Volume 21 (2002), number 2 pp. 137–148

COMPUTER GRAPHICS *forum*

Universal Rendering Sequences for Transparent Vertex Caching of Progressive Meshes

A. Bogomjakov and C. Gotsman

Computer Science Department, Technion—Israel Institute of Technology, Haifa 32000, Israel
alexb@cs.technion.ac.il, gotsman@cs.technion.ac.il

Abstract
We present methods to generate rendering sequences for triangle meshes which preserve mesh locality as much as possible. This is useful for maximizing vertex reuse when rendering the mesh using a FIFO vertex buffer, such as those available in modern 3D graphics hardware. The sequences are universal in the sense that they perform well for all sizes of vertex buffers, and generalize to progressive meshes. This has been verified experimentally.

Keywords: Rendering sequence, Transparent vertex caching, Triangle strips, Progressive meshes, Space-filling curves

ACM CSS: I.3.2 Graphics Systems—*Distributed/network graphics*; E.4 Coding and Information Theory—*Data compaction and compression*

1. Introduction and Previous Work

One of the trends in contemporary computer graphics applications is the use of more and more polygons in order to increase image realism. This trend is partially fuelled by recent developments in graphics hardware, particularly the appearance of the Graphics Processing Unit (GPU) on low-end display adaptors. This means that not only the scan conversion is done by the graphics adaptor, but also the three-dimensional (3D) geometric projections and shading operations. Hence, processing of the scene geometry is no longer a bottleneck as it was in the past.

In order to process geometry as rapidly as possible, the GPUs (e.g. the NVidia GeForce 1 and 2) maintain a FIFO vertex cache of fixed size, through which processed vertices travel. While rendering a typical 3D mesh on a per-triangle basis, each vertex may have to be processed more than once, since each vertex participates in six triangles on average. Processing a cached vertex can be significantly faster than processing an uncached vertex. Thus, to maximize benefit from the cache, the mesh triangles, hence also the associated vertices, must be rendered in an order which somehow preserves locality. This ordering of the triangles is called the mesh rendering sequence. A good rendering sequence will minimize the average number of cache misses per triangle, also known as the Average Cache Miss Ratio (ACMR) which for a k-entry vertex cache is defined as:

$$\mathrm{ACMR}(k) = \frac{\text{number of cache misses during rendering}}{\text{number of triangles}}.$$

This value can be anywhere between 0.5 and 3.0, since the number of triangles in a typical 3D mesh is approximately double the number of vertices and at least one cache miss is incurred for each vertex. Figure 1 shows a mesh and a possible rendering sequence. Note that the sequence is not necessarily continuous, i.e. triangles adjacent in the rendering sequence are not necessarily adjacent in the mesh.

3D meshes are usually specified, for example, in the ASCII VRML 2.0 file format, as a list of triangles in an arbitrary order, where each triangle is specified as three indices into a list of vertices. Simple-minded renderers send these triangles to the graphics pipeline in the order specified in the file, hence achieving mediocre rendering performance. More sophisticated renderers use the triangle strips technique, which renders the triangle mesh using a FIFO vertex cache of size 2, which is a standard part of

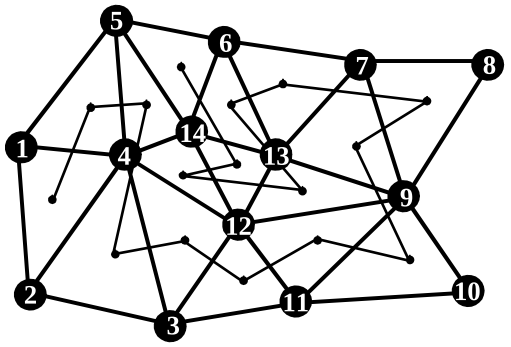


Figure 1: *A triangle mesh containing* 14 *vertices and* 16 *triangles and a possible rendering sequence. Triangles are numbered in rendering sequence order. The rendering sequence is* ⟨1, 2, 4⟩, ⟨1, 4, 5⟩, ⟨4, 5, 14⟩, ⟨2, 3, 4⟩, ⟨3, 4, 12⟩, ⟨3, 11, 12⟩, ⟨9, 11, 12⟩, ⟨9, 10, 11⟩, ⟨7, 9, 13⟩, ⟨7, 8, 9⟩, ⟨6, 7, 13⟩, ⟨6, 13, 14⟩, ⟨9, 12, 13⟩, ⟨4, 12, 14⟩, ⟨12, 13, 14⟩, ⟨5, 6, 14⟩. *There are* 25 *cache misses (*ACMR = 1.56*) for a cache of size* 4 *and* 14 *cache misses (the minimum,* ACMR = 0.88*) for a cache of size* 16*.*

legacy 3D hardware. Algorithms to generate triangle strips were described by Akeley *et al.* [1], Evans *et al.* [2], Xiang *et al.* [3] and Stewart [4]. However, due to the limited size of the cache, it is provably not possible to reduce the ACMR below 1.0 in this case. Deering [5] first proposed a hardware model where a larger vertex cache is allowed, which he called generalized triangle meshes, but did not supply algorithms to generate the appropriate rendering sequences. Chow [6] later provided algorithms, as did Bar-Yehuda and Gotsman [7] and Lin and Yu [8]. Deering's hardware design has since been implemented in Sun Microsystems Elite3D graphics hardware series [9] and the generalized mesh representation is an important component of the compressed geometry format of the Java3D API [10]. Recently, Mitra and Chiueh [11] also proposed an architecture based on two vertex buffers and a breadth-first traversal algorithm for generating rendering sequences for it.

The published algorithms, however, used a non-FIFO vertex cache, i.e. a cache which could be explicitly controlled by the user, which is non-existent in today's general-purpose low-end GPUs. Realizing this, Hoppe [12] proposed an algorithm to generate rendering sequences for a so-called transparent FIFO cache, and experimentally showed that for any given cache size, his algorithm generates rendering sequences whose ACMR is not significantly worse than those generated by Chow's algorithm. However, a major problem with all the algorithms, including Hoppe's, is that the cache size must be known in advance, i.e. a different rendering sequence is generated for any cache size, and using it to render a mesh when a smaller cache is present may provide far from optimal results.

Another drawback of all the existing algorithms is that they do not generalize well for progressive meshes. Progressive meshes differ from fixed-resolution meshes in that a vertex removal order is imposed on them, usually for reasons of geometric approximation. As each vertex is removed in turn from the mesh, the resulting hole is retriangulated. Progressive meshes are useful in rendering a mesh at a resolution which can continuously vary depending, say, on viewing parameters. A sequence of update records would indicate the vertex removals and retriangulations to perform in order to achieve the desired polygon count. Since the mesh is constantly changing, the rendering sequence must also change with it. The only works we are aware of in this respect are those of El-Sana *et al.* [13] and Stewart [4] on maintaining simple triangle strips for progressive meshes.

