(B)  OUT[p]
   }
}
```

- Designated block Entry with OUT[Entry]=∅
- pred(B) = predecessor nodes of B in the control flow graph

Available Expressions Analysis

$[x := a+b]^1;$
$[y := a*b]^2;$
while $[y > a+b]^3$ (
 $[a := a + 1]^4;$
 $[x := a + b]^5$
)

(a+b) always available
at label 3

For each program point, which expressions <u>must</u> have already been computed, and not later modified, on <u>all paths</u> to the program point

Some required notation

blocks : Stmt → P(Blocks)

blocks($[x := a]^{lab}$) = $\{[x := a]^{lab}\}$

blocks($[skip]^{lab}$) = $\{[skip]^{lab}\}$

blocks(S1; S2) = blocks(S1) $\cup$ blocks(S2)

blocks(if $[b]^{lab}$ then S1 else S2) = $\{[b]^{lab}\} \cup$ blocks(S1) $\cup$ blocks(S2)

blocks(while $[b]^{lab}$ do S) = $\{[b]^{lab}\} \cup$ blocks(S)

FV: (BExp $\cup$ AExp)→ Var

Variables used in an expression

AExp(a) = all non-unit expressions in the arithmetic expression a

similarly AExp(b) for a boolean expression b

Available Expressions Analysis

- Property space
  - in_{AE}, out_{AE}: Lab $\longrightarrow \wp(\text{AExp})$
  - Mapping a label to set of arithmetic expressions available at that label

- Dataflow equations
  - Flow equations – how to join incoming dataflow facts
  - Effect equations - given an input set of expressions S, what is the effect of a statement

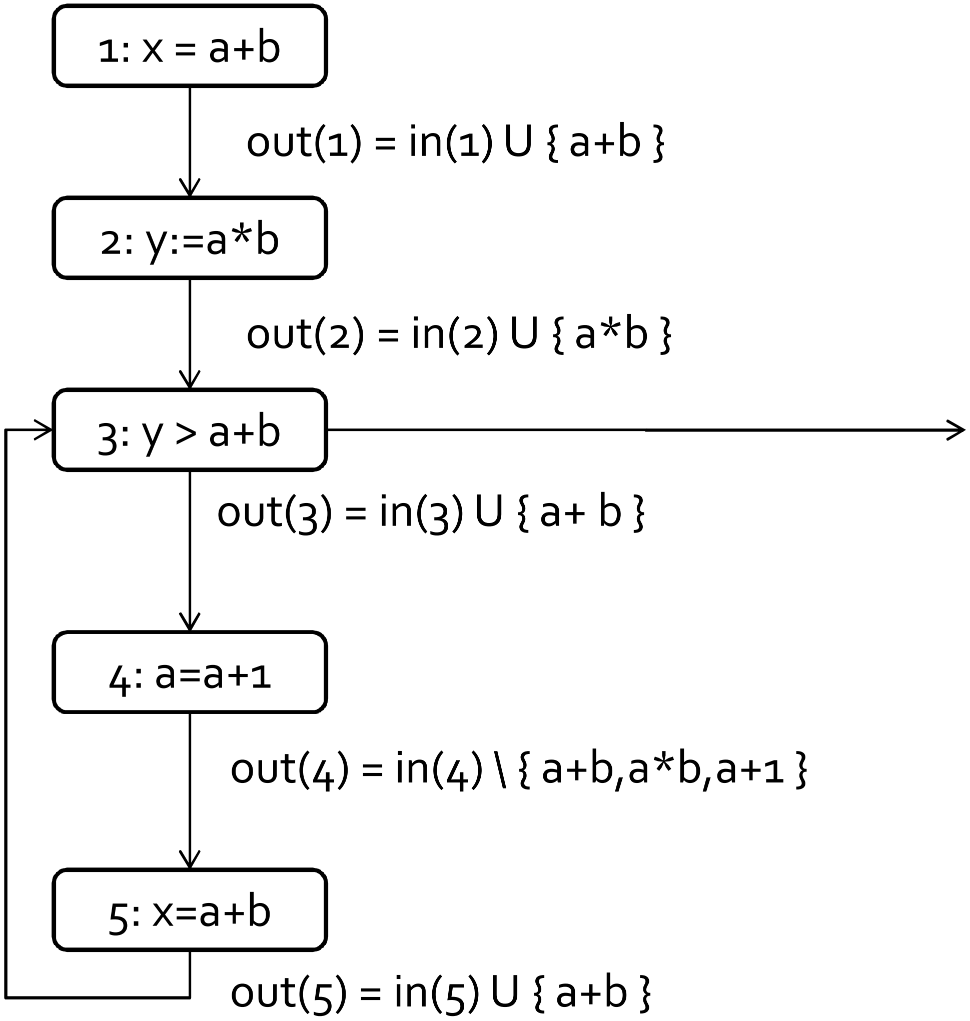
Available Expressions Analysis

- in_{AE} (lab) =
  - $\varnothing$ when lab is the initial label
  - $\bigcap$ { out_{AE} (lab') | lab' $\in$ pred(lab) } otherwise
- out_{AE} (lab) = …

Block	**out (lab)**
$[x := a]^{lab}$	in(lab) \ { a' $\in$ AExp | x $\in$ FV(a') } U { a' $\in$ AExp(a) | x $\notin$ FV(a') }
$[\text{skip}]^{lab}$	in(lab)
$[b]^{lab}$	in(lab) U AExp(b)

From now on going to drop the AE subscript when clear from context

Transfer Functions



1: x = a+b

out(1) = in(1) U { a+b }

2: y:=a*b

out(2) = in(2) U { a*b }

3: y > a+b

out(3) = in(3) U { a+ b }

4: a=a+1

out(4) = in(4) \ { a+b,a*b,a+1 }

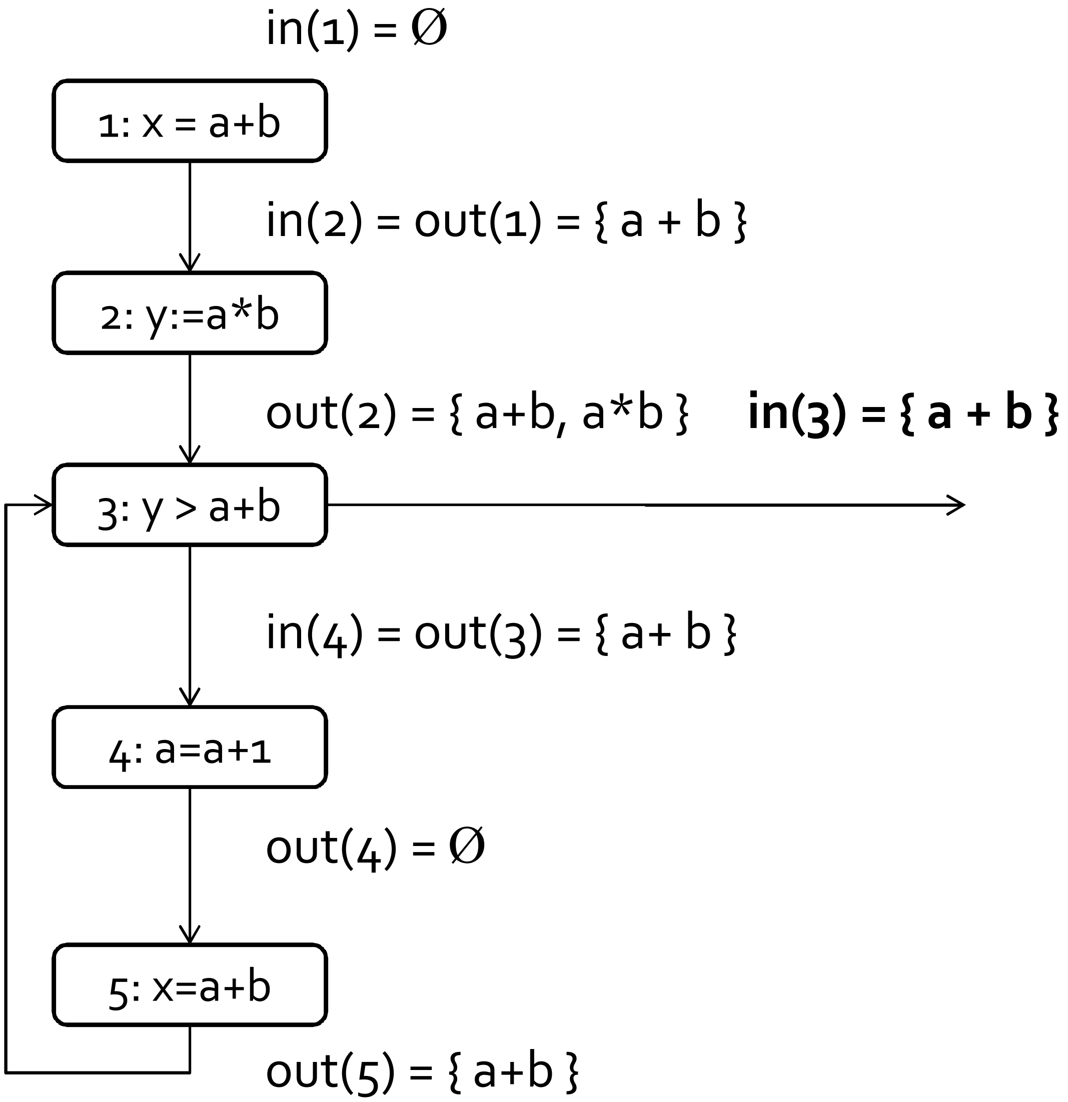
5: x=a+b

out(5) = in(5) U { a+b }

in(1) = $\emptyset$
in(2) = out(1)
in(3) = out(2) $\cap$ out(5)
in(4) = out(3)
in(5) = out(4)

[x := a+b]^1;
[y := a*b]^2;
while [y > a+b]^3 (
  [a := a + 1]^4;
  [x := a + b]^5
)

Solution



in(1) = Ø

1: x = a+b

in(2) = out(1) = { a + b }

2: y:=a*b

out(2) = { a+b, a*b } **in(3) = { a + b }**

3: y > a+b

in(4) = out(3) = { a+ b }

4: a=a+1

out(4) = Ø

5: x=a+b

out(5) = { a+b }

Kill/Gen

Block	**out (lab)**
$[x := a]^{lab}$	$in(lab) \setminus \{ a' \in AExp \mid x \in FV(a') \} \cup \{ a' \in AExp(a) \mid x \notin FV(a') \}$
$[skip]^{lab}$	$in(lab)$
$[b]^{lab}$	$in(lab) \cup AExp(b)$

Block	**kill**	**gen**
$[x := a]^{lab}$	$\{ a' \in AExp \mid x \in FV(a') \}$	$\{ a' \in AExp(a) \mid x \notin FV(a') \}$
$[skip]^{lab}$	$\emptyset$	$\emptyset$
$[b]^{lab}$	$\emptyset$	$AExp(b)$

$$out(lab) = in(lab) \setminus kill(B^{lab}) \cup gen(B^{lab})$$

B^{lab} = block at label lab

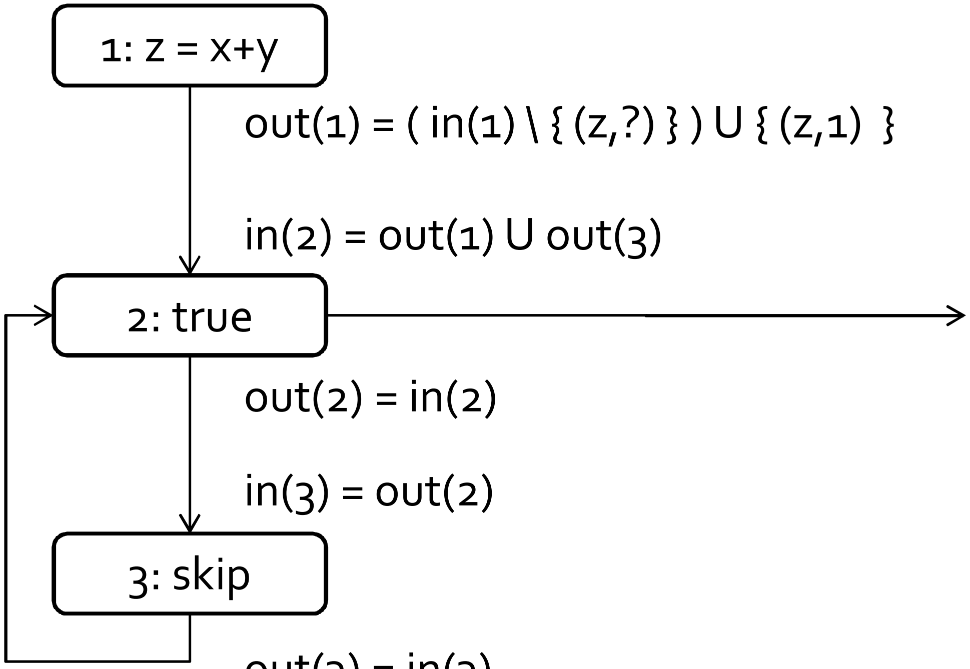
Reaching Definitions Revisited

Block	**out (lab)**
$[x := a]^{lab}$	$in(lab) \setminus \{ (x,l) \mid l \in Lab \} \cup \{ (x,lab) \}$
$[skip]^{lab}$	$in(lab)$
$[b]^{lab}$	$in(lab)$

Block	**kill**	**gen**
$[x := a]^{lab}$	$\{ (x,l) \mid l \in Lab \}$	$\{ (x,lab) \}$
$[skip]^{lab}$	$\emptyset$	$\emptyset$
$[b]^{lab}$	$\emptyset$	$\emptyset$

For each program point, which assignments <u>may</u> have been made and not overwritten, when program execution reaches this point along <u>some path</u>.

