22.11.2010

תאוריה של מ״ן - הרצאה שלישית.

Want to proof:

המושג של גרף המילטוני אינו גדיר בלוגיקה מסדר ראשון.

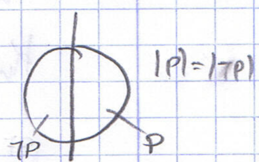
Hamiltonicity is not FOL definable (even with order).

First proof idea:

Have to find graphs $H_n \equiv_n^{FOL} G_n$ such that

H_n is hamiltonian, G_n is not Hamiltonian.

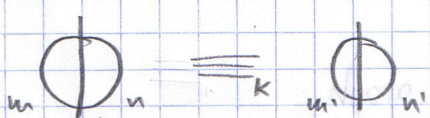
Second proof idea:



הכוונה לומר שמספר האלמנטים ב-P ומ-¬P זהה (הם מאותו גודל).

lemma:

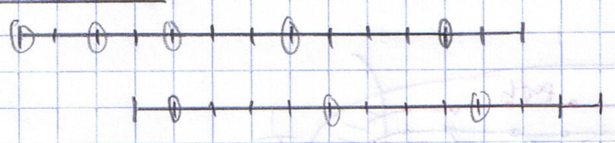
Equi-cardinality is not FOL-definable.



if $m, n, m', n' \geq k$ we have this equivalence.

(ההוכחה נעשה באהרנפויכט — משחק שלמדנו).

With order:

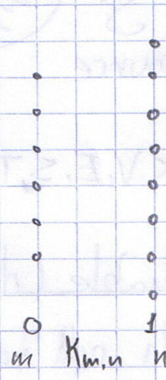


lemma:

Make words into graphs.

$$\overset{P}{\underset{m}{000000}}\ \overset{\neg P}{\underset{n}{11111111}}$$

$E(x,y) \iff P(x) \wedge \neg P(y)$



מכל '0' יש קשת לכל '1' והפוך - כאשר הקשתות הן מכוונות.

מתי $K_{m,n}$ הינו המילטוני?

$K_{m,n}$ hamiltonian $\iff m=n$.

Take translation scheme

$\Phi : \langle x=x, P(x) \wedge \neg P(y), x<y \rangle$

Φ^*: words over $\{0,1\}$ $\rightsquigarrow$ Bipartite graphs. (1993 19)

Assume Θ_{HAM} defines hamiltonian graphs $\Phi^\#(\Theta_{HAM})$.

$$\Phi^*(w) = K_{m,n} \models \Theta_{HAM} \iff w \models \Phi^\#(\Theta_{HAM}) \iff$$

number of '0' in w equals number of '1' in w.

Eulerian is not FOL definable.

K_n clique with n-vertices.

K_n is Eulerian $\iff$ n is odd.

Now assume Θ_{euler} defines Eulerian graphs.

Φ: $\langle x = x, x \neq y \rangle$

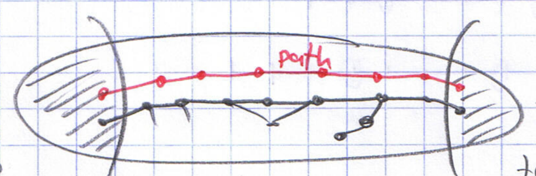
Φ^*: Set $\rightsquigarrow$ Complete graph.

$\Phi^\#(\Theta_{euler})$.

Reachability.

$F \subset V^2$
$S \subset V$
$T \subset V$

S source



T target

Graph $\langle V, E, S, T \rangle$

D-reachable (deterministic).

On path all vertices have degree 2.

Reachable (Non-deterministic \ ND-reachable).

There exists a path from some point in S to some point in T.

How do we check this?

D-reachable.

Deterministic log space.

ND-reachable.

Non-deterministic log space.

Path Systems.

V - Verticies

$E \subseteq V^3$

$S, T \subseteq V$

$S \subseteq PTC$ (path transitic closure)

סגור לכללים של הם'

$(x,y) \in PTC$
and there is
z with $E(x,y,z)$
$\Rightarrow z \in PTC$.

checkable in polynomial time.

ספרי לוגיקה של מערכת ___ ם' מוכיחות:

D-Log space	L
ND-Logspace	NL
polynomial time	P
Non-polynomial time	NP

$$L \underset{(?)}{\subseteq} NL \underset{(?)}{\subseteq} P \underset{(?)}{\subseteq} NP$$

Definition:

(Boolean queries)

Q - some query.

S - set of queries.

Q is complete for S if for every $Q_0 \in S$ there is a FOL translation scheme Φ such that for every instance on order structures of (suitable for Q_0)

$$\Phi^*(A) \models Q \iff A \models Q_0 = I^\#(Q)$$

Theorem:

1. D-reachability is complete for logspace on ordered structures.
2. ND-reachability is complete for Nlogspace on ordered structures.
3. Path systems is complete for Polynomial time on ordered structures.