This paper introduces methods for generating universal mesh rendering sequences. These sequences preserve locality on all scales, and hence may be used as rendering sequences with a FIFO cache of any size. We also show how, thanks to the universality of the sequences, these rendering sequences may be adapted to progressive meshes without a significant performance penalty. In a sense, the rendering sequences generated by our algorithms are analogous to the so-called discrete space-filling curves [14–16], highly regular constructions applicable only to uniform grid structures. The simplest application of these sequences, once computed, is to list a triangle mesh in the order dictated by the sequence in an ASCII VRML file, so even simple renderers may benefit from them.

Space-filling curves are classics dating back to Hilbert, Peano and other mathematicians. See the book by Sagan [14] for a complete treatise on the subject. These curves are actually traversals of the cells in a (multidimensional) grid, which preserve locality in some sense. Quantification of the notion of locality-preservation has also been the focus of recent attention, and a variety of measures have been proposed [15,16], tailored to specific applications. The essence of our work is to generalize this notion to general irregular triangle meshes, where the classical methods fail. Attempts at such constructions have been made by Bartholdi and Goldsman [17], but the effectiveness of the construction was not quantified in their work. Note that these traversals depend only on the *connectivity* of the mesh, and not on its geometry, i.e. the coordinates in space of the vertices.

It might be argued that the precise connectivity of a mesh is just an artefact of the specific method used to create the mesh, hence it would be reasonable to allow modification of the connectivity in order to generate good rendering sequences more easily, as long as the geometric shape of the mesh is preserved. Some applications even perform remeshing, which modifies the number and the geometry of the mesh vertices in order to achieve a more regular connectivity. While this is true in some applications, we make the more stringent assumption that the connectivity cannot be changed at all.

It might also be argued that if the algorithm computing the rendering sequence is fast enough, it could be run on the fly immediately before rendering, eliminating the need for a precomputed universal rendering sequence suitable for all cache sizes, since at this point the vertex cache size of the rendering hardware is known, and a rendering sequence tailored to the cache size (such as Hoppe's [12]) can be used. This might be true in theory, but in a typical client–server scenario, where the server is very powerful, and the client very weak (e.g. a PDA), it is very important to reduce the computation load on the client to a minimum, hence a universal rendering sequence precomputed and stored at the server, is advantageous.

From a theoretical point of view, Bar-Yehuda and Gotsman [7] have shown that a vertex cache of size $\Theta(\sqrt{n})$ is required in order to render an n-vertex triangle mesh with the minimum ACMR of 0.5. Conversely, given a cache of size k, they show that the ACMR is $0.5 + \Omega(1/k)$. These bounds apply to the case of a controllable cache, so a (more restricted) FIFO cache can perform no better. In general, the objective is to generate a rendering sequence, such that when each triangle is rendered, hopefully as many of the triangle vertices as possible will be present in the cache. If not—these count as cache misses.

The remainder of this paper is organized as follows. Section 2 presents two algorithms for generating universal rendering sequences, which are generalized to progressive meshes in Section 3. In Section 4 we present experimental evidence that the rendering sequences generated by our algorithm indeed perform well. We conclude in Section 5.

A preliminary version of this paper appeared in GI'2001 [18].

2. Generating Universal Rendering Sequences

In this section we present two algorithms to generate universal rendering sequences. The first is inspired by classical space-filling curve constructions, and the second is obtained as a solution to an optimization problem.

2.1. The recursive cut algorithm

We know that the classical space-filling curves on rectangular grids (of sizes which are powers of two) have good locality properties, so as a first experiment it would be interesting to see how these curves perform as rendering sequences. It is easy to use the classical Hilbert curve to produce a rendering sequence for the regular triangle grid (instead of a rectangular grid). See Figure 2(b). The graph in Figure 5(a) shows the ACMR incurred by this rendering sequence as a function of the cache size. The Hilbert construction proceeds as follows: partition the mesh into four (identical) quarters. Render all triangles in the first quarter (recursively), then all triangles in the second quarter (recursively), etc. The recursion terminates when the mesh contains only a few triangles. Care is exercised so that the last triangle rendered in the first quarter is adjacent to the first triangle rendered in the second quarter, and similarly for the third and fourth quarters. This is possible due to the regular structure of the mesh, and thus guarantees a continuous curve. The Hilbert curve construction inspires the following analogous recursive procedure for irregular triangle meshes: partition (bisect) the mesh into two approximately equal submeshes. Render the first submesh, then render the second submesh. Make sure, though, that the exit point from the first submesh is close to the entry point into the second submesh. To be more precise, we need to find a balanced edge-cut of the mesh *dual graph*, i.e. a set of edges of the dual graph such that removing them from the mesh results in two disconnected sets of faces of approximately the same size. It is even better if the edge-cut contains only a small number of edges, because most of the cache misses will ultimately be incurred along the edge-cut. After finding a bisection the procedure fixes the exit point of one submesh and the entry point of the other, then it recursively proceeds to render the first submesh and then the second submesh. The entry and exit points are found by taking one of the cut edges as the gate for transition from one submesh to the other. Since the edge is taken from the cut, its incident triangles are guaranteed to be in different submeshes. The heuristic for choosing the gate edge maximizes the topological distance between the entry and the exit points inside each of the two submeshes, see Figure 3. This leaves more freedom for finding good bisections when each of these submeshes is recursively bisected.

Finding a balanced edge-cut of a graph is a much-studied problem in itself, with applications in parallel processing, numerical computation and VLSI, to name a few. Rather than reinvent the wheel, we used the excellent METIS software package [19]. METIS is able to find balanced edge-cuts in time linear in the mesh size. Figures 2(c) and 4(a) show the rendering sequences generated using this algorithm on two meshes, and Figure 5 the associated performance graphs. The first mesh is the regular triangulated grid mentioned before, where each vertex has degree 6.

The second mesh is irregular, with vertices of degrees anywhere between 4 and 8. Since the rendering sequences depend on mesh connectivity alone, we visualize the irregular mesh using the graph drawing procedure of Tutte [20], where the boundary vertices are mapped to a circle, and each of the interior vertices is placed at the centroid of its neighbours (this drawing is generated by iteratively solving a system of linear equations for the planar coordinates of the vertices). To compare, Figure 5 shows the performance curves generated by this algorithm for the meshes. For the regular mesh, the rendering sequence generated by the recursive cut algorithm is not much worse than the Hilbert curve. It is also possible to use the simple "raster snake" curve, which also seems to give reasonable results. Not surprisingly, a completely random rendering sequence performs dismally.