Why solution with smallest sets?



in(1) = { (x,?),(y,?),(z,?) }

out(1) = (in(1) \ { (z,?) }) U { (z,1) }

in(2) = out(1) U out(3)

out(2) = in(2)

in(3) = out(2)

out(3) = in(3)

in(1) = { (x,?),(y,?),(z,?) }
in(2) = out(1) **U** out(3)
in(3) = out(2)

[z := x+y]^1;
while [true]^2 (
  [skip]^3;
)

After simplification: in(2) = in(2) U { (x,?),(y,?),(z,1) }

Many solutions: any superset of { (x,?),(y,?),(z,1) }

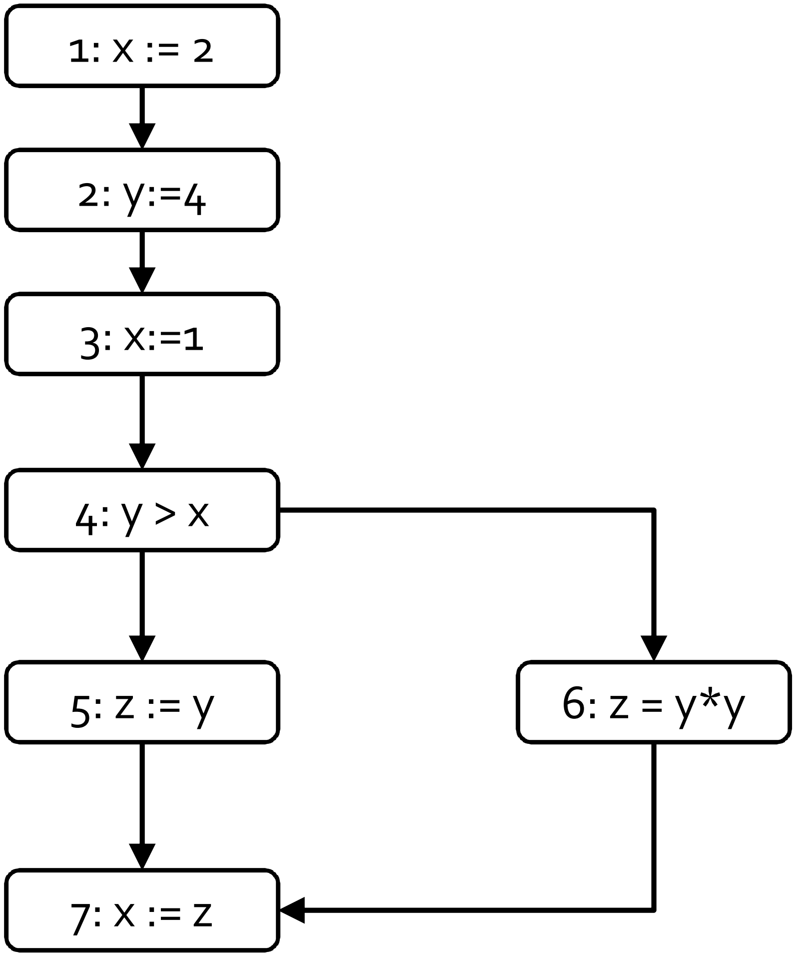
Live Variables

[x :=2]^1;
[y:=4]^2;
[x:=1]^3;
(if [y>x]^4 then [z:=y]^5
else [z:=y*y]^6);
[x:=z]^7

For each program point, which variables <u>may</u> be live at the exit from the point.

Live Variables

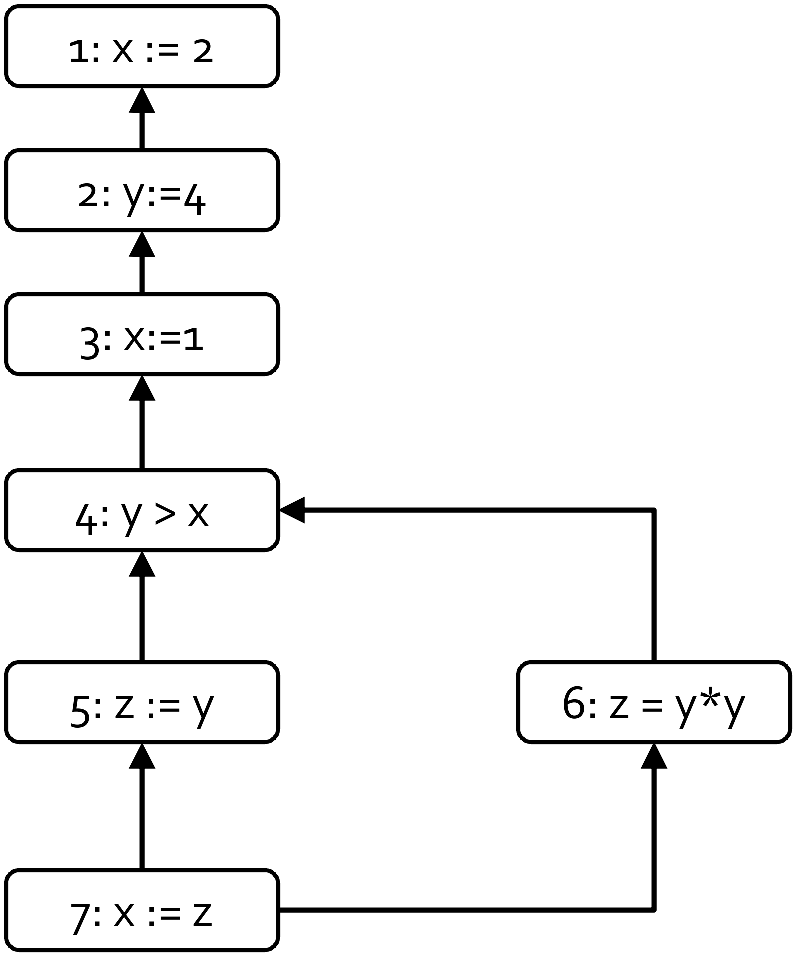
$[x := 2]^1$;
$[y := 4]^2$;
$[x := 1]^3$;
(if $[y > x]^4$ then $[z := y]^5$
else $[z := y*y]^6$);
$[x := z]^7$



Live Variables

$[x := 2]^1$;
$[y := 4]^2$;
$[x := 1]^3$;
(if $[y > x]^4$ then $[z := y]^5$
else $[z := y * y]^6$);
$[x := z]^7$

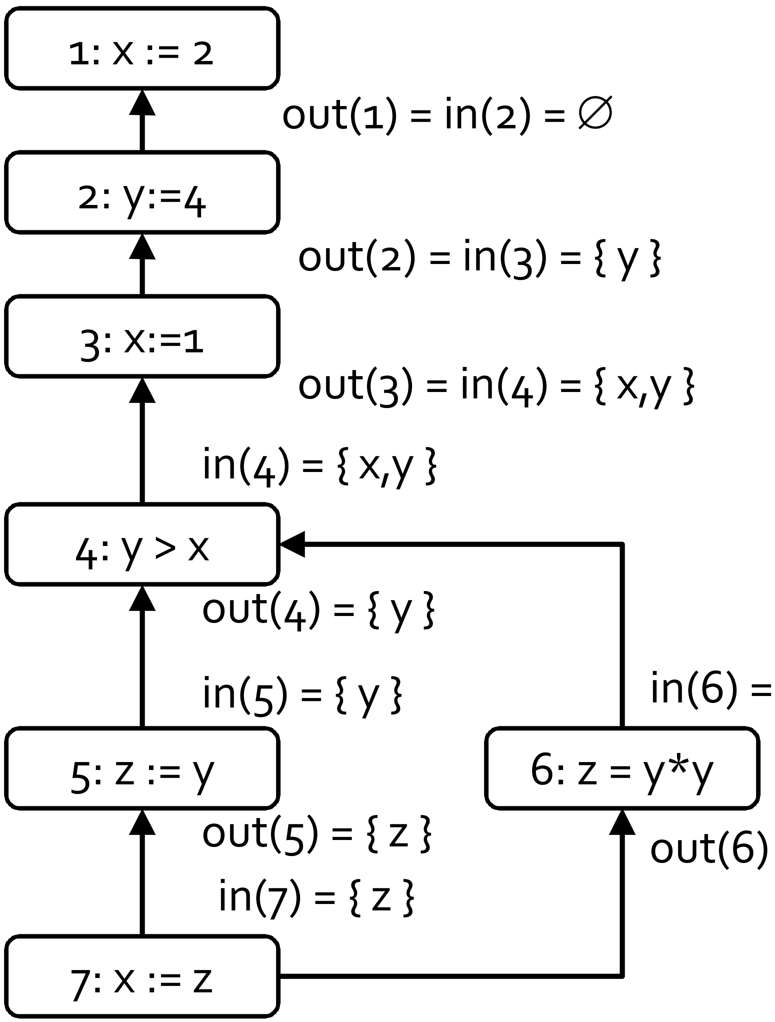
Block	kill	gen
$[x := a]^{lab}$	{ x }	{ FV(a) }
$[skip]^{lab}$	Ø	Ø
$[b]^{lab}$	Ø	FV(b)



Live Variables: solution

$[x := 2]^1$;
$[y := 4]^2$;
$[x := 1]^3$;
(if $[y > x]^4$ then $[z := y]^5$
else $[z := y * y]^6$);
$[x := z]^7$

Block	kill	gen
$[x := a]^{lab}$	{ x }	{ FV(a) }
$[skip]^{lab}$	Ø	Ø
$[b]^{lab}$	Ø	FV(b)



in(1) = ∅

out(1) = in(2) = ∅

out(2) = in(3) = { y }

out(3) = in(4) = { x,y }

in(4) = { x,y }

out(4) = { y }

in(5) = { y }

in(6) = { y }

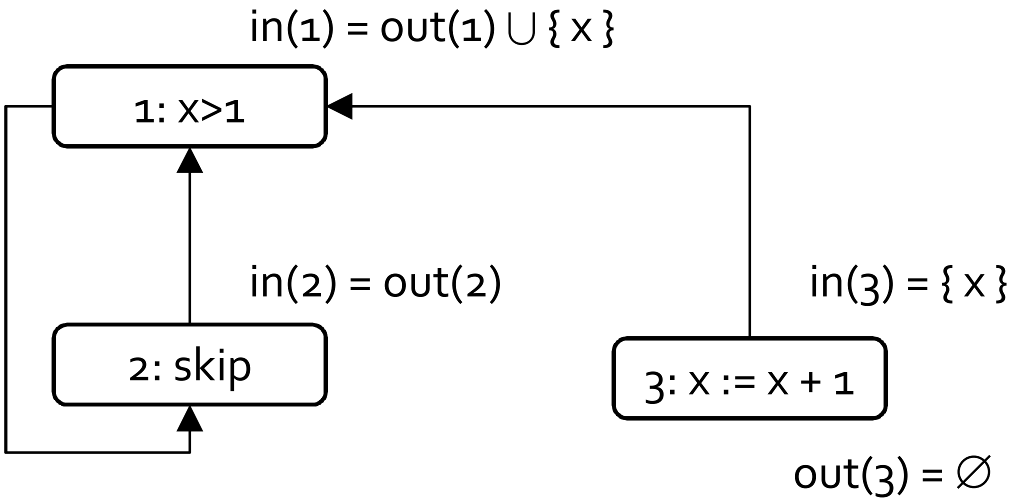
out(5) = { z }

out(6) = { z }

in(7) = { z }

out(7) = ∅

Why solution with smallest set?



in(1) = out(1) ∪ { x }

in(2) = out(2)

in(3) = { x }

out(3) = ∅

out(1) = in(2) **U** in(3)
out(2) = in(1)
out(3) = ∅

```
while [x>1]¹ (
  [skip]²;
)
[x := x+1]³;
```

After simplification: in(1) = in(1) U { x }

Many solutions: any superset of { x }

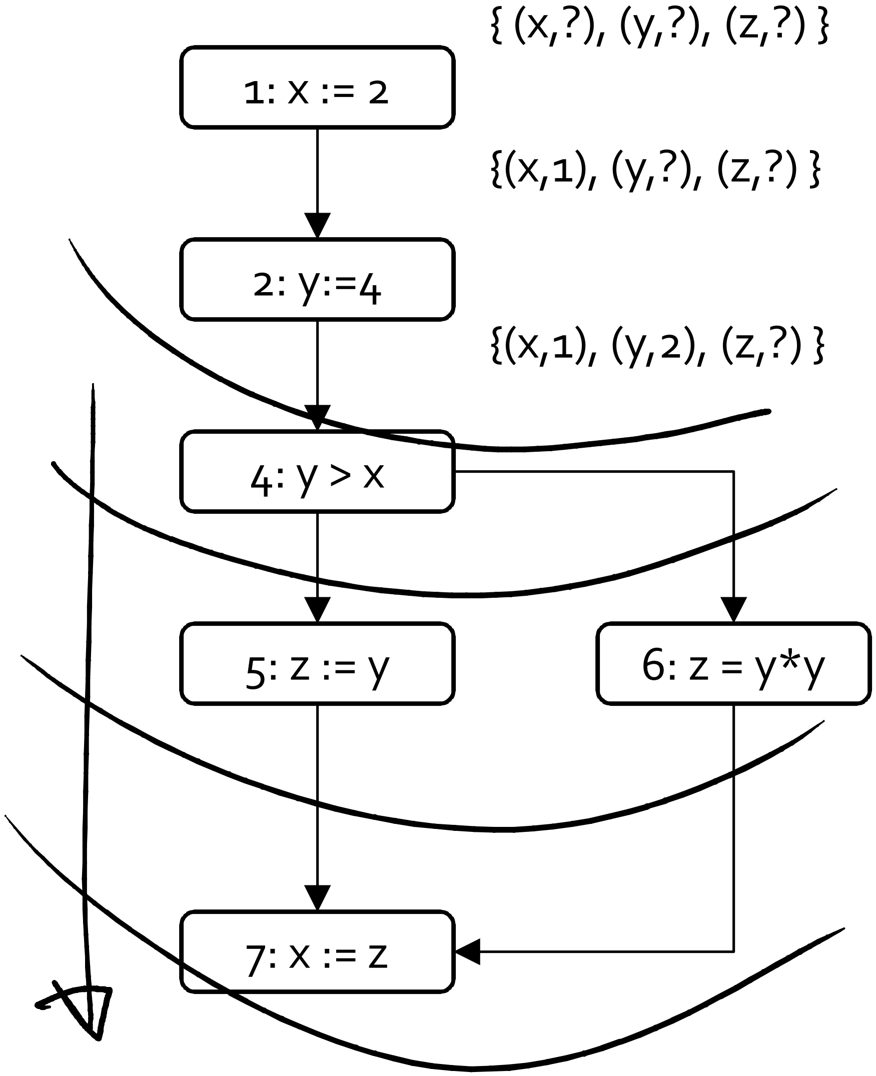
Monotone Frameworks

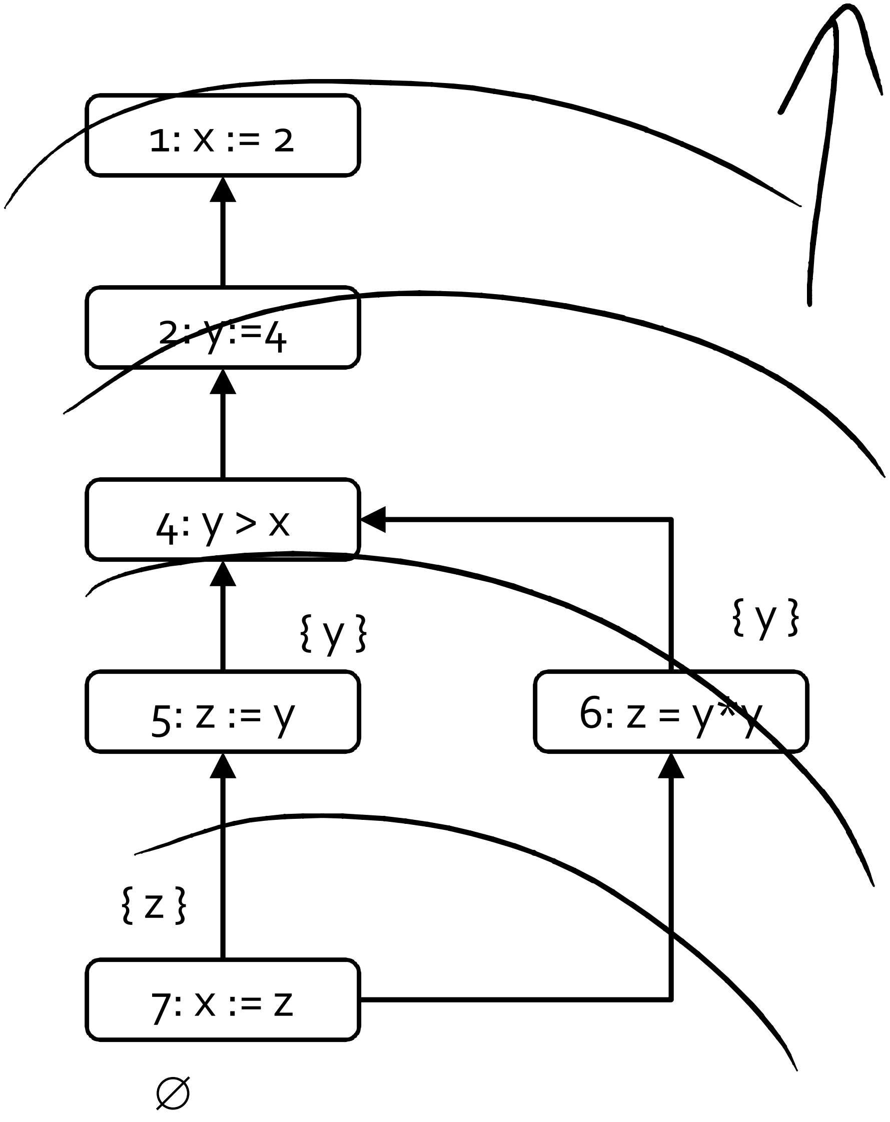
$$\text{In(lab)} = \begin{cases} \text{Initial} & \text{when lab} \in \text{Entry labels} \\ \sqcup \{ \text{out(lab')} \mid \text{(lab',lab)} \in \text{CFG edges} \} & \text{otherwise} \end{cases}$$

$$\text{out(lab)} = f_{lab}(\text{in(lab)})$$

- $\sqcup$ is $\cup$ or $\cap$
- CFG edges go either forward or backwards
- Entry labels are either initial program labels or final program labels (when going backwards)
- Initial is an initial state (or final when going backwards)
- f_{lab} is the transfer function associated with the blocks B^{lab}

Forward vs. Backward Analyses





Must vs. May Analyses

- When $\sqcup$ is $\cap$ - must analysis
  - Want largest sets that solves the equation system
  - Properties hold on all paths reaching a label (exiting a label, for backwards)

- When $\sqcup$ is $\cup$ - may analysis
  - Want smallest sets that solve the equation system
