Andrzej Mostowski
November 1, 1913 - August 22, 1975
The Legacy of Andrzej Mostowski

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The teachers ...

Kasimierz Kuratowski
1896-1980

Alfred Tarski
1901-1983
... and their joint books

- K. Kuratowski and A. Mostowski
  *Set Theory*

- A. Tarski, A. Mostowski and R. Robinson
  *Undecidable Theories*
  Amsterdam 1959
An anomaly of the ordered pair

In the first edition of *Set Theory* the following exercise can be found:

Show that there are no non-empty sets $A$ and $B$ such that $A \cap B = A \times B$!

- If the *cartesian product* were defined by an *abstraction principle* in the Fregian sense, indeed no such sets could exist.
- However, with the definition of the *ordered pair* (by Kuratowski, or any other definition), such sets can be constructed by transfinite induction.

**Conclusion:**
Defining the ordered pair always has **side effects**.
Translation schemes and interpretability

In the book *Undecidable Theories* undecidability is often proved by reductions to arithmetic via interpretations. These ideas are also used and further developed in


Other early influences in Poland

From left to right:

W. Sierpinski, 1882-1969, S. Mazurkiewicz, 1888-1945,
J. Lukasiewicz, 1878-1956, S. Lesniewski, 1886-1939,
A. Lindenbaum, 1904-1941(?)
Vienna 1936-1937

The period in Vienna left a deep impact on A. Mostowski.

His interest in Set Theory and Metamathematics has its origin here.

A. Mostowski would both do research and expository work in these fields.

Kurt Gödel, 1906-1978
Zürich 1937-1938

H. Hopf, 1894-1971
G. Polya 1887-1985
P. Bernays, 1888-1977
... and even Physics.

Wolfgang Pauli, 1900-1958
The true humanist scholar of the age of reason

A. Mostowski’s introduction to science and scholarship was his apprenticeship with the true giants of their time.

They exemplified true universal humanist principles (as I understand them):

- Be a humble and responsible servant to science, and to humanity!
- Always respect your fellow human, be it a student, a colleague, or a layperson!
- Don’t abuse the privilege society has entrusted on you!
- Be always ready to learn from others and from your own mistakes!
- Look at human knowledge as a whole!

And so A. Mostowski adopted these principles from his teachers and tried to pass them on to the next generation.
Macabre Memories of Zürich

This relief stands next to the entrance of the police department in Zürich, where A. Mostowski had to register as a foreign resident every few months.

How appropriate he thought: Three decapitated persons, holding their heads in their hands, decorate the entrance to the lion’s den.

Well, they represent the martyrs Felix and Regula, and their servant, the city Saints of Zürich, early Christians who were decapitated by the Romans, and have no connections to the office in the building.
Elitism vs Professionalism and High Standards

- Elitims today is based on exclusion rather than professional authority.

- Mathematics, and science in general, in Mostowski’s time, was elitist, but elitisms was based, not on exclusion but on professional standards.

- In contrast to today, professional standards were very high, and there was a consensus about it.

- Students had to internalize these standars, and had to know by themselves whether they already had reached that level.

A. Mostowski treated his students as equal humans, but instilled in them the respect for professional standards.
The Legacy, I: Two reports
The Legacy, I: Foundational studies in 1955

A. Mostowski was mostly concerned with foundational studies:


The present stage of investigations on the foundation of mathematics opened at the time when the theory of sets was introduced. The abstractness of that theory and its departure from the traditional stock of notions which are accessible to experience, as well as the possibility of applying many of its results to concrete classical problems, made it necessary to analyze its epistemological foundations. This necessity became all the more urgent at the moment when antinomies were discovered. However, there is no doubt that the problem of establishing the foundations of the theory of sets would have been formulated and discussed even if no antinomy had appeared in the set theory.
Thirty Years of Foundational Studies 1966

In 1964 he ends his lecture


as follows:

We stop here our presentation of what we consider as the most important results in the recent development of logic and the foundation of mathematics. The rate of development of these domains is presently so rapid that many new excellent results will certainly appear before these lectures will come to the hands of prospective readers. Let us hope that these new results will not only bring new interesting insights into the details but also allow us to form a sound judgement about outstanding problems in the philosophy of mathematics which have been waiting so long for a final solution.
The Fragmentation of Mathematical Logic

The last monograph to cover all of set theory, recursion theory and model theory up to current research level was J. Shoenfield’s Mathematical Logic published first in 1967.