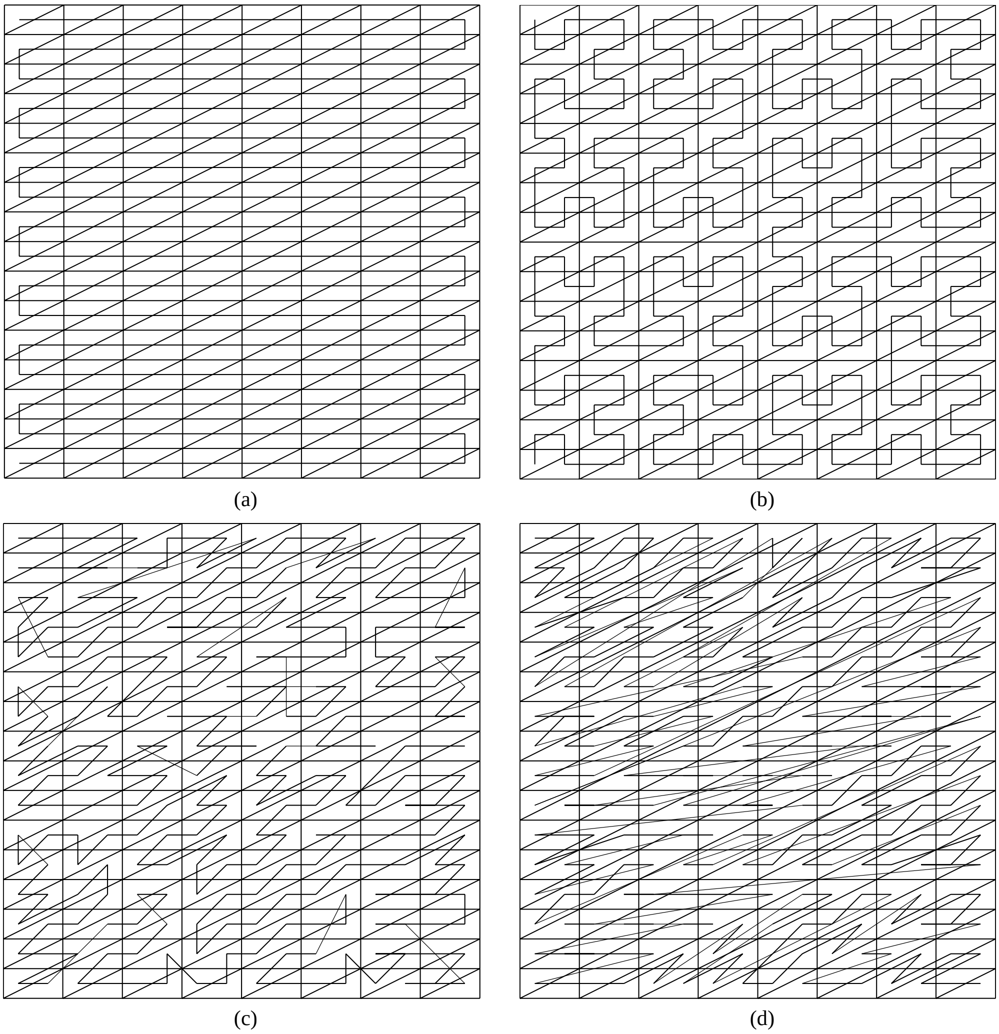


Figure 2: *Possible rendering sequences for a regular triangle mesh generated from a* 16×8 *regular square grid. Note that all vertices but the boundary ones have degree* 6*. Thin lines denote "jumps" in the sequence between non-neighbouring triangles. (a) Simple "snake" raster. (b) Hilbert space-filling curve. (c) Sequence generated by the recursive cut algorithm. (d) Sequence generated by the MLA algorithm.*

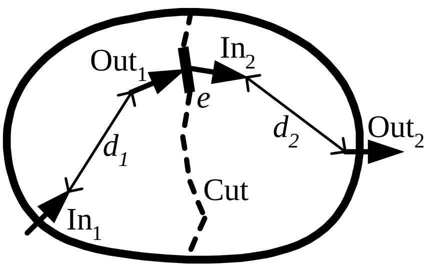


Figure 3: *The exit and entry points* Out_1 *and* In_2 *are found by taking the edge e from the edge-cut to be the gate, such that the topological distances* d_1 *and* d_2 *are maximized.*

2.2. Performance of the recursive cut algorithm

We now show that the recursive cut algorithm generates rendering sequences whose ACMR is the best possible asymptotically: $0.5 + O(1/k)$.

We know that for any planar graph with m nodes there exists a (balanced) separator of size $O(\sqrt{m})$ [21]. This means that any bisection of an m-node dual graph produced by METIS will have an edge-cut of length bound by $a\sqrt{m}$, where a is some constant. Bar-Yehuda and Gotsman [7] showed that for a minimum time rendering (ACMR $= 0.5$) of an m-triangle mesh a vertex buffer of size $O(\sqrt{m})$ is always sufficient. This means that there exists a constant b