  - Properties hold at least on one path reaching a label (existing a label, for backwards)

Example: Reaching Definition

- L = $\wp$(Var×Lab) is partially ordered by $\subseteq$
- $\sqcup$ is $\cup$
- L satisfies the Ascending Chain Condition because Var × Lab is finite (for a given program)

Example: Available Expressions

- L = $\wp$(AExp) is partially ordered by $\supseteq$
- $\sqcup$ is $\cap$
- L satisfies the Ascending Chain Condition because AExp is finite (for a given program)

Analyses Summary

	Reaching Definitions	**Available Expressions**	**Live Variables**
L	$\wp$(Var x Lab)	$\wp$(AExp)	$\wp$(Var)
$\sqsubseteq$	$\subseteq$	$\supseteq$	$\subseteq$
$\sqcup$	$\cup$	$\cap$	$\cup$
$\perp$	$\varnothing$	AExp	$\varnothing$
Initial	{ (x,?) | x $\in$ Var}	$\varnothing$	$\varnothing$
Entry labels	{ init }	{ init }	final
Direction	Forward	Forward	Backward
F	{ f: L $\rightarrow$ L | $\exists$k,g : f(val) = (val \ k) U g }		
f_{lab}	f_{lab}(val) = (val \ kill) $\cup$ gen		

Analyses as Monotone Frameworks

- Property space
  - Powerset
  - Clearly a complete lattice

- Transformers
  - Kill/gen form
  - Monotone functions (let’s show it)

Monotonicity of Kill/Gen transformers

- Have to show that $x \sqsubseteq x'$ implies $f(x) \sqsubseteq f(x')$
- Assume $x \sqsubseteq x'$, then for kill set k and gen set g $(x \setminus k) \cup g \sqsubseteq (x' \setminus k) \cup g$

- Technically, since we want to show it for all functions in F, we also have to show that the set is closed under function composition

Distributivity of Kill/Gen transformers

- Have to show that $f(x \sqcup y) \sqsubseteq f(x) \sqcup f(y)$
- $f(x \sqcup y) = ((x \sqcup y) \setminus k) \cup g$
  $= ((x \setminus k) \sqcup (y \setminus k)) \cup g$
  $= (((x \setminus k) \cup g) \sqcup ((y \setminus k) \cup g))$
  $= f(x) \sqcup f(y)$

- Used distributivity of $\sqcup$ and $\cup$
  - Works regardless of whether $\sqcup$ is $\cup$ or $\cap$

Points-to Analysis

- Many flavors
- PWHILE language

p ∈ PExp pointer expressions

a ::= x | n | a1 op_a a2 | &x | *x | nil

S ::= $[x := a]^{lab}$
  | $[skip]^{lab}$
  | S1;S2
  | if $[b]^{lab}$ then S1 else S2
  | while $[b]^{lab}$ do S
  | x = malloc

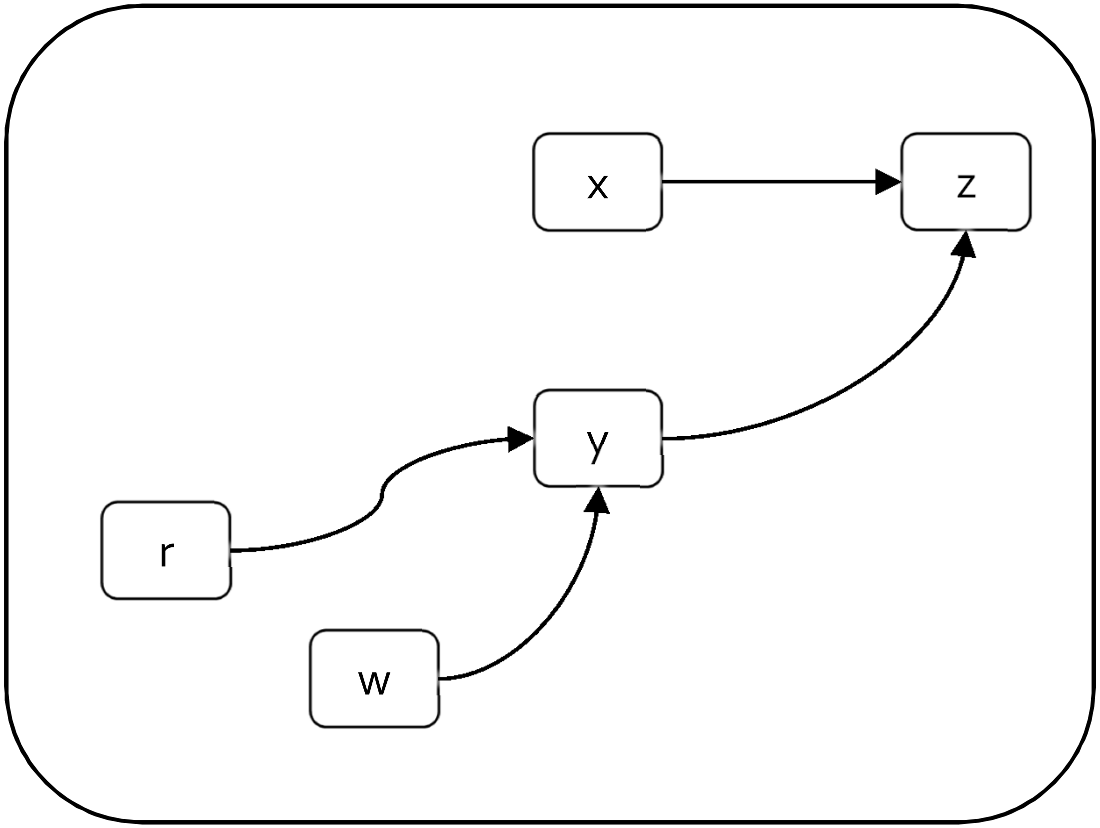
Points-to Analysis

- Aliases
  - Two pointers p and q are aliases if they point to the same memory location
- Points-to pair
  - (p,q) means p holds the address of q
- Points-to pairs and aliases
  - (p,q) and (r,q) means that p and r are aliases

- Challenge: no a priori bound on the set of heap locations

Terminology Example

[x := &z]^1
[y := &z]^2
[w := &y]^3
[r := w]^4



Points-to pairs: (x,z), (y,z), (w,y), (r,y)
Aliases: (x,y), (r,w)

(May) Points-to Analysis

- Property Space
    - $L = (\wp(\mathrm{Var} \times \mathrm{Var}), \subseteq, \cup, \cap, \emptyset, \mathrm{Var} \times \mathrm{Var})$

