Local Search Algorithms for the Maximum Carpool Matching Problem

Gilad Kutiel [1] Dror Rawitz [2]

[1]Technion

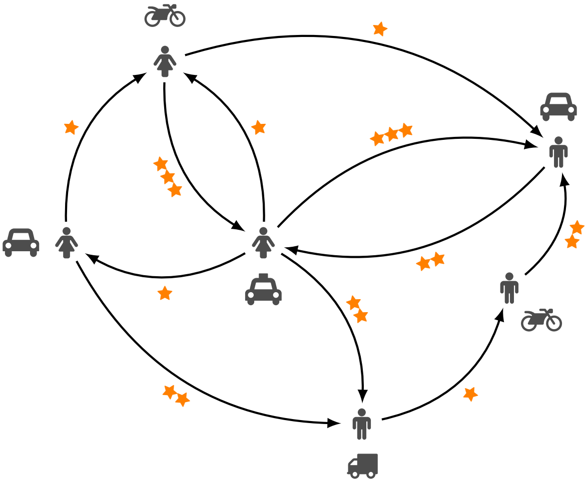
[2]Bar Ilan University

ESA 2017

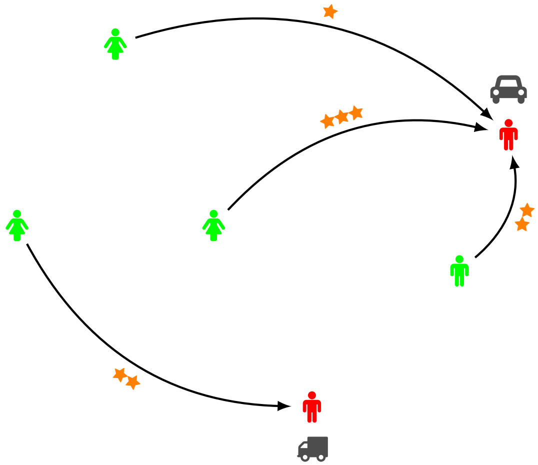
Maximum Carpool Matching (MCM)

Maximum Carpool Matching (MCM)

Maximum Carpool Matching (MCM)



Maximum Carpool Matching (MCM)



Max ★★★

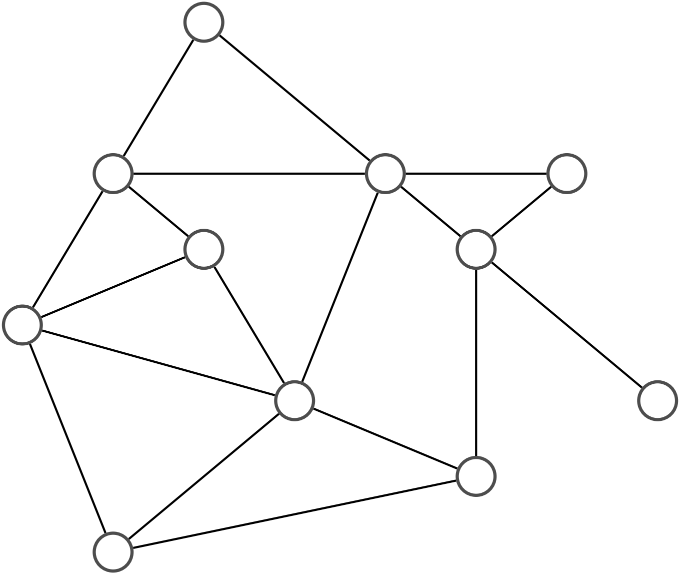
s.t.

Maximum Carpool Matching (MCM) cont.

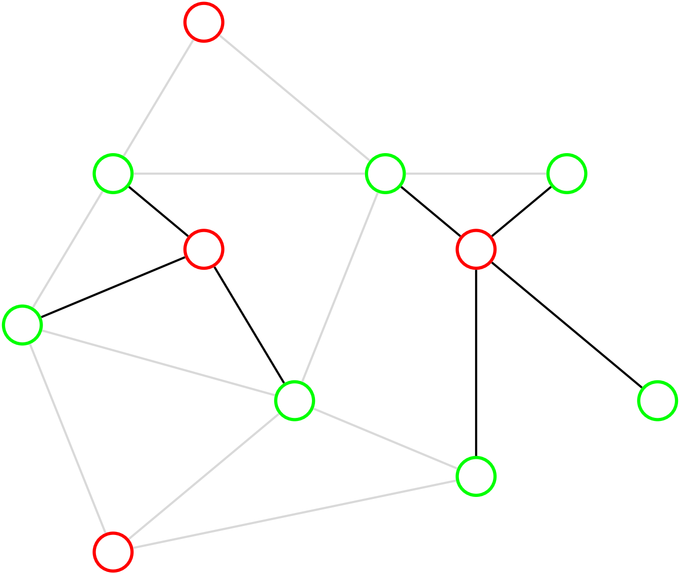
Max ★★★

s.t.