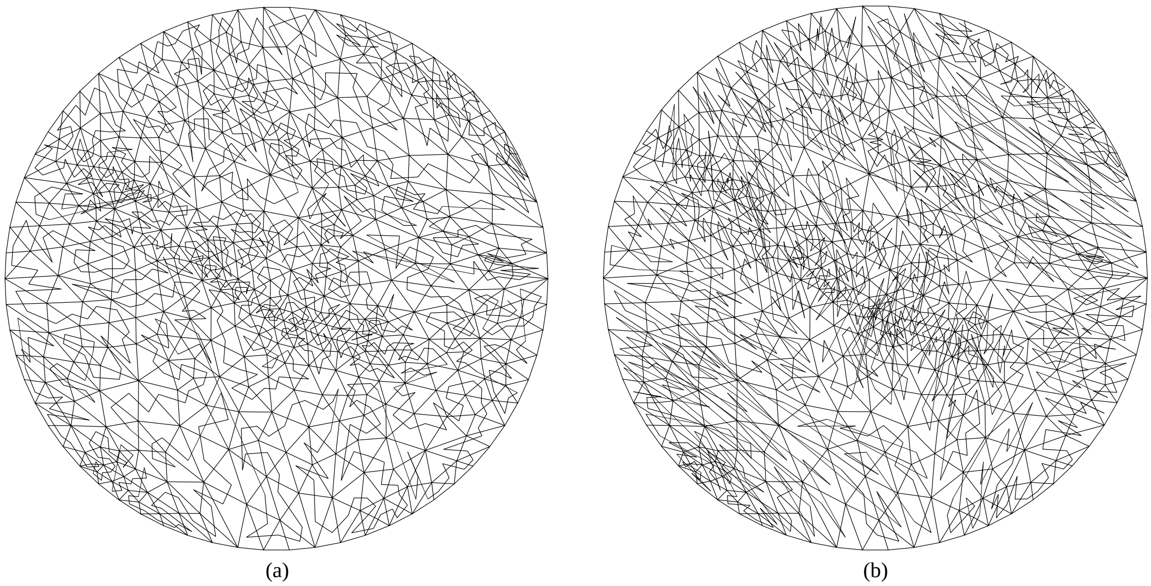


Figure 4: *Possible rendering sequences for an irregular mesh of* 509 *vertices and* 950 *triangles. Thin lines denote "jumps" in the sequence between non-neighbouring triangles. (a) Sequence generated by the recursive cut algorithm. (b) Sequence generated by the MLA algorithm.*

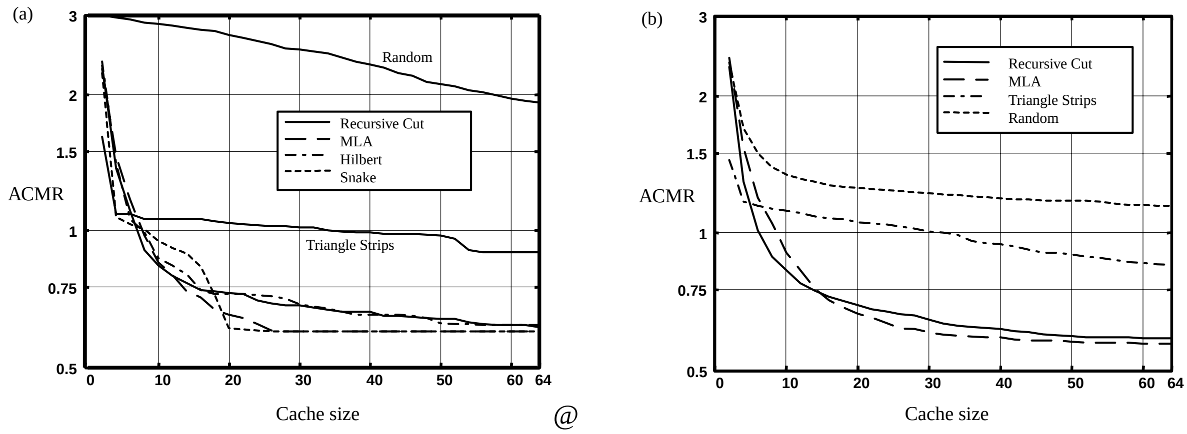


Figure 5: *Performance of different rendering sequences as a function of cache size. The random rendering sequence is the ordering of the triangles as they happened to appear in the VRML IndexedFaceSet shape in the input file. (a) Regular triangle grid (as in Figure* 2*). Note how the snake raster is no worse, and sometimes even better, than the other sequences on very small or very large cache sizes. (b) Irregular triangle grid (as in Figure* 4*).*

such that any mesh of size bk^2 can be rendered in minimum time using a k-entry vertex buffer. So when the recursion of our algorithm reaches levels where the submesh size is less than bk^2, these submeshes are not likely to cause additional cache misses, because their edge-cuts will be "swallowed" by the cache.

The total number of additional cache misses can be estimated by calculating the approximate sum of the edge-cut lengths for all the relevant submesh sizes. At recursion level i a submesh has approximately $m/2^i$ triangles. We are interested only in those i that satisfy:

$$\frac{m}{2^i} \geqslant bk^2$$

or

$$i \leqslant \log \frac{m}{bk^2} \equiv N. \tag{1}$$

Now recursion level i consists of 2^i submeshes with about $m/2^i$ triangles each, so the sum of the relevant edge-cuts is

$$\sum_{i=0}^{N} 2^i a\sqrt{\frac{m}{2^i}} = a\sqrt{m}\sum_{i=0}^{N} 2^{i/2} = a\sqrt{m}\frac{(2^N)^{1/2}-1}{\sqrt{2}-1}.$$

Substituting N from (1),

$$a\sqrt{m}\frac{(\frac{m}{bk^2})^{1/2}-1}{\sqrt{2}-1} \leqslant \frac{am}{\sqrt{b}(\sqrt{2}-1)} \cdot \frac{1}{k}.$$

And since $m \simeq 2n$, the estimated ACMR is:

$$\text{ACMR}(k) \simeq \frac{1}{m} \cdot \left(n + \frac{am}{\sqrt{b}(\sqrt{2}-1)} \cdot \frac{1}{k}\right) = 0.5 + O(1/k) \quad (2)$$

which is the best asymptotic behaviour we could expect.

2.3. The minimum linear arrangement approach

Space-filling curves in general, and rendering sequences in particular, are attempts to impose a one-dimensional (1D) ordering on a set of higher-dimensional elements. Indeed, space-filling curves are used for reducing higher-dimensional problems to 1D ones without losing too much of the spatial correlations present in the data, e.g. in image compression [22].

Following this argument, the problem of generating an efficient rendering sequence may be cast as an instance of the Minimum Linear Arrangement (MLA) problem on hypergraphs. A hypergraph is a pair $\langle V, HE\rangle$ where V is a vertex set, and HE a set of hyperedges connecting sets of vertices. A graph is a special case of a hypergraph where every hyperedge connects just two vertices. The MLA problem requires that the n vertices of the hypergraph be mapped to the integers $1, \ldots, n$, such that the sum of the hyperedge lengths is minimal. The length of a hyperedge $e = [v_1, \ldots, v_k]$ is defined to be

$$L(e) = \max(m(v_1), \ldots, m(v_k)) - \min(m(v_1), \ldots, m(v_k))$$

where $m : V \longrightarrow 1, \ldots, n$ is the mapping function. This means that all vertices participating in a hyperedge should be mapped in close proximity. The MLA is a member of the class of geometric embedding problems [23], where combinatorial structures, e.g. graphs, are embedded in a geometric domain, such that they optimize some geometric measure, e.g. distance. Here the graph is embedded into a 1D grid.