- Transfer functions

Statement	**out(lab)**
$[p = \&x]^{lab}$	$in(lab) \cup \{ (p,x) \}$
$[p = q]^{lab}$	$in(lab)\ U \{ (p,x) \mid (q,x) \in in(lab) \}$
$[*p = q]^{lab}$	$in(lab)\ U \{ (r,x) \mid (q,x) \in in(lab)$ and $(p,r) \in in(lab) \}$
$[p = *q]^{lab}$	$in(lab)\ U \{ (p,r) \mid (q,x) \in in(lab)$ and $(x,r) \in in(lab) \}$

(May) Points-to Analysis

- What to do with malloc?
- Need some static naming scheme for dynamically allocated objects

- Single name for the entire heap
  - $[\![[p = \text{malloc}]^{lab}]\!] (S) = S \cup \{ (p,H) \}$
- Name based on static allocation site
  - $[\![[p = \text{malloc}]^{lab}]\!] (S) = S \cup \{ (p,lab) \}$

(May) Points-to Analysis

$\emptyset$

[x :=malloc]^1;

{ (x,H) }

[y:=malloc]^2;

{ (x,H), (y,H) }

(if [x==y]^3 then

{ (x,H), (y,H) }

[z:=x]^4

{ (x,H), (y,H), (z,H) }

else

[z:=y]^5

{ (x,H), (y,H), (z,H) }

);

{ (x,H), (y,H), (z,H) }

Single name H for the entire heap

Allocation Sites

- Divide the heap into a fixed partition based on allocation site
- All objects allocated at the same program point represented by a single “abstract object”

AS1	AS2	AS3	AS1
AS2	AS3	AS3	AS2

(May) Points-to Analysis

∅

$[x := malloc]^1$; // A1

{ (x,A1) }

$[y := malloc]^2$; // A2

{ (x,A1), (y,A2) }

(if $[x == y]^3$ then

{ (x,A1), (y,A2) }

$[z := x]^4$

{ (x,A1), (y,A2), (z,A1) }

else

$[z := y]^5$

{ (x,A1), (y,A2), (z,A2) }

);

{ (x,A1), (y,A2), (z,A1), (z,A2) }

Allocation-site based naming (using A_{lab} instead of just “lab” for clarity)

Weak Updates

Statement	**out(lab)**
[p = &x]^{lab}	in(lab) ∪ { (p,x) }
[p = q]^{lab}	in(lab) **U** { (p,x) | (q,x) ∈ in(lab) }
[*p = q]^{lab}	in(lab) **U** { (r,x) | (q,x) ∈ in(lab) and (p,r) ∈ in(lab) }
[p = *q]^{lab}	in(lab) **U** { (p,r) | (q,x) ∈ in(lab) and (x,r) ∈ in(lab) }

∅

[x :=malloc]^1; // A1

{ (x,A1) }

[y:=malloc]^2; // A2

{ (x,A1), (y,A2) }

[z:=x]^3;

{ (x,A1), (y,A2), (z,A1) }

[z:=y]^4;

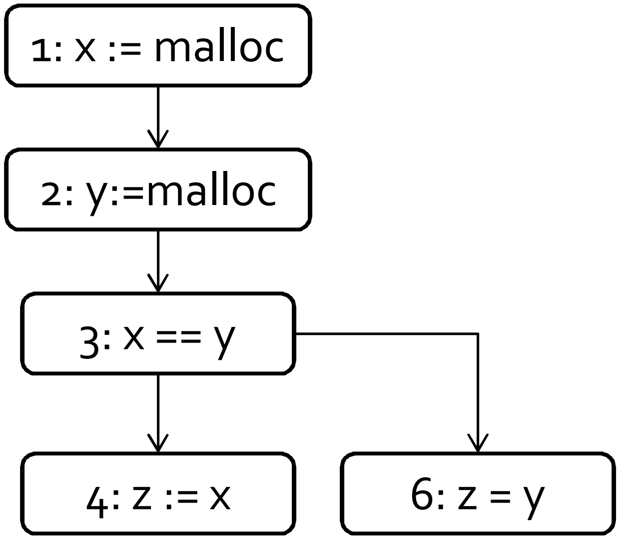
{ (x,A1), (y,A2), (z,A1), (z,A2) }

(May) Points-to Analysis

- Fixed partition of the (unbounded) heap to static names
    - Allocation sites
    - Types
    - Calling contexts
    - …
- What we saw so far – flow-insensitive
    - Ignoring the structure of the flow in the program

Flow-sensitive vs. Flow-insensitive Analyses

```
[x :=malloc]¹;
[y:=malloc]²;
(if [x==y]³ then
   [z:=x]⁴
else
  [z:=y]⁵
);
```



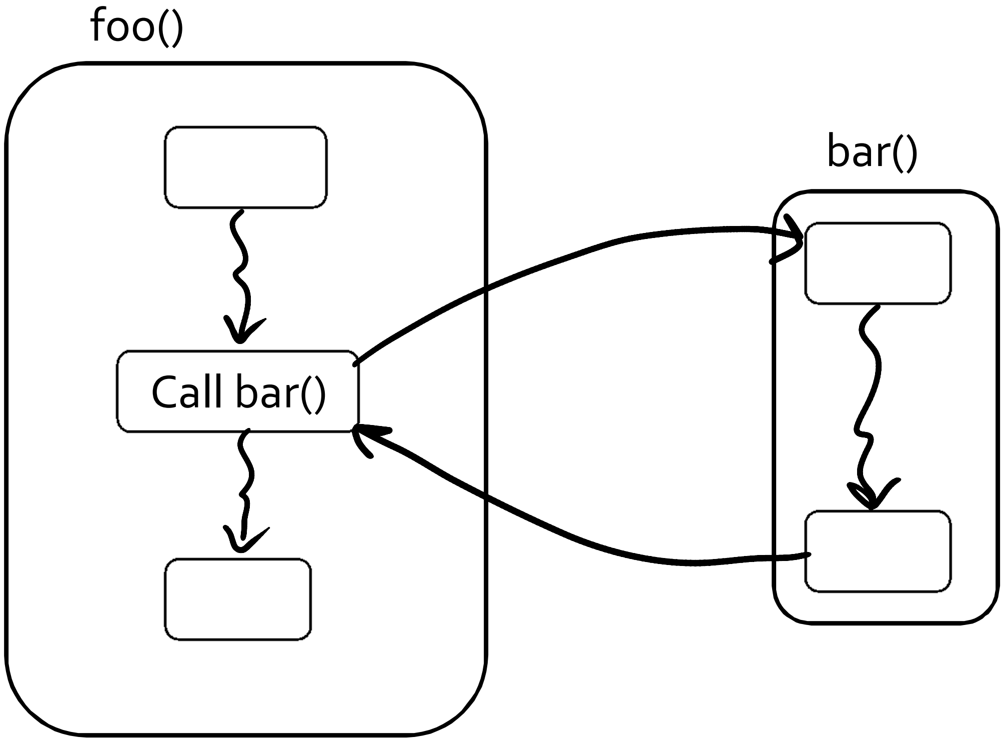
- Flow sensitive: respect program flow
  - a separate set of points-to pairs for every program point
  - the set at a point represents possible may-aliases on some path from entry to the program point
- Flow insensitive: assume all execution orders are possible, abstract away order between statements

So far…

- Intra-procedural analysis

- How are we going to deal with procedures?