Maximum Spanning Star Forest (MSSF)



Maximum Spanning Star Forest (MSSF)



Max 

s.t. 

Hardness

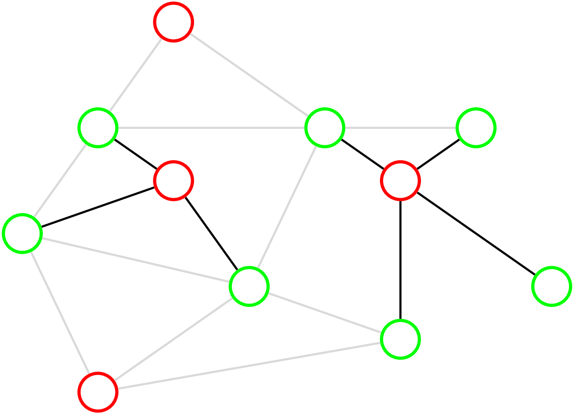
- The problems are APX-hard

Hardness

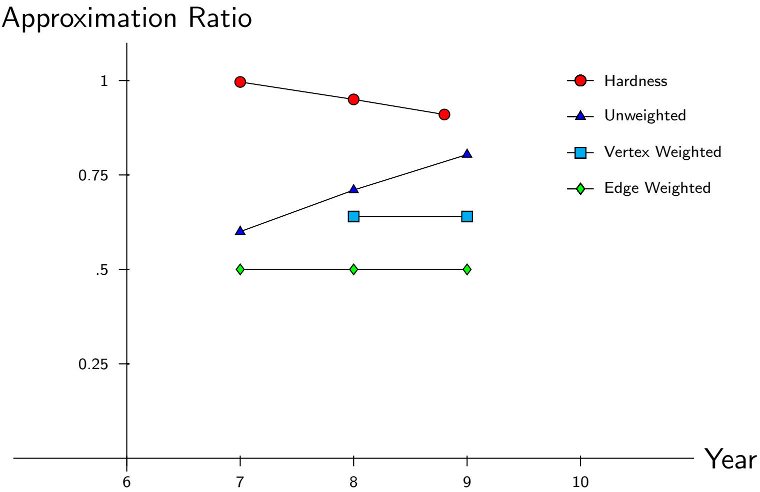
- The problems are APX-hard
- $|\text{Dominating Set}| + |\text{SSF}| = |V|$

Hardness

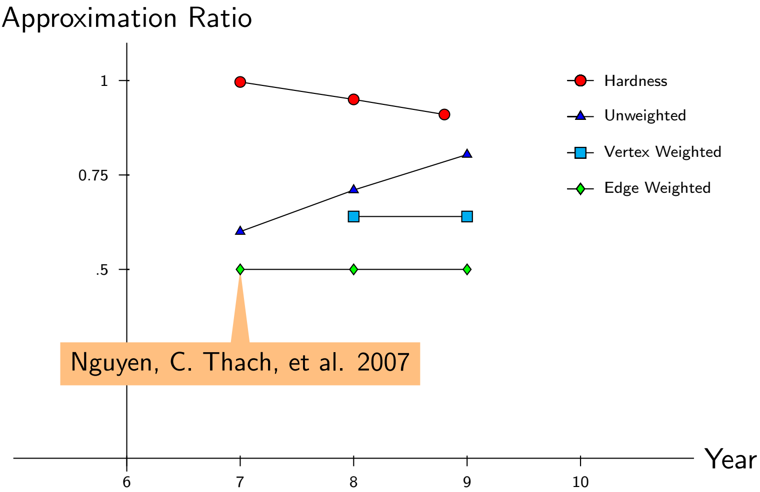
- The problems are APX-hard
- $|\text{Dominating Set}| + |\text{SSF}| = |V|$



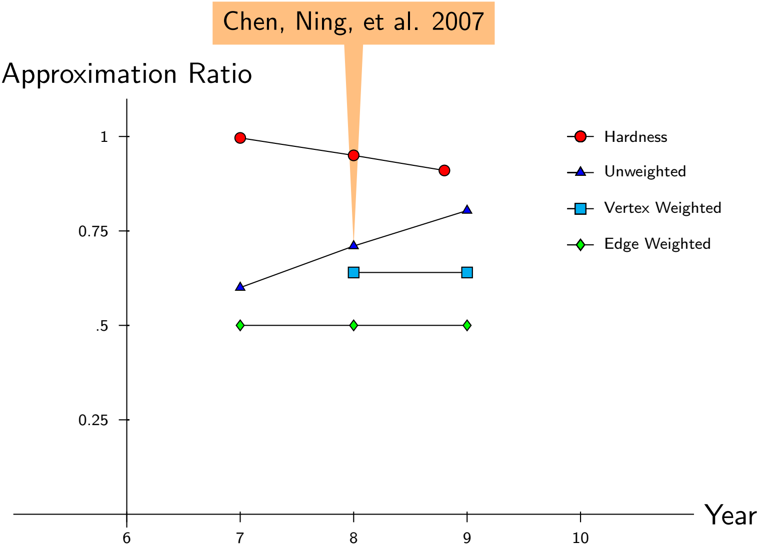
Previous Work (MSSF)