Our mesh rendering problem may be cast as an instance of the MLA as follows: the hypergraph vertices correspond to the mesh triangles, and the hyperedges correspond to the mesh vertices, i.e. a hyperedge relates all mesh triangles incident on the same mesh vertex. The meaning of edge length in our context is the distance in the rendering sequence between the first and last triangles incident on the mesh vertex.

The MLA problem is known to be NP-Hard, hence efficient algorithms have been proposed to approximate the minimum. We use that of Bar-Yehuda *et al.* [24], which approximates the minimum in $O(n^{2.2})$ time and $O(n)$ space, where n is the number of vertices. The time complexity may be reduced by adjusting some algorithmic parameters, at the expense of the output quality.

We use an MLA solver as a "black box", which gives an output that approximates a solution to our problem. This means that any MLA algorithm can be used and whenever a better one is available we can benefit from it immediately.

Figures 2(d) and 4(b) show the rendering sequences generated by the MLA algorithm on our two test meshes, and Figure 5 compares the sequence performance with those generated by other methods. In particular, it is interesting to compare to the performance of a rendering sequence obtained from a leading triangle stripper [3]. While the ACMR of that sequence is reasonable, it is nowhere near optimal, since it was designed specifically for a cache of size two.

3. Application to Progressive Meshes

Many real-time 3D graphics applications, especially those dealing with large scenes, employ variants of the progressive mesh technique (also known as continuous level-of-detail) [25] to increase rendering performance. In a nutshell, this means that the polygon count of the scene is adjusted on the fly to adapt the geometric scene resolution to the rendered image resolution. For example, it is wasteful to render hundreds of polygons when the contribution of those polygons to the rendered image is less than one pixel.

Typical progressive mesh algorithms operate in two stages: in a preprocessing stage, a data structure is built containing information pertaining to the operations performed in order to increase or decrease the polygon count. The polygon count may be changed by a variety of methods, such as edge collapses [25] or vertex removals [26]. We consider the more general vertex removal method. The decision as to which vertex to remove at any given resolution level is usually based on geometric criteria, such as geometric approximation error relative to the original model. In essence, this means that at each level the vertex which least damages the geometric shape of the model is removed first. The resulting data structure usually consists of a sequence of update records which record at each resolution which vertex is to be removed, and how the resulting hole is to be retriangulated. When increasing resolution, the same record indicates the vertex to be inserted to the mesh, and how the mesh connectivity is adjusted accordingly. During rendering, this information is used on the fly to adjust the mesh resolution according to some user (and view) dependent criteria.

Since the update records of the progressive mesh are generated offline, long before the rendering, this is a given for the rendering sequence generator. The mesh cannot be modified on the fly to optimize other criteria, such as the performance of the rendering sequence. Hence, given this sequence of update records, we must generate not only a rendering sequence for the highest resolution, but also a sequence of corresponding updates to the rendering sequence, so that it continues to perform well also at lower resolutions.

We experimented with two rendering sequence update methods. Assume a vertex is to be removed from the mesh, and that vertex is incident on triangles whose positions in the rendering sequence are $k_1, \ldots, k_n$. In a manifold closed mesh, the resulting hole will have $n - 2$ edges, hence the vertex removal will eliminate n triangles from the mesh, and retriangulating the hole will generate $n - 2$ new triangles. The simplest way to update the rendering sequence is to arbitrarily assign the $n - 2$ new triangles to the first $n - 2$ indices of the n now available. This makes sense, as the new $n - 2$ triangles fill in the same area as the old n triangles filled in the past. Thus the locality will be somewhat preserved. We call this the simple update algorithm. However, this will probably be suboptimal, and the assignment of these new triangles among the freed indices can be optimized to better preserve the locality. Towards this end, we developed the smart update algorithm, whose pseudo-code appears below. Note that we do not attempt more global updates to the rendering sequence, and the places of all mesh triangles not affected by the vertex removal are not changed in the rendering sequence. Figure 6 shows a model at three levels of resolution, the rendering sequence for the highest resolution, and the rendering sequences generated for the two lower resolutions by the smart update algorithm.

Psuedo-code of the Smart Update Algorithm.

```
L - List of adjacency lists
L(t) - List of triangles that have common
    vertex with triangle t
S - Rendering sequence
S(t) - Index of triangle t in S
t1..tn - Triangles removed from mesh
T1..Tn-2 - Triangles inserted into mesh
S' - Updated rendering sequence

Input: S, t1..tn, T1..Tn-2
Output: S'

// Initialise S'
S' = S;
for i = 1 to n
  S'(ti) = empty;
endfor

// Assignment due to two common vertices

for i = 1 to n-2
  for j = 1 to n
    if Ti and tj have exactly 2 common
    vertices then
      S'(Ti) = S(tj);
      S(tj) = empty;
    endif
  endfor
endfor

// Initialisation of adjacency lists

for i = 1 to n-2
  if S'(Ti) is empty
    for j = 1 to n
      if S(tj) is not empty and Ti and tj
      have at least one common vertex
        add tj to L(Ti);
         endif
    endfor
  endif
endfor

// Assignment due to one common vertex

while there exists at least one non-empty list
   in L
  let L(T) be a list with minimum length;
  let t = head of L(T);
  S'(T) = S(t);
  S(t) = empty;
  empty L(T);
  remove all occurrences of t from
  other lists in L;
endwhile

// Assign the remaining faces to whatever
   places are left

for i = 1 to n-2
  if S'(Ti) is empty
    for j = 1 to n
      if S(tj) is not empty
        S'(Ti) = S(tj);
        S(tj) = empty;
      endif
  endif
endfor
```

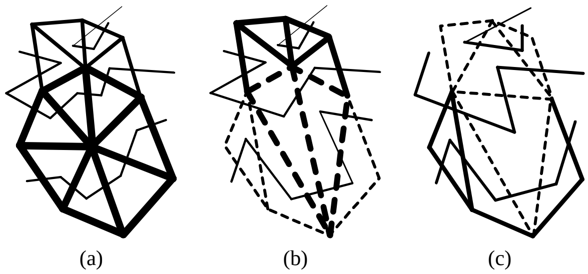


Figure 6: *Smart update of a rendering sequence during geometric resolution reduction. Thick lines mark the edges of the "star" whose centre the fat vertex—is to be removed. Dashed lines mark the edges of the retriangulated "hole". (a) High resolution. (b) One vertex removed. (c) Two vertices removed.*