- Inter-procedural analysis

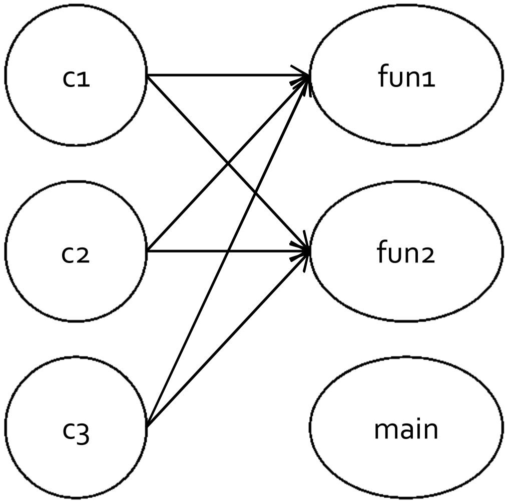
Interprocedural Analysis



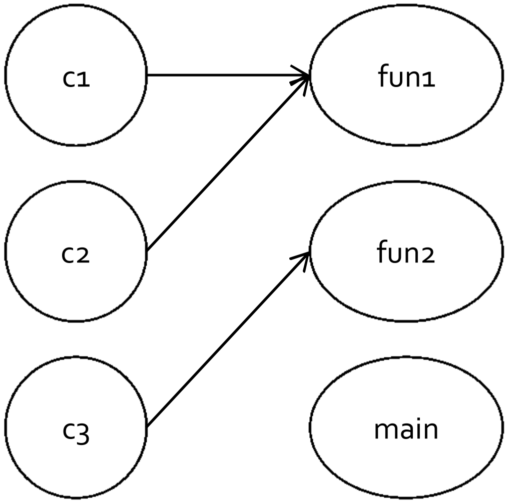
- The effect of calling a procedure is the effect of executing its body

Call Graph

```
int (*pf)(int);

int fun1 (int x) {
  if (x < 10)
    return (*pf) (x+l); // C1
  else
    return x;
}

int fun2(int y) {
  pf = &fun1;
  return (*pf) (y) ; // C2
}

void main() {
  pf = &fun2;
  (*pf )(5) ; // C3
}
```



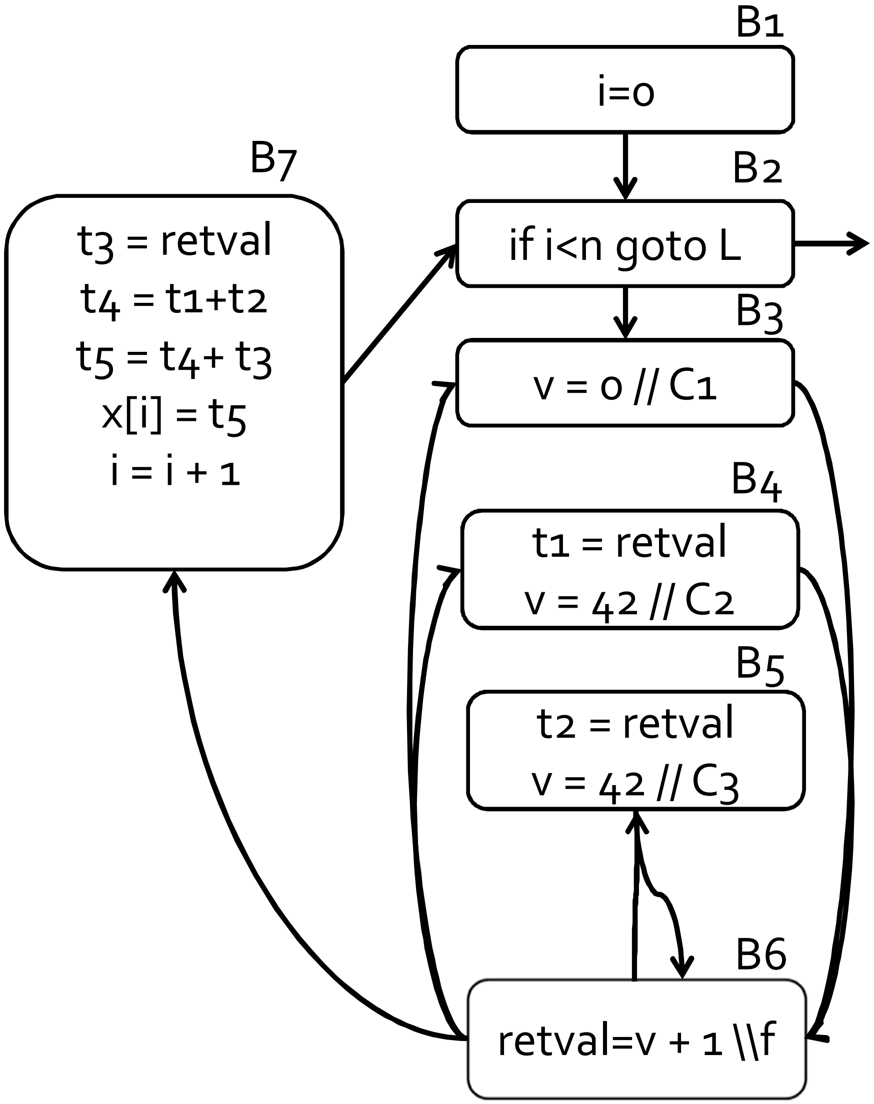
type based



more precise

Context Sensitivity

```
main() {
 for (i = 0; i < n; i++) {
   t1 = f(0);    // C1
   t2 = f(42); // C2
   t3 = f(42); // C3
   X[i] = t1 + t2 + t3;
 }
}

int f(int v)
  return (v+1);
}
```



B1

i=0

B2

if i<n goto L

B3

v = 0 // C1

B4

t1 = retval

v = 42 // C2

B5

t2 = retval

v = 42 // C3

B6

retval=v + 1 \\f

B7

t3 = retval

t4 = t1+t2

t5 = t4+ t3

x[i] = t5

i = i + 1

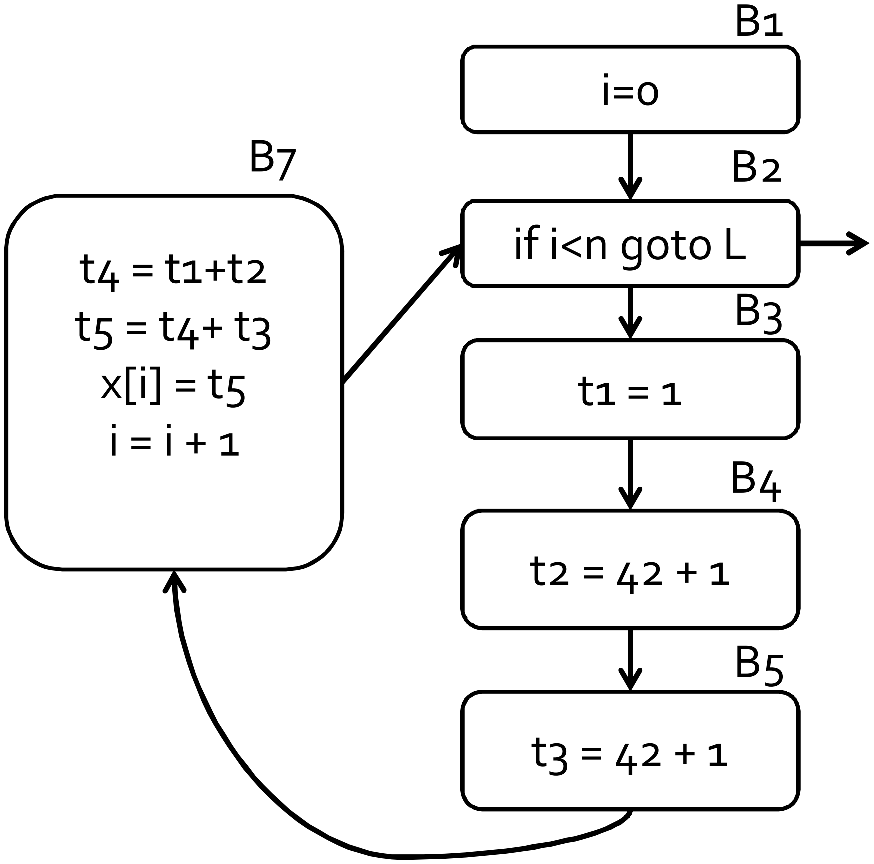
Solution Attempt #1

- Inline callees into callers
    - End up with one big procedure
    - CFGs of individual procedures = duplicated many times
- Good: it is precise
    - distinguishes different calls to the same function
- Bad
    - exponential blow-up, not efficient
    - doesn’t work with recursion

```
main() { f(); f(); }
f() { g(); g(); }
g() { h(); h(); }
h() { ... }
```

Inlining

```
main() {
 for (i = 0; i < n; i++) {
   t1 = f(0);    // C1
   t2 = f(42); // C2
   t3 = f(42); // C3
   X[i] = t1 + t2 + t3;
 }
}

int f(int v)
  return (v+1);
}
```



B1

i=0

