– extended abstract –

Efficiently Exploring a Continuous Unknown Domain by an Ant-Inspired Process

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Abstract

An ant-inspired method is described for exploring a continuous unknown planar region by a group of robots having limited sensors and no explicit communication. Such a method has applications in robotics where a robot with limited sensing capabilities but with the ability to leave marks on the ground has to cover a closed region for purposes of cleaning a dirty floor, painting a wall, or demining a mine-field. We formalize the problem and suggest the *mark and cover* (MAC) rule of motion to solve it using temporary markers ("pheromones") as means of navigation and indirect communication. The convergence of the algorithm is proved, and its cover time is shown to be the asymptotically optimal $O(A/a)$, A being the total area and a - the area covered by the robot in a single step.

1 Introduction

Exploring unknown terrain is an important issue in robotics. The problem has already been investigated, and several methods have been suggested and implemented. Most of those methods, however, rely on sophisticated, expensive and fragile systems of sensors (e.g. odometers, infra-red sensors, ultrasound radar or GPS), and/or sophisticated mapping algorithms. In this paper we suggest a minimalist approach; we wish to achieve the goal of covering with a minimum of sensing and computing, even if some performance reduction is implied. Our algorithm uses a marking mechanism and mark-sensitive sensors. In mathematical terms, our aim is to find a local rule of motion that will cause the robot to follow a *space-covering curve*, such that every point of the given region should be in some prespecified r-neighborhood of the robot's trail.

Existing methods for graph search (e.g. BFS, DFS) cannot be used for our purpose since no vertices or edges exist in our setting; a robot can move to arbitrary points on the continuum, while the BFS and DFS algorithms assume a discrete and finite set of possible locations.

Some **related work** has already been done in various areas:

- **Robotic covering:** In previous work ([9],[8]) a discrete problem of graph-exploration was solved using markers. More recently, the problem of covering a tiled floor was addressed in two different ways: In [18] the dirt on the floor served as memory to help the robot's navigation, while in [20] and [21] a vanishing trace was used for that purpose. In [3] the issue of inter-robot communication is addressed in the context of various missions, among them *grazing* - i.e. visiting every point of a region for purposes of food-fetching. A reactive model of behavior is presented, and simulation shows that detailed communication does not contribute too much to the performance. In [2] many experimental works are presented for planetary exploration by autonmous robots. Heuristic navigation methods are given in [11] for path planning of an autonomous mobile cleaning robot, and in [13] for a robot exploration and mapping strategy. However no rigorous analysis is given in the above references. In [12] an algorithm is presented for exploration of an undersea terrain, using exact location sensors and internal mapping. Practical implementations of covering algorithms have been demonstrated in [22] and [14]. in [22] a set of robots is described that helps to clean a railway station, using magnetic lines on the floor as guidelines. This method seems to work well, but is limited to pre-mapped regions. In [14] a cooperation of a team of robots is created by an explicit level of inter-robot communication. Each robot can choose one of multiple possible behaviors, according to its specific conditions. In one of these behaviors the robot plays the role of a janitorial service man, by cleaning the dust around it.

- **SLNMs - Short-Lived Navigational Marks:** The idea of using marked trails for searching is inspired by the Greek myth of Ariadne, the grand-daughter of Zeus, who used a thread to escape from Dedalus's maze [17]. One way to mark a trail is by using odor, like the pheromones used by various insects. Experiments with an insect-inspired robot are reported in [15], where the robot has an odor marking and detection system. Another way of marking is by heating the floor, and several experiments were reported in [5]. Other researchers (e.g. [4]) used robot-placed landmarks to help path planning in the presence of dynamic obstacles. In the current work, however, we'll assume using traces so that our robots can be rather cheap and simple.

- **Off-line covering:** An off-line version of the problem as well as approximation algorithms for it are presented in [1]. The related (NP-hard) problem of optimal watchman route is to find the shortest path in a polygon such that every point of the polygon is visible from a point of the path. This problem is investigated in [7]. The goal there is to design a minimum-length path that will see each and every point in a given (i.e. known in advance) polygon.

The problem we address is different from those addressed in other works in that we confine our robots to (at most) local sensing, such that the group is a decentralized one, and adding or deleting robots does not introduce a need to change the protocol. Also we evaluate the performance of our algorithm by upper bounds on the cover time.