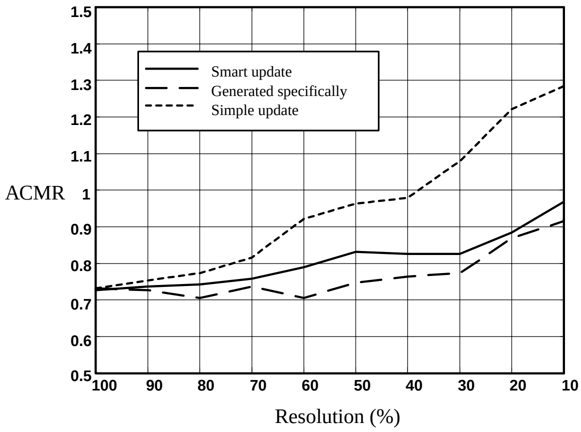


Figure 7: *Performance of rendering sequence of irregular mesh (of Figure 4) during geometric resolution reduction, comparing our update algorithms to what would have been obtained had the rendering sequence been generated specifically for each resolution. Vertex cache size* = 16. *The mesh simplification history, i.e. the sequence of vertex removals and retriangulations, was generated by a commercial simplifier available from* `www.virtue3d.com`.

It is obvious that the rendering sequence, updated as the resolution is decreased, will accumulate distortions such that after many updates it might cease to perform well. The key to the effectiveness of the procedure is graceful degradation of the performance. A good way to quantify the degradation is to compare the ACMR of the updated rendering sequence to that of a rendering sequence generated (using one of our methods) specifically for the given resolution. Figure 7 plots the ACMR of the irregular mesh of Figure 4 as a function of the triangle count, for a cache of size 16, and compares it to the ACMR that would have resulted had the rendering sequence been computed independently at each resolution. As is to be expected, the simple update algorithm performs quite poorly, accumulating significant distortion by the time a large number of vertices are removed. In contrast, the smart update algorithm performs almost as well as rendering sequences generated specifically for the mesh at that resolution, degrading very gracefully.

Table 1: *Characteristics of the meshes used in our tests*

Mesh	Number of vertices	Number of triangles	Recursive cut runtime (s)
Regular32	561	1,024	0.3
Bunny (simplified)	1,092	2,084	0.6
Flipper	6,179	12,337	5.4
Face	12,530	24,118	9.5
Horse	19,851	39,698	18
Buddha	32,316	67,240	27
Bunny	34,834	69,451	32

4. Experimental Results

We claim that both methods presented above for generating rendering sequences are universal in the sense that they perform well for all cache sizes, and degrade gracefully when applied to progressive meshes with given simplification histories. To quantify this, we have run our algorithms on a variety of test meshes, and measured the ACMR empirically for different values of cache size and mesh resolution. Some of the vital statistics and algorithm runtimes on a 550 MHz Pentium III PC with 128 MB RAM for our test meshes appear in Table 1.

Our experiments have shown that the ACMR is indeed a good measure of expected rendering sequence performance. Two totally different rendering sequences for the same model having the same ACMR exhibit identical rendering speeds, and a sequence with a smaller ACMR always gives better performance.

Figure 8 shows plots of the ACMR of the test meshes, as a function of both cache size and mesh resolution, for the two rendering sequence generators described in Section 2. Quite surprisingly, the recursive cut algorithm generates rendering sequences which perform almost identically for all meshes, both as a function of cache size and as a function of mesh resolution. The MLA algorithm generates rendering sequences which exhibit some variance in the ACMR between meshes.

Figures 8(a) and (c) show also the ACMR measured by Hoppe [12] for his algorithm run on the bunny4000 model (the famous Stanford bunny decimated to 4000 vertices). The values are slightly better than those of our rendering

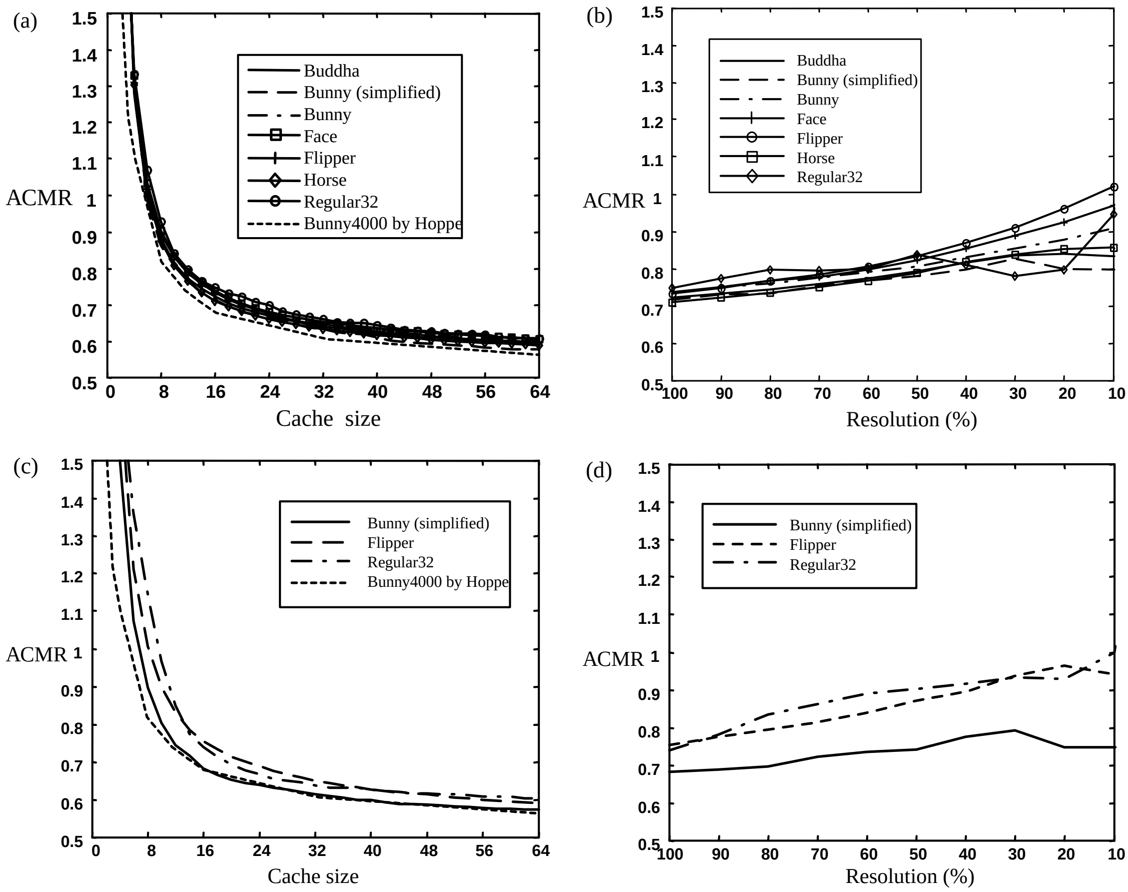


Figure 8: *ACMR results on test meshes of Table* 1. *(a), (b) Recursive cut algorithm. (c), (d) MLA algorithm. The cache size in (b) and (d) is 16.*