B2

if i<n goto L

B3

t1 = 1

B4

t2 = 42 + 1

B5

t3 = 42 + 1

B7

t4 = t1+t2

t5 = t4+ t3

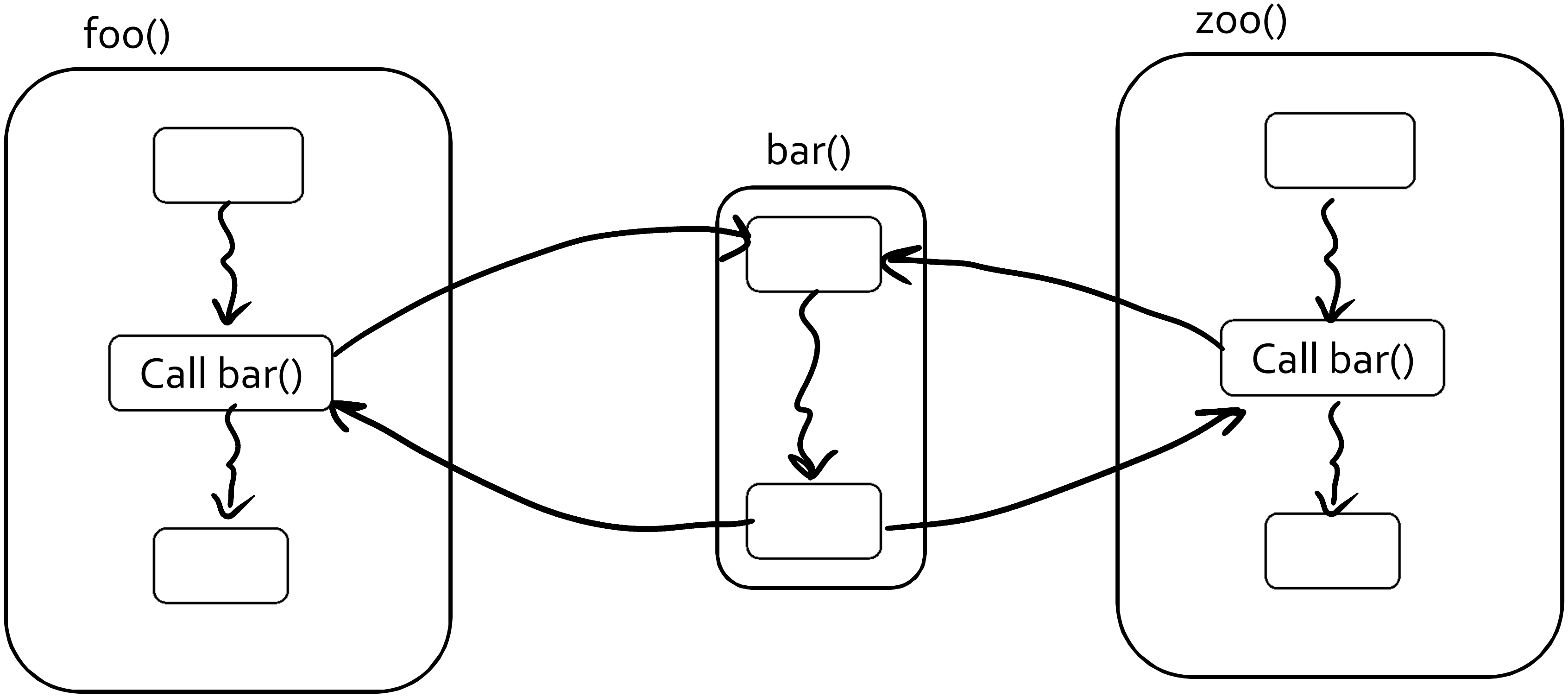
x[i] = t5

i = i + 1

Solution Attempt #2

- Build a “supergraph” = inter-procedural CFG
- Replace each call from P to Q with
  - An edge from point before the call (call point) to Q’s entry point
  - An edge from Q’s exit point to the point after the call (return pt)
  - Add assignments of actuals to formals, and assignment of return value
- Good: efficient
  - Graph of each function included exactly once in the supergraph
  - Works for recursive functions (although local variables need additional treatment)
- Bad: imprecise, “context-insensitive”
  - The “unrealizable paths problem”: dataflow facts can propagate along infeasible control paths

Unrealizable Paths

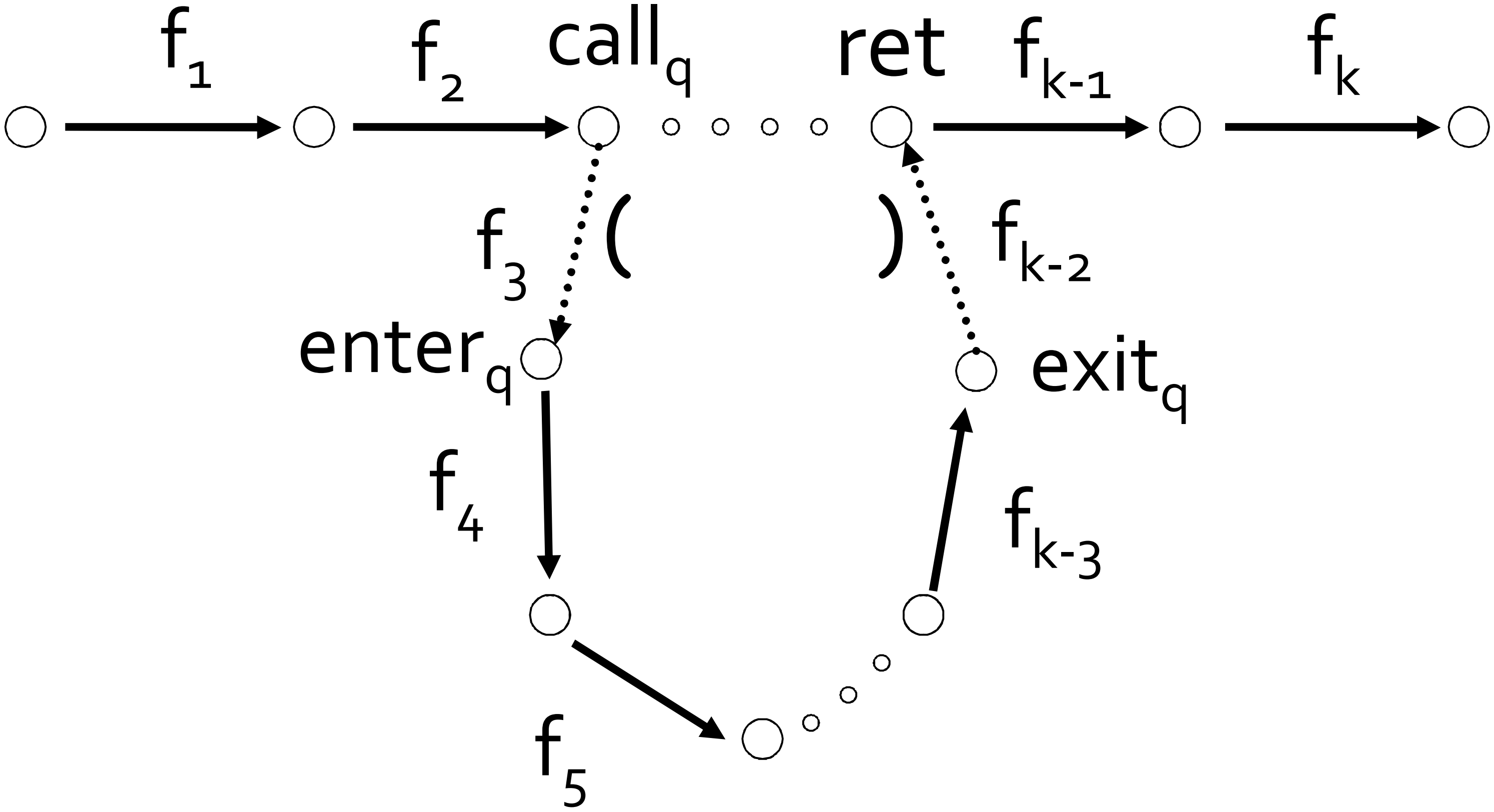


Interprocedural Analysis

```
begin
proc p() is$^1$
  [x := a + 1]$^2$
end$^3$
[a=7]$^4$
[call p()]$^5_6$
[print x]$^7$
[a=9]$^8$
[call p()]$^9_{10}$
[print a]$^{11}$
end
```

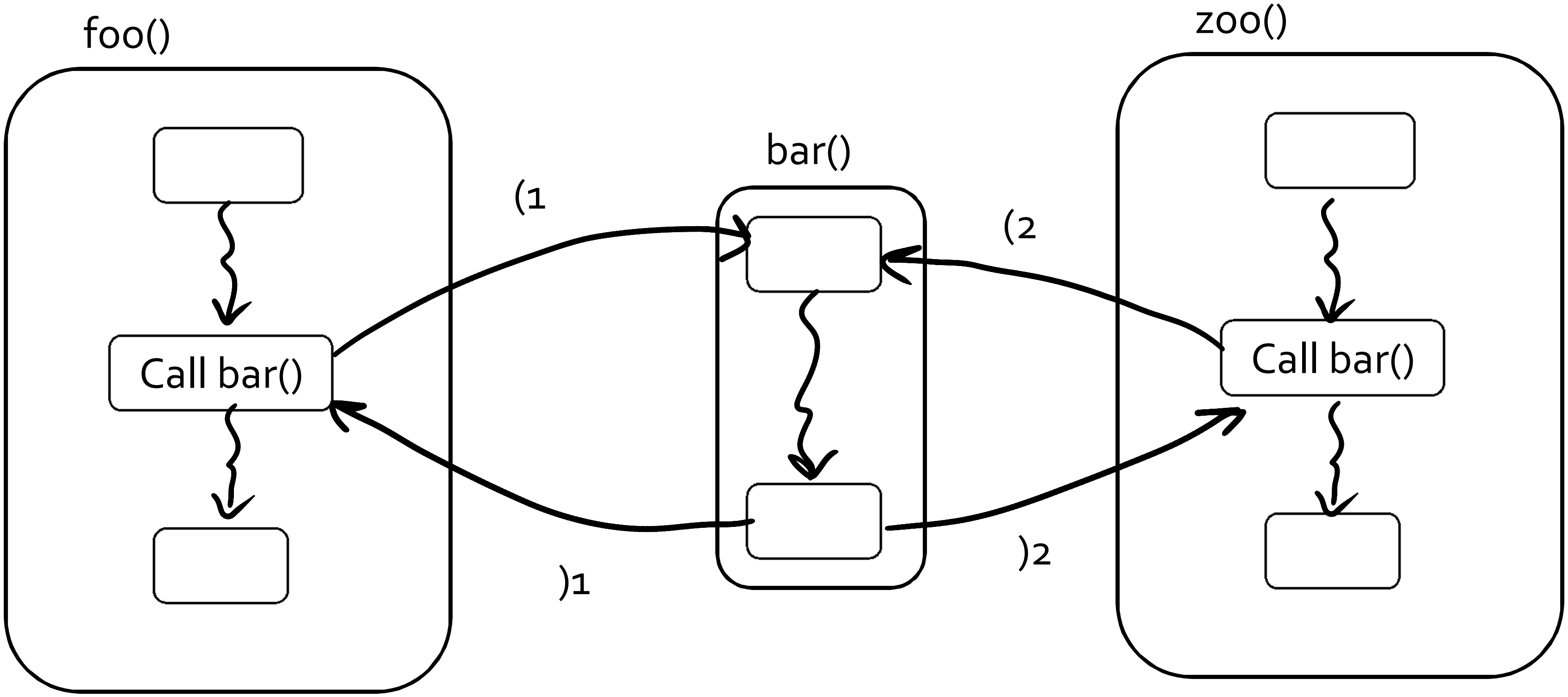
- Extend language with begin/end and with [call p()]$^{clab}_{rlab}$
- Call label clab, and return label rlab

IVP: Interprocedural Valid Paths



- IVP: all paths with matching calls and returns
  - And prefixes

Valid Paths



Interprocedural Valid Paths

- **IVP** set of paths
  - Start at program entry
- Only considers matching calls and returns
  - aka, valid

- Can be defined via context free grammar
  - matched ::= matched $(_i$ matched $)_i$ | ε