2 MAC (Continuous Ant Walk) - A deterministic, mark-based algorithm for the Continuous Covering Problem

Consider a group of robots with very limited sensors that cooperate to cover a continuous, bounded, connected region. The robots have no means to calculate their locations or to communicate with each other directly. Their only means of communication are the marks they draw on the floor (e.g. heat or smell), which enable them to recognize a location as "already visited". The intensity of the mark enables the robot to distinguish recently visited from previously visited points. The task of the robot(s) is to cover the whole region using the ability to sense previously marked points in its near neighborhood. If an odometric sensor of high precision is available, visited locations can be recorded in the robot's memory rather than marked on the floor. Our assumed robot covers a disc of radius r around its center, and then moves a distance of $r' > r$ in an arbitrary direction. Thus, the robot visits a sequence of points $Z = z_1, z_2, \ldots, z_t$ up to time t. Once at point z_i, the robot may leave there a mark, the intensity of which will be denoted by $s(z_i)$. We also assume that each (continuous) point z in R (the region to be explored) is dynamically assigned a value $\sigma(z)$ which is the average of the marked points in its neighborhood, i.e. $\sigma(z) = \max_{u \in B_r(z)} s(u)$, where $B_r(z)$ is a disc of radius r centered at z. The goal of our robot is to cover the region, i.e. to follow the sequence Z such that $|z_{i+1} - z_i| = r$ for all i and R is covered, i.e. $R = \bigcup_{t=0}^{T} B_r(z(t))$. We now consider a reactive rule of motion such that a set of robots that obey this rule will eventually cover the region in the sense defined above. The algorithm is called MAC for "Mark And Cover", and is formalized as follows:

```
/* MAC - Mark And Cover */
/* r = covering radius, r_s = sensing radius */
Rule MAC(z: current location)
A) cover B_r(z);
B) if there is a point z' ∈ B_{r_s}(z) such that z' is yet uncovered
           (i.e. there is no marked point in B_r(z'))
      then go to z', while marking the line z → z';
      else
C)            if there is a point z' ∈ B_{r_s}(z) such that the line z' → z is marked
                /* line direction is detected by intensity */
                then            /* backtrack */
D)                          go to z';
E)            else STOP (R is covered).
end MAC.
```

See Figure 1 for an example.

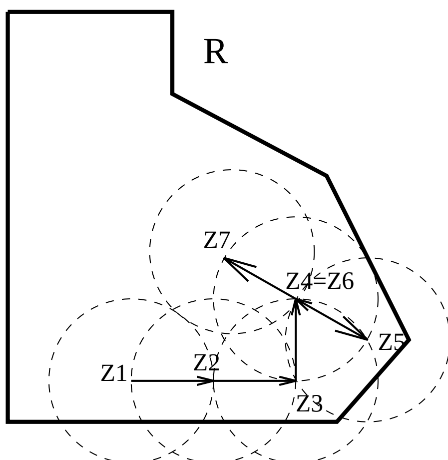

Figure 1: Seven steps of the MAC procedure (bold arrows), using the "stay straight" heuristic to resolve ties. Note the step back from Z_5 to Z_4, due to the total coverage of the r-circle around Z_5.

The intuition behind this algorithm is that in a "forward" motion (steps B and C) the robot finds a new, uncovered area and covers it. In the process, it marks the path from the previous point to the new point. The direction of this mark can later be recognized since, if the line $z \rightarrow z'$ has been marked, the intensity at z' will always be stronger than in z. Once no new point exists in the near neighborhood (i.e. in $B_{r_s}(z)$), the robot "backtracks" using the marked path (step E), or, if no uncovered point nor backward mark exist around z, the robot finally stops (step F), being aware that its mission has been completed.

2.1 Performance Analysis of the MAC rule

How efficient is this process ? In order to evaluate ther cover time, we first prove an upper bound on the number of points in a run of the algorithm:

Lemma 1 *If a region R has area A and perimeter P, then the graph G(V, E) whose vertices are the visited points and whose edges are the lines marked by the* MAC *algorithm has at most* $2\sqrt{3}\frac{A+rP+\pi r^2}{r^2}$ *edges.*

The proof (as well as the other proofs) is deferred to the Appendix. Next we show that MAC indeed covers the region:

Lemma 2 *The set* $z_1, z_2, \ldots, z_T$ *of vertices defined by the* MAC *algorithm covers the region R, i.e.,*

$$R = \bigcup_{i=1}^{T} B_r(z_i).$$

Now, as the edge-set of G is bounded and each edge is traversed twice, one can show that T, the cover time, is bounded, too:

Theorem 1 *The time needed to cover a region R with area A and perimeter P by the* MAC *algorithm, denoted* T^{MAC}*, is bounded as follows:*

$$\frac{2A}{(\sqrt{3}/2 + 2\pi/3)r^2} - 2 \leq T^{\mathsf{MAC}} \leq 2\frac{A + rP + r^2}{r^2}.$$

3 Discussion

The problem of continuous covering has various implications for both theory and practice. The algorithm discussed in this paper can serve as an inspiration for further research in several directions, some of which are mentioned below.

Cooperation: An important property of the MAC process is that it enables many robots to cooperate, all oberying the same rule as defined above. This way each robot contributes implicitely to a global plan, without planning by itself. This makes our system *modular* in the sense that one does not have to rearrange the team in order to add or remove members from it - just add the new robot and it will join the effort; there is no need for additional hardware or communication protocols. As can be seen, collisions are not a problem under the MAC protocol (assuming the hardware is not vulnerable to bumps), since a robot can consider his fellow as a "wall", although some performance-degradation is likely to occur if too many of the robots are cluttered in a small neighborhood. Also, there is no danger of deadlock since a robot never "waits" - it always has a possible step to do (unless it got mechanically stuck). Recall that according to the rule, a marked line never intersects neither itself nor other robot's thread, hence several robots will perform at least as good as one. See Figures 2, 3, for examples of multiagent simulations.