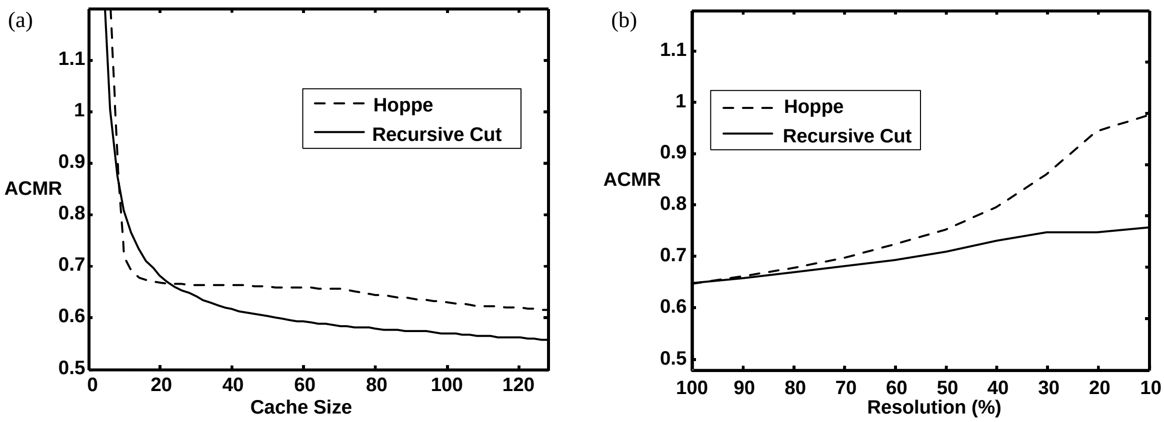
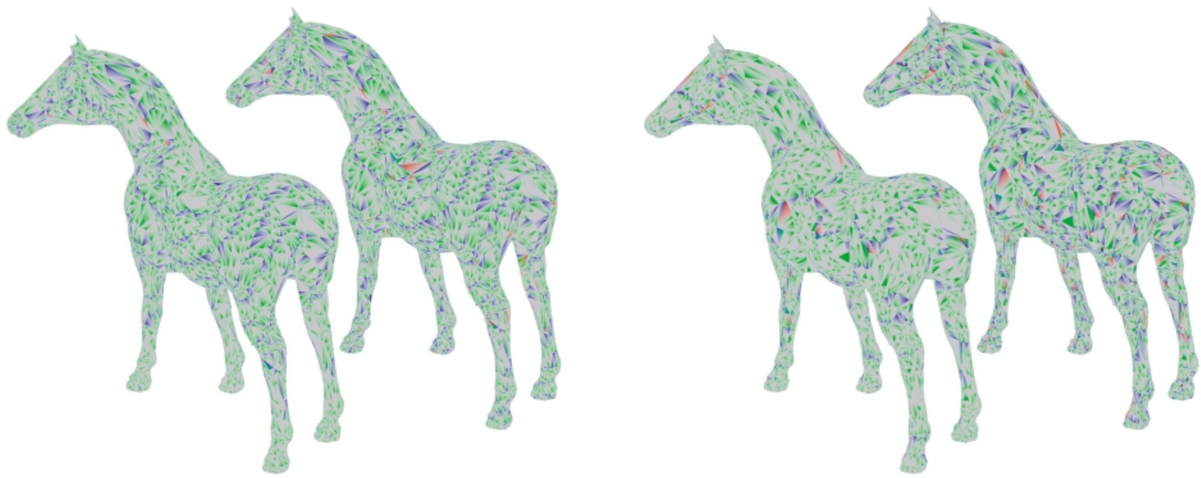


Figure 9: *Comparison of our universal rendering sequences with sequences optimized for a specific cache size, as measured on the horse model. Hoppe's rendering sequence was generated using DirectX 8.0* [27]*, optimized for a vertex cache with* 16 *entries. (a) Using one mesh rendering sequence (generated at highest mesh resolution) for different cache sizes. (b) Performance at different resolutions, using the same (smart) update algorithm for both sequences. The cache size is* 22*, where both rendering sequences have identical ACMRs at full resolution, as seen in (a).*

(a) (b)

Figure 10: *Visualization of cache misses when rendering the simplified bunny model using vertex cache of size* 16*. Colour of a corner codes the number of cache misses for the corner's vertex when rendering the corresponding triangle. Each vertex will always have exactly one green corner. The goal is to reduce the number of blue and red corners by maximizing the number of grey ones. (a) Original rendering sequence as appeared in VRML file, ACMR =* 2.65*. (b) Using rendering sequence generated by the recursive cut algorithm, ACMR =* 0.71*.*



(a) (b)

Figure 11: *Visualization of cache misses (colour coded as in Figure* 10*) when rendering the horse model using vertex cache of size* 14 *(right-hand model in each pair) and* 70 *(left-hand in each pair). (a) Hoppe's rendering sequence generated for cache of size 16.* ACMR(14) = 0.7*;* ACMR(70) = 0.63*. (b) Recursive cut universal rendering sequence.* ACMR(14) = 0.73*;* ACMR(70) = 0.58*.*

sequences. Recall, however, that Hoppe generates a different sequence for every cache size. Considering that we use just one universal sequence, our results are very competitive.

Of course, universality comes at the cost of optimality. Hoppe's rendering sequences optimized for a given cache size will always perform better than ours on that cache size. However, when using precisely the same rendering sequence with different cache sizes and mesh resolutions (under the same update algorithm), our sequences are superior, as illustrated by Figures 9 and 11.