  - valid ::= valid $(_i$ matched | matched
    - *paths* can be defined by a regular expression

The Join-Over-Valid-Paths (JVP)

- vpaths(n) all valid paths from program start to n
- JVP[n] $= \sqcup\{[\![e_1, e_2, \ldots, e]\!]$ (initial)
  $(e_1, e_2, \ldots, e) \in$ vpaths(n)$\}$
- JVP $\sqsubseteq$ JFP
  - In some cases the JVP can be computed
  - (Distributive problem)

Sharir and Pnueli ‘82

- Call String approach
  - Blend interprocedural flow with intra procedural flow
  - Tag every dataflow fact with call history

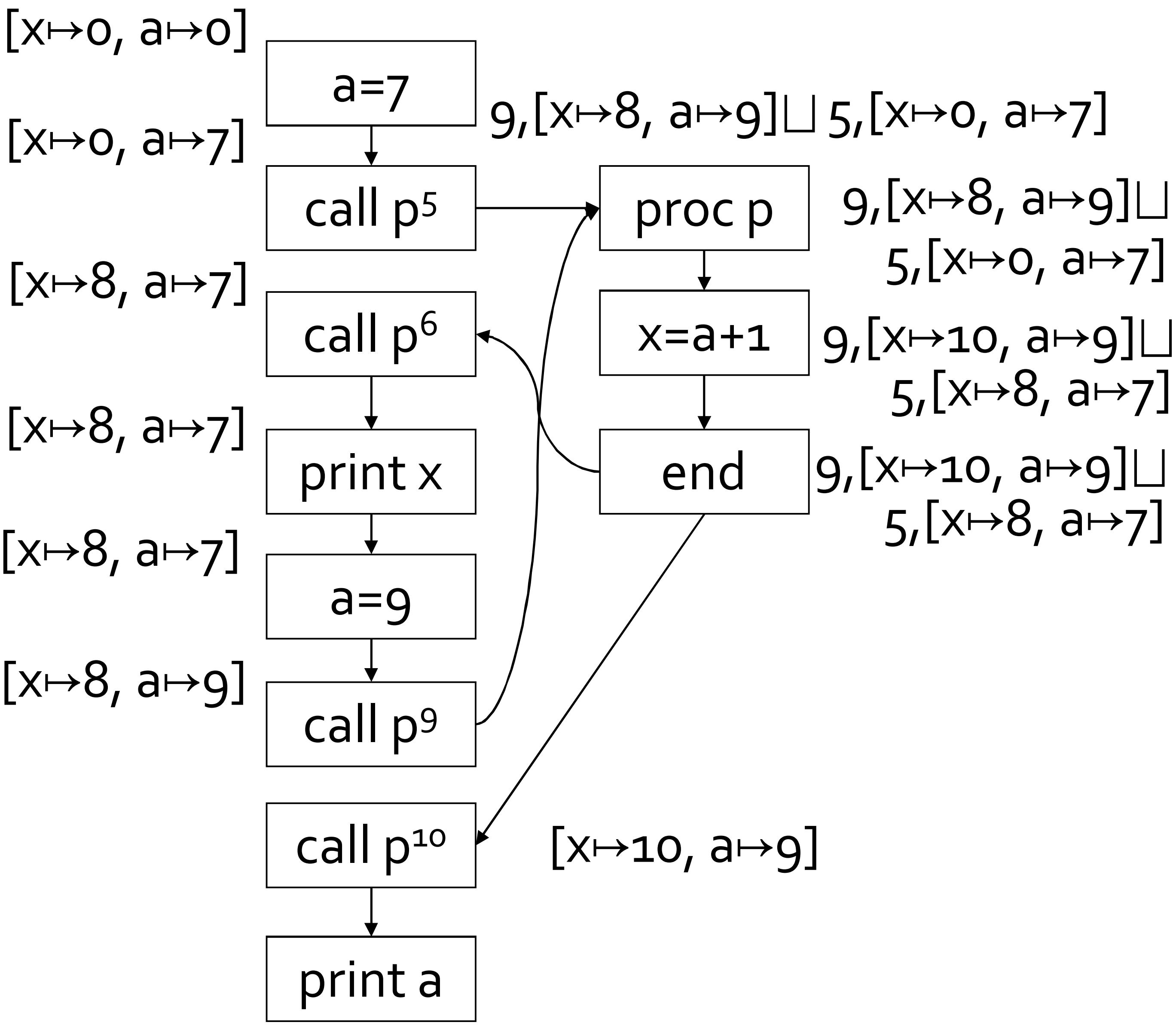
- Functional approach
  - Determine the effect of a procedure
    - E.g., in/out map
  - Treat procedure invocations as “super ops”

The Call String Approach

- Record at every node a pair (l, c) where l ∈ L is the dataflow information and c is a suffix of unmatched calls

- Use Chaotic iterations
- To guarantee termination limit the size of c (typically 1 or 2)
- Emulates inline (but no code growth)
- Exponential in size of c

```
begin
proc p() is$^1$
   [x := a + 1]$^2$
end$^3$
[a=7]$^4$
[call p()]$^5_6$
[print x]$^7$
[a=9]$^8$
[call p()]$^9_{10}$
[print a]$^{11}$
end
```



[x↦0, a↦0]

a=7

9,[x↦8, a↦9]⊔ 5,[x↦0, a↦7]

[x↦0, a↦7]

call p^5

proc p

9,[x↦8, a↦9]⊔
5,[x↦0, a↦7]

[x↦8, a↦7]

call p^6

x=a+1

9,[x↦10, a↦9]⊔
5,[x↦8, a↦7]

[x↦8, a↦7]

print x

end

9,[x↦10, a↦9]⊔
5,[x↦8, a↦7]

[x↦8, a↦7]

a=9

[x↦8, a↦9]

call p^9

call p^{10}

[x↦10, a↦9]

print a

begin^{0}

proc p() is^{1}

if [b]^{2} then (

[a := a -1]^{3}

[call p()]$^{4}_{5}$