Actual performance: Our analysis gives an upper bound, but the exact time of covering depends on the number of points generated in the process, which in turn depends on the starting point(s), the shpae of the region, and the heuristic used in step C of the algorithm to choose the next point among several uncovered points in $B_{r_s}(z)$. In the final version of the paper, simulation examples will be shown that demonstrate those dependencies for various numbers of participating robots.

Imprecise positioning: E.g., in our MAC algorithm we assume that the robot is able to either recognize a previously-layed trace, or recall a previous location from its memory, based on some

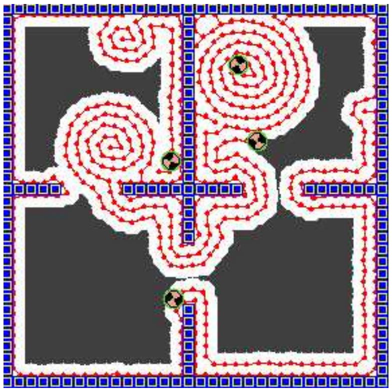

Figure 2: Four MACers in four rooms - preliminary stage.

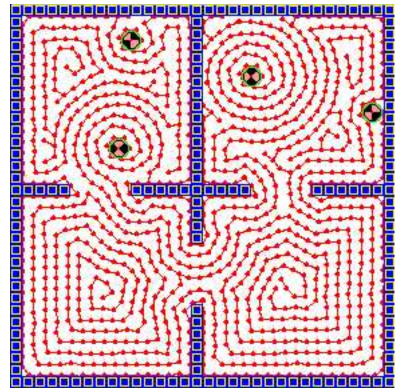

Figure 3: Four MACers in four rooms - final stage. Note that their traces are four sparate trees.

positioning mechanism (e.g. odometric or GPS). Those positioning systems are both expensive and are prone to errors. It seems, however, that in order to guarantee coverage we only need our positioning system to be unique, but not necessarily precise; i.e. if the position is z and our systems returns $P(z)$ as a position, we only need to have $P(z) \neq P(z')$ if $z \neq z'$. The open question here is to define the precise conditions on the positioning function $P(z)$ which allow a MAC-like algorithm to cover the region.

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Appendix - Proofs

Proof of Lemma 1 :
According to the algorithm, a point can never become a vertex if it is less than r units apart from any existing marked point. Hence if we draw a rhombus made of two $(30°, 120°, 30°)$ triangles around each edge in E, no two rhombuses will intersect; otherwise it is implied that a third visited point exists in the r-vicinity of a vertex of the rhombus. See Figure 4 for an example. The area of

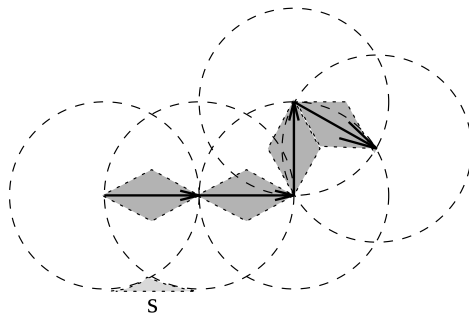

Figure 4: Four steps of the MAC procedure (bold arrows), the covered area (dashed line) and the unique area around each edge (gray rhombus). Note that only the circles around the stopping points are covered, not around the edges; for example the area s is yet uncovered in this case. Also note that an angle between two edges in the path cannot be smaller than 60^o, hence all rhombuses are disjoint.

such a unique quadrilateral region is $r^2/2\sqrt{3}$, so if we had a region with no boundary (e.g. a torus or a sphere), we would have that $|E| \leq \frac{2\sqrt{3}A}{r^2}$. However in most realistic situations there are some additional edges near the boundary of R; the number of such edges can be bounded from above by assuming that the region R has been expanded by a strip of width r. The area of such a strip does not exceed $S = r(P + \pi r)$, Hence we get $|E| \leq 2\sqrt{3}\frac{A+rP+\pi r^2}{r^2}$. □

Proof of Lemma 2 :
Assume, on the contrary, that upon termination there still exist uncovered points in R. Consider z_c, the uncovered point which is closest to a point of the set V, say to z_i. Now the distance $|z_c - z_i|$ cannot be less or equal to r_s, or otherwise no backtracking from z_i was possible (i.e. z_c should have been detected during one of the visits to z_i). Hence $|z_c - z_i| > r_s$; however only an r-neighborhood of z_i is covered, so (since $r < r_s$) there must be an uncovered point in R which is closer to the set V than z_c, in cotradiction to our assumption. □

Proof of Theorem 1 :
The robot goes over an edge per unit of time. According to the algorithm, each edge is traversed exactly twice, and from Lemma 1 we know that the number of edges is bounded above by $2\frac{A+rP+r^2}{r^2}$. Hence, the upper bound results. The lower bound comes from the fact that each step adds at most $(\sqrt{3}/2 + 2\pi/3)r^2$ to the covered area. Hence there must be at least $A/((\sqrt{3}/2 + 2\pi/3)r^2)$ vertices in G before covering, and since the graph $G(V, E)$ is connected, there are at least $|V| - 1$ edges in E, each of which is traversed twice. □