By now the unified view of Mathematical Logic remains the privilege of the lucky few. For most of us Mathematical Logic may mean one of the following:

- A very specialized highly technical branch of pure mathematics: Model Theory, Set Theory, Computability Theory, with each of them fragmented into subdisciplines.

- An highly sophisticated engineering discipline, which uses logical tools to model situations in computer science, computer engineering and artificial intelligence with an eye on commercial applications.

- A branch of highly technical philosophy, with applications to epistemology, ethics, and the foundations of specialized reasoning.

I am afraid A. Mostowski would have found all these developments both intriguing and deplorable.
The Legacy, II: Scientific Internationalisms

A. Mostowski was totally dedicated to Science without borders, and in the time of the Cold War, a main bridge builder for scientists on both sides of the Iron Curtain.

1959 Main initiator and organizer of the conference Infinitistic Methods in Warsaw, under the auspices of the International Mathematical Union and the Mathematical Institute of the Polish Academy of Sciences.

1964–1968 Vice president of the Section Logic, Methodology and Philosophy of Sciences, of the International Union of the History and Philosophy of Science.

1971–1975 President of the above.

1972 Co-initiator of the International Banach Center in Warsaw and director of the Logic Year at this center.

The 1959 and 1972 events in Warsaw were landmark events and left their impact beyond expectation.
Infinitistic Methods 1959
Infinitistic Methods 1959, Participants and Organizers

**CSSR:** L. Rieger (Prague)  
**France:** P. Février and D. Lacombe * (Paris). R. Fraïssé (Alger)  
**Germany:** G. Asser (Berlin), P. Lorenzen (Hamburg)  
**GB:** G. Kreisel (Reading)  
**Hungary:** L. Kalm’ar (Szeged), R. Péter and J. Surányi (Budapest)  
**Israel:** A. Robinson (Jerusalem)  
**Roumania:** E. Beth and A. Heyting (Amsterdam)  
**Switzerland:** P. Bernays, E. Specker and G. Müller (Zürich)  
**USA:** J.W. Addison * (Ann Arbor), W. Boone (Urbana), S.C. Kleene (Madison), S. MacLane and D. Scott (Chicago), R. Montague (Los Angeles), C. Spector (Columbus), L. Henkin, A. Tarski * and R. L. Vaught (Berkeley)  
**USSR:** J.T. Medvedev *, P.S. Novikov and A.J. Sragović (Moscow).

**Bold face:** Authors, *: Speakers only.  
**Colored:** Organizing committee

Authors who did not attend:  
R. MacDowell (Yellow Springs, USA), A.S. Essénine-Volpine (Moscow, USSR)
International long term visitors in Warsaw 1959-1974

We quote from S. Krajewski and M. Srebrny (2005)

A longer visit was thus, inter alia, received by:

J.W. Addison, M. Benda, M. Boffa, M. Dickmann,
E. Frederikson, D. Giorgetta, P. Hinman, F.V. Jensen,
R. Kowalski, E.G.K. Lopez-Escobar, M. Machover,
J.A. Makowsky, K.C. Ng, K. Prikry, L. Rieger,
H. Sayeki, Y. Suzuki, B.F. Wells, G. Wilmers,
... P. Hajek and P. Vopenka ...

and many more ... H. van Barendregt, e.g.
The Legacy, III: Read and Disseminate

A true scholar reads and discusses other people’s work.