[a := a + 1]^{6}

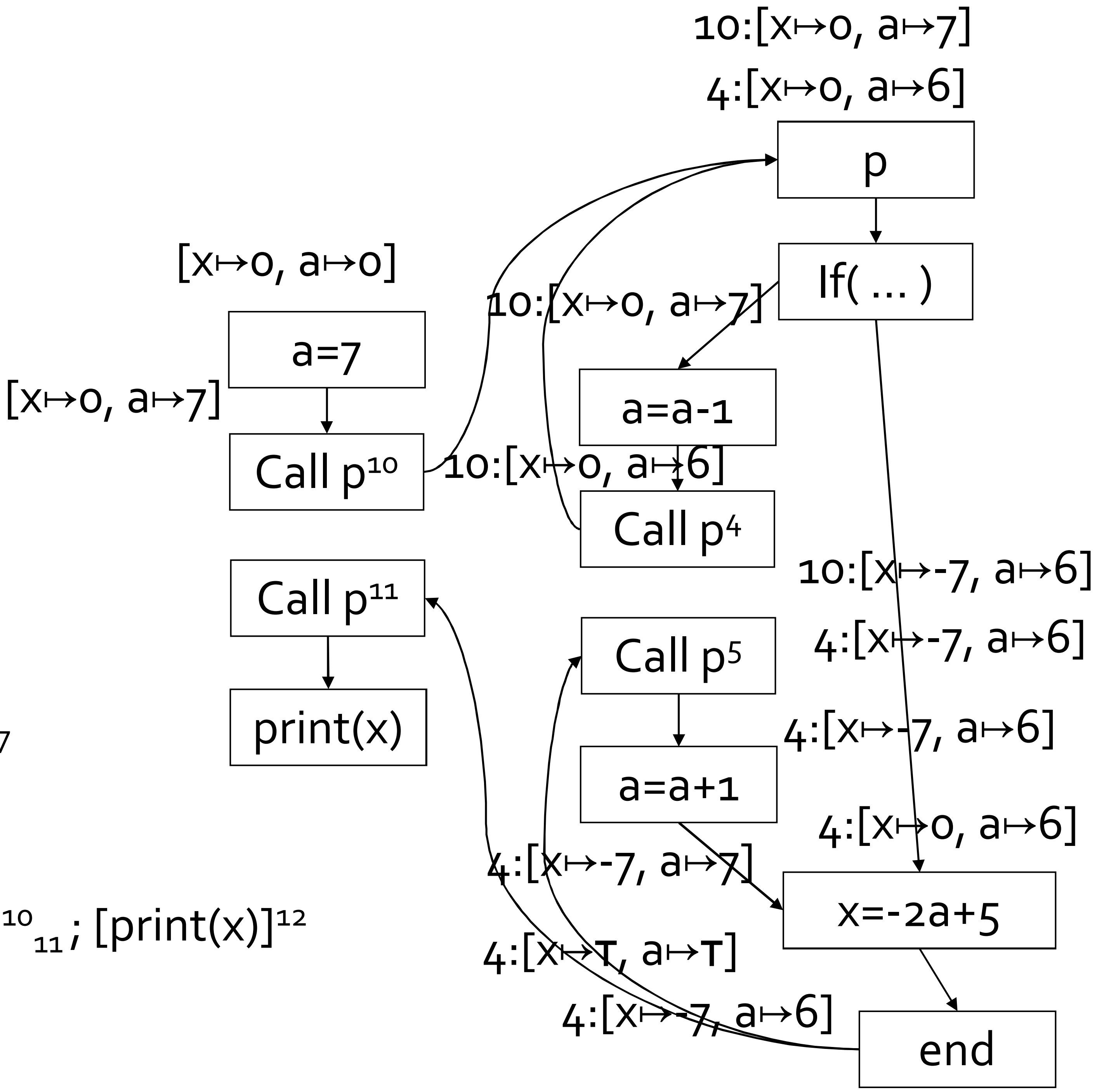
)

[x := -2* a + 5]^{7}

end^{8}

[a=7]^{9} ; [call p()]$^{10}_{11}$; [print(x)]^{12}

end^{13}

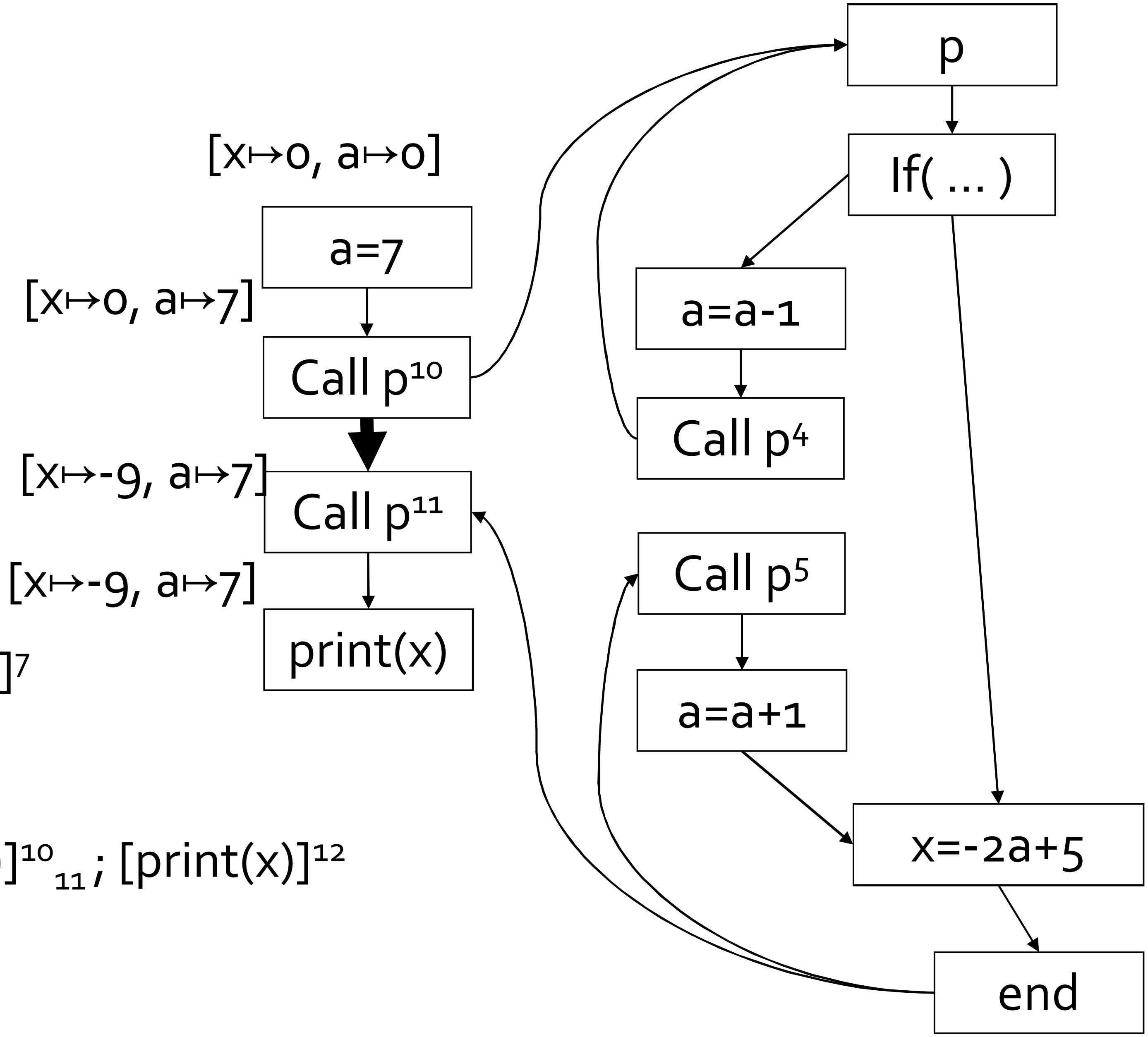


The Functional Approach

- The meaning of a procedure is mapping from states into states
- The abstract meaning of a procedure is function from an abstract state to abstract states

$\lambda e.[x \mapsto -2e(a)+5,\ a \mapsto e(a)]$

```
begin
 proc p() is$^1$
    if [b]$^2$  then (
     [a := a -1]$^3$
     [call p()]$^4_5$
     [a := a + 1]$^6$
   )
  [x := -2* a + 5]$^7$
end$^8$
[a=7]$^9$ ; [call p()]$^{10}_{11}$; [print(x)]$^{12}$
end
```



$[x \mapsto 0,\ a \mapsto 0]$

$[x \mapsto 0,\ a \mapsto 7]$

$[x \mapsto -9,\ a \mapsto 7]$

$[x \mapsto -9,\ a \mapsto 7]$

a=7

Call p^{10}

Call p^{11}

print(x)

p

If(…)

a=a-1

Call p^4

Call p^5

a=a+1

x=-2a+5

end

$\lambda e.[x \mapsto -2e(a)+5,\ a \mapsto e(a)]$

begin

proc p() is^1

if [b]^2 then (

[a := a -1]^3

[call p()]$^4{}_5$

[a := a + 1]^6

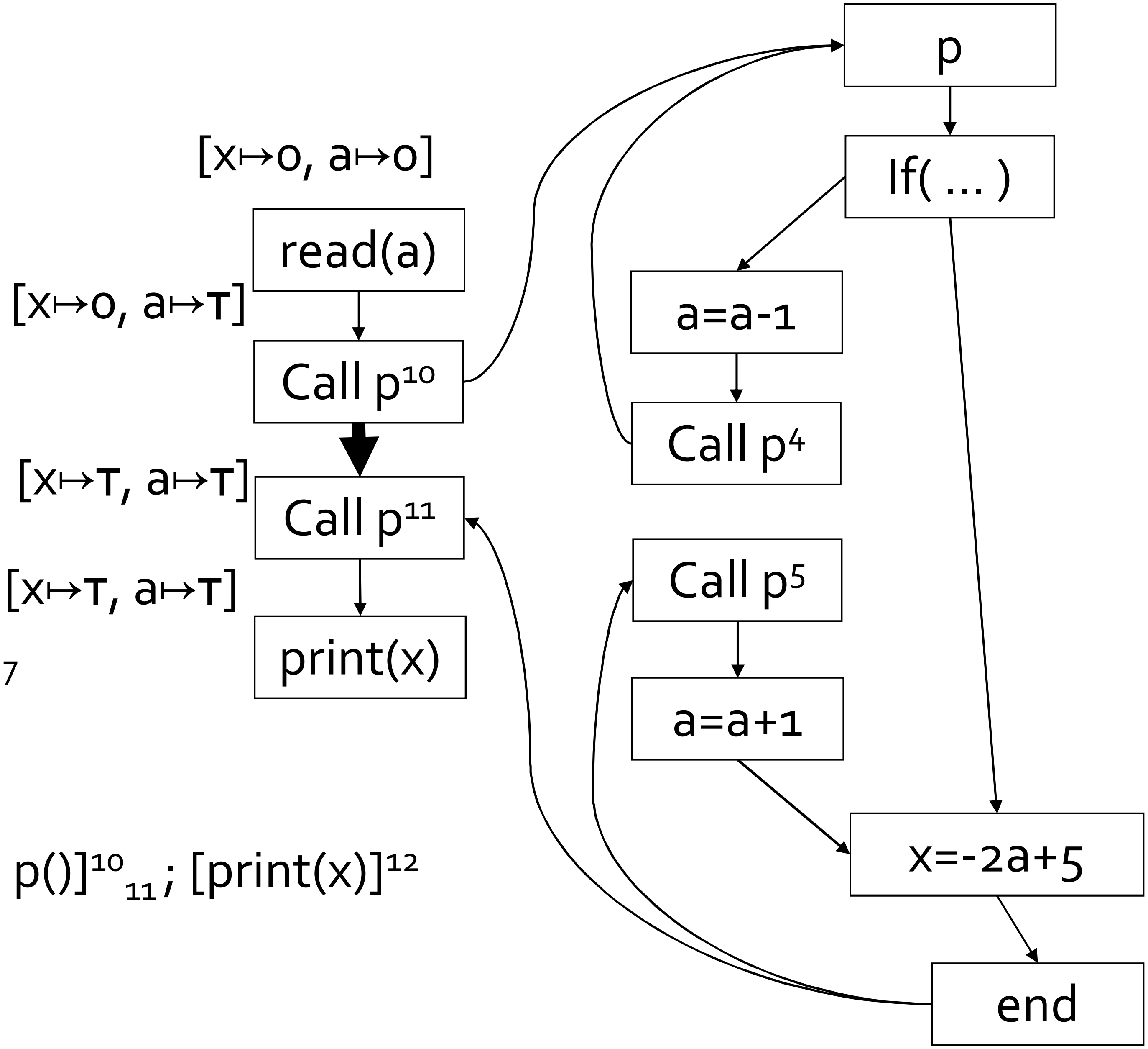
)

[x := -2* a + 5]^7

end^8

[read(a)]^9 ; [call p()]$^{10}{}_{11}$; [print(x)]^{12}

end



Functional Approach: Main Idea

- Iterate on the abstract domain of functions from L to L
- Two phase algorithm
  - Compute the dataflow solution at the exit of a procedure as a function of the initial values at the procedure entry (functional values)
  - Compute the dataflow values at every point using the functional values
- Computes JVP for distributive problems