Table 2 shows the actual frame/s rendering rates measured on a ASUS GeForce 2 graphics card containing a vertex buffer with ten entries, using our rendering sequence. This is compared to the frame rates achieved when an arbitrary rendering sequence is used, and the frame rates achieved by triangle stripping. It is evident that the frame rate speedup is

Table 2: *Rendering speed-ups using vertex buffer. The ACMR was measured by simulation of a vertex cache of size* 10*. The rendering speed was measured on the ASUS GeForce* 2 *GTS, which has an effective vertex cache size of 10 entries. "Original"—arbitrary rendering sequence (as appeared in original VRML file). "Rec. cut"—rendering sequence as generated by our recursive cut algorithm. "Triangle Strips"—rendering sequence generated by the triangle stripper* [3]*, and rendered using the OpenGL triangle strip primitive.*

Mesh	ACMR			Rendering speed (frames/s)		
	Original	Rec. cut	Triangle strips	Original	Rec. cut	Triangle strips
Buddha	1.00	0.81	1.08	227	240	236
Bunny	1.00	0.83	1.08	228	229	185
Bunny (simplified)	2.76	0.81	1.06	1299	2439	2222
Face	1.34	0.83	1.10	418	680	546
Flipper	1.84	0.83	1.10	508	1299	787
Horse	2.83	0.81	1.09	98	327	205

quite consistent with that predicted by the ratio between the corresponding ACMRs, which can be up to factor 3.

5. Summary and Conclusion

This work describes two (related) methods to generate rendering sequences for meshes which should be very useful in the age of 3D hardware with vertex buffers. They may be precomputed once per mesh and then used for any cache size. These rendering sequences have also been shown to be useful in progressive mesh applications and apply to all types of 3D meshes, including non-manifold and non-genus-0 meshes.

Software executables demonstrating the concepts described in this paper may be found at the URL: `http://www.cs.technion.ac.il/~gotsman/caching`. An interesting result is that the rendering sequences seem to perform equally well on meshes of all sizes (as a function of cache size). This seems to be another positive indication of the "universality" of the sequences.

Future work will include improvements of the algorithms, optimization of the implementations, and more sophisticated updates to the progressive rendering sequence.

Acknowledgements

Thanks to Reuven Bar-Yehuda, Guy Even, Jon Feldman and Seffi Naor for helpful inputs on mesh partitioning and embedding algorithms, and making available their software for the MLA. The bunny and buddha meshes are courtesy of the Stanford University Computer Graphics Laboratory.

This work was partially supported by Israel–German fund (GIF) grant I-627-45.6/99 and the Technion VP for Research Fund.

References

1. K. Akeley, P. Haeberli and D. Burns. The tomesh.c program. Available on SGI computers and developers toolbox CD. 1990.
2. F. Evans, S. Skiena and A. Varshney. Optimizing triangle strips for fast rendering. In *Visualization '96 Proceedings*, pages 319–326. IEEE, 1996.
3. X. Xiang, M. Held and J. Mitchell. Fast and effective stripification of polygonal surface models. In *Symposium on Interactive 3D Graphics Proceedings*, pages 71–78. ACM, 1999.
4. J. Stewart. Triangle strips for continuous level-of-detail meshes. *Graphics Interface 2001 Proceedings*.
5. M. Deering. Geometry compression. *Computer Graphics (SIGGRAPH '95 Proceedings)*, 13–20, 1995.
6. M. Chow. Optimized geometry compression for real-time rendering. In *Visualization '97 Proceedings*, pages 347–354. IEEE, 1997.
7. R. Bar-Yehuda and C. Gotsman. Time/space tradeoffs for polygon mesh rendering. *ACM Transactions on Graphics*, 15(2):141–152, 1996.
8. G. Lin and T. P.-Y. Yu. A non-recursive algorithm for minimum-time rendering of meshes with arbitrary genus, RPI. 2001 Preprint.
9. Sun Microsystems Elite3D Series. Available on www at URL: `http://www.sun.com/desktop/products/Graphics/elite3djtf.html`.
10. H. Sowizral, K. Rushforth and M. Deering. *The Java 3D API Specification*. 2nd edn. Sun Microsystems Press (Java Series), 2000.

11. T. Mitra and T. Chiueh. A breadth-first approach to efficient mesh traversal. In *Workshop on Graphics Hardware Proceedings*. ACM, 1998.

12. H. Hoppe. Optimization of mesh locality for transparent vertex caching. *Computer Graphics (SIGGRAPH '99 Proceedings)*, 269–276, 1999.

13. J. El-Sana, E. Azanli and A. Varshney. Skip strips: maintaining triangle strips for view-dependent rendering. In *Visualization '99 Proceedings*. IEEE, 1999.

14. H. Sagan. *Space-filling Curves*. Springer, New York, 1994.

15. A. Orni. Measuring the locality of space-filling curves, *MSc Thesis*, Computer Science Department, Technion—Israel Institute of Technology, 1998.

16. C. Gotsman and M. Lindenbaum. On the metric properties of discrete space-filling curves. *IEEE Transactions on Image Processing*, 5(5):794–797, 1996.

17. J. J. Bartholdi and P. Goldsman. A continuous spatial index of a triangulated surface, *PhD Thesis*, Industrial Engineering Department, Georgia Institute of Technology, 1999.

18. A. Bogomjakov and C. Gotsman. Universal rendering sequences for transparent vertex caching of progressive meshes. *Graphics Interface 2001 Proceedings*, 81–90, 2001.

19. G. Karypis and V. Kumar. METIS—a software package for partitioning unstructured graphs, partitioning meshes, and computing fill-reducing orderings of sparse matrices. Version 4, University of Minnesota. Available on www at URL `http://www-users.cs.umn.edu/~karypis/metis/main.shtml`.

20. W. T. Tutte. How to draw a graph. *Proceedings of London Math Society*, 10:304–320, 1960.

21. R. Lipton and R. Tarjan. A separator theorem for planar graphs. *SIAM Journal of Applield Mathematics*, 36(2):177–189, 1979.

22. A. Lempel and J. Ziv. Compression of two-dimensional data. *IEEE Transactions on Information Theory*, 32(1):2–8, 1986.

23. M. D. Hansen. Approximation algorithms for geometric embeddings in the plane with applications to parallel processing problems. In *Proceedings of the Conference on Foundations of Computer Science*, pages 604–609. IEEE, 1989.

24. R. Bar-Yehuda, G. Even, J. Feldman and J. Naor. Computing an optimal orientation of a balanced decomposition tree for linear arrangement problems. *Journal of Graph Algorithms and Applications*, 5(4):1–27, 2001. Software available at `http://www.eng.tau.ac.il/~guy/Minla`.

25. H. Hoppe. Progressive meshes. *Computer Graphics (SIGGRAPH '96 Proceedings)*, 99–108, 1996.

26. W. J. Schroeder, J. A. Zarge and W. E. Lorensen. Decimation of triangle meshes. *Computer Graphics (SIGGRAPH '92 Proceedings)*, 65–70, 1992.

27. Microsoft DirectX 8 Series. Available on www at URL: `http://www.microsoft.com/directx`.