A. Mostowski excelled in both. He

- followed the developments of foundational studies,
- discussed important papers in his seminars, and
- wrote expository and explanatory papers to spread progress made by others.

Most remarkably on incompleteness results, undecidability and forcing.
The Legacy, IV: His most influential papers

A. Mostowski lived before the Publish or Perish craze. He nevertheless has an H-index of approximately 25.
His five most quoted works are:


[101 citations] Über die Unabhängigkeit des Wohlordnungssatzes vom Ordnungsprinzip, FM 1939.

Work on Models of Arithmetic, Definability, and Hierarchies

The citation numbers do not reflect

- The impact of his expository writing,
- to what extent his publications were READ, and
- to what extent his papers triggered new lines of investigations.

In particular, this concerns his work on models of (also second order) arithmetic, definability of sets of natural numbers, and hierarchy theory.
Andrzej Ehrenfeucht

1932-

Mostowski’s most visible and prolific Ph.D. student.

- Andrzej Ehrenfeucht, On Theories Categorical in Power, FM 1957

- Andrzej Ehrenfeucht, An Application of Games to the Completness Problem for Formalized Theories, FM 1960

Ehrenfeucht’s paper with Mostowski, and the two papers cited on this slide mark the beginning of Classification Theory.

The work is further developed by R. Vaught and M. Morley, and then S. Shelah entered the world of model theory.

The rest is history!
Per (Pelle) Lindström

1936-2009

- Per Lindström, *First Order Predicate Logic with Generalized Quantifiers*, Theoria, 1966,

- Per Lindström, *On extensions of elementary logic*, Theoria, 1969

It was A. Mostowski’s paper on generalized quantifiers which inspired P. Lindström.

Lindström told me in 1980 that he really was looking for a new non-trivial application of the Ehrenfeucht-Fraïssé Theorem, which he had independently (re-)discovered by himself.

The Axiom of Choice

Following a suggestion of A. Lindenbaum to

extend and systematize the method

of A. Fränkel's proof of independence of the Axiom of choice, A. Mostowski had three fundamental papers on the Axiom of Choice:

- (with A. Lindenbaum)  
  Über die Unabhängigkeit des Auswahlaxioms und einger seiner Folgerungen (1938)

- Über die Unabhängigkeit des Wohlordnungssatzes vom Ordnungsprinzip, FM 1939.

- Axiom of choice for finite sets, JSL 1948.
Nothing can shatter mathematical truth ...

In 1973, while I was in Warsaw, A. Mostowski came disturbed from a Ph.D. exam of a very gifted student.

How can it be, that the student failed so terribly, not even able to formulate the Fundamental Theorem of Algebra?

We suggested that he might have had a blackout.

Blackout ??!!???. If you wake me up in the middle of deep sleep, point a gun at me and ask me what is the Fundamental Theorem of Algebra, I would calmly tell you, and also prove it.
Thomas Mann and Mathematics

Thomas Mann's wife Katya was the daughter of Alfred Pringsheim, a mathematician. She initially studied mathematics herself.

Thomas Mann was A. Mostowski’s favorite author, and he read him in German. For Thomas Mann, and maybe also for A. Mostowski, mathematics was the spiritual equivalent to passion.

I tell them that if they will occupy themselves with the study of mathematics they will find in it the best remedy against the lusters of the flesh.

*Thomas Mann (The Magic Mountain)*
Thank you Panie Profesorze,

thank you Professor Mostowski,

for all you have taught us!

... and thank YOU those who are present for your attention!
Credits of photographs

- P. Lindström’s picture was taken from the announcement of the University of Gothenburg of the *The Lindström Lectures*
  http://www.flov.gu.se/english/research/logic/lindstrom-lectures/
  The picture is owned by P. Lindström’s widow.

- A. Ehrenfeucht’s picture was taken from Computer Science, University of Colorado Boulder
  https://www.colorado.edu/cs/users/andrzej

All other photographs are in the public domain.