The Cues in “ Dependent Multiple Cue Integration for Robust Tracking” are Independent

Ido Leichter, Member, IEEE, Michael Lindenbaum, Member, IEEE, and Ehud Rivlin, Member, IEEE

Abstract—A methodology for integrating multiple cues for tracking was proposed in several papers. These papers claim that, unlike other methodologies, conditional independence of the cues is not assumed. This brief communication refutes this claim and points out other major problems in the methodology.

Index Terms—Bayesian tracking, multiple cue integration.

1 CONDITIONAL INDEPENDENCE OF CUES
A methodology for integrating multiple cues for tracking has been proposed several times in [1]–[4]. In all these papers, it is claimed that this methodology does not assume that the cues are conditionally independent (conditional on the tracked state). Each of these papers consequently claims that this methodology is better than those that do assume conditional independence of the cues. However, as is shown in what follows, the methodology does assume conditional independence of all the cues as well.

In the methodology, there are \( F \) state vectors \( x_1, \ldots, x_F \) and \( F \) corresponding measurements \( z_1, \ldots, z_F \), all related to a single frame in the image sequence. \( X_{i:j} \) denotes the set of state vectors \( x_i, \ldots, x_j \), and \( Z_{i:j} \) denotes the set of measurements \( z_i, \ldots, z_j \). The methodology assumes (e.g., (1) in [1]) that these random variables are ordered such that “the dependence is only in one direction” (quote from [1]–[4])

\[
p(z_i|X_{1:F}, Z_{1:i-1}, Z_{i+1:F}) = p(z_i|X_{1:i}, Z_{1:i-1}), \\
p(x_i|Z_{1:F}, X_{1:i-1}, X_{i+1:F}) = p(x_i|Z_{1:i}, X_{1:i-1}).
\]

At first sight it may appear that this setting is more general than that where the measurements are conditionally independent. However, a short derivation (see Appendix) reveals that this setting implies that each measurement is independent of all others given the state vectors (in fact, given only the “preceding” state vectors):

\[
p(z_i|X_{1:F}, Z_{1:i-1}, Z_{i+1:F}) = p(z_i|X_{1:i}).
\]

This shows that the first equation in (1) contains redundant dependencies. Moreover, from (2) it follows immediately that (see Appendix)

\[
p(Z_{1:F}|X_{1:F}) = \prod_{i=1}^{F} p(z_i|X_{1:i}).
\]

That is, all the measurements are assumed to be conditionally independent — similarly to other papers that rely on this assumption.

In the general outline of the algorithm provided in Sec. 3.3 of [1], it appears that no joint likelihoods of two or more measurements \( z_i \) are involved. This is another indication that the measurements are treated as being conditionally independent.

2 OTHER PROBLEMS
Apart from the above flaw in the definition of the methodology’s setting, there is another problem in the setting itself. Symmetrically to (2), each state vector is assumed to be conditionally independent of all others given measurements:

\[
p(x_i|Z_{1:F}, X_{1:i-1}, X_{i+1:F}) = p(x_i|Z_{1:i}).
\]

This is a very strict assumption that is usually unsatisfied. For instance, the example application of the methodology in [1] included the state vectors of the bounding box and the object contour. Obviously, these two state vectors are not independent, even when conditioned on the related, typically noisy measurements.

Finally, the methodology involves the approximation of the functions \( p_i^t = p(x_i^t|X_{1:i-1}, \ldots) \) (Eq. (4) in [1]). No justification for this approximation was provided. In particular, for \( i \geq 2 \), each function \( p_i^t \) is a function of multiple state vectors \( x_1^t, \ldots, x_i^t \). However, the approximation of each of the functions \( p_i^t \) is a function (or a sampling thereof) of \( x_i^t \) alone. This raises doubts about the appropriateness of the approximation, and thus about the correctness of the overall implementation of the Bayesian filtering under the proposed model as well.

APPENDIX
First, we prove that (1) implies (2) and (4).

Applying Bayes’ rule on the right-hand side of the first equation in (1) yields

\[
p(z_i|X_{1:F}, Z_{1:i-1}, Z_{i+1:F}) = \frac{p(Z_{1:i-1}|z_i, X_{1:i}) p(z_i|X_{1:i})}{p(Z_{i+1:F}|X_{1:F})}.
\]
From the product rule for probabilities, the right-hand side equals
\[
\frac{\prod_{j=1}^{i-1} p(z_j|z_{1:j-1}, x_i, x_{1:i}) p(z_i | x_{1:i})}{p(z_{1:i-1} | x_{1:i})}, \tag{6}
\]
which, from the first equation in (1), equals
\[
\frac{\prod_{j=1}^{i-1} p(z_j|z_{1:j-1}, x_{1:i}) p(z_i | x_{1:i})}{p(z_{1:i-1} | x_{1:i})}. \tag{7}
\]
Based again on the product rule for probabilities, the product of \(i-1\) terms in the numerator equals the denominator. Thus, the above equals \(p(z_i | x_{1:i})\), which proves (2). Eq. (4) follows symmetrically from the second equation in (1).

Next, we prove (3).

From the product rule for probabilities
\[
p(z_{1:F}|x_{1:F}) = \prod_{i=1}^{F} p(z_i|z_{1:i-1} x_{1:F}). \tag{8}
\]
Applying (2) on the probability density functions in the right-hand side yields (3). 

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\textbf{REFERENCES}