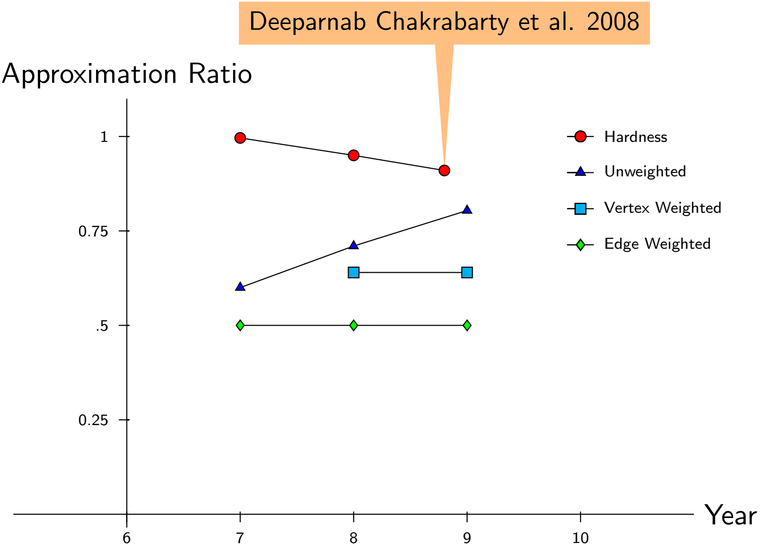
Previous Work (MSSF)



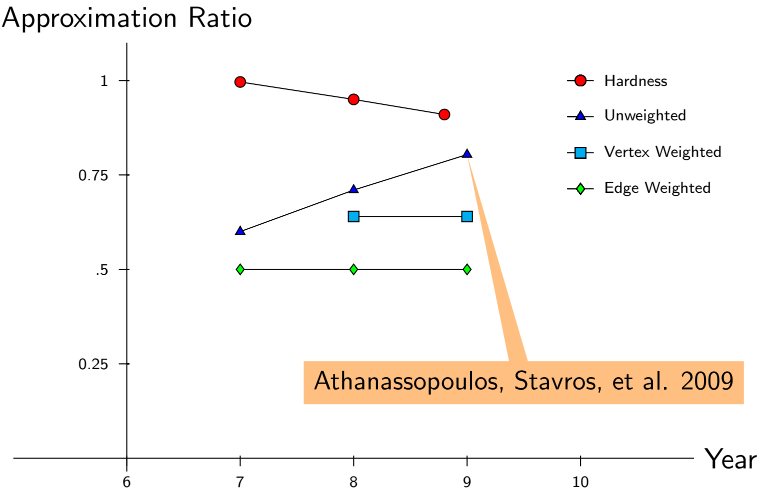
Previous Work (MSSF)



Previous Work (MSSF)



Previous Work (MSSF)



Previous Work (MCM)

- The algorithms for MSSF do not generalize to MCM

Previous Work (MCM)

- The algorithms for MSSF do not generalize to MCM
- Heuristics + experiments

Previous Work (MCM)

- The algorithms for MSSF do not generalize to MCM
- Heuristics + experiments
  - Solution's quality matters

Previous Work (MCM)

- The algorithms for MSSF do not generalize to MCM
- Heuristics + experiments
  - Solution's quality matters
- 1/3-approximation

Previous Work (MCM)

- The algorithms for MSSF do not generalize to MCM
- Heuristics + experiments
  - Solution's quality matters
- 1/3-approximation
- 1/2-approximation (unweighted)

Our Results

- APX-hard for bounded degree graphs

Our Results

- APX-hard for bounded degree graphs
- MCM as submodular maximization (David Adjashvili)

Our Results

- APX-hard for bounded degree graphs
- MCM as submodular maximization (David Adjashvili)
  - $1/2$-approximation

Our Results

- APX-hard for bounded degree graphs
- MCM as submodular maximization (David Adjashvili)
    - $1/2$-approximation
- $(1/2 - \varepsilon)$-approximation

Our Results

- APX-hard for bounded degree graphs
- MCM as submodular maximization (David Adjashvili)
  - $1/2$-approximation
- $(1/2 - \varepsilon)$-approximation
- Group Carpool (not submodular)

Our Results

- APX-hard for bounded degree graphs
- MCM as submodular maximization (David Adjashvili)
  - $1/2$-approximation
- $(1/2 - \varepsilon)$-approximation
- Group Carpool (not submodular)
  - $(1/2 - \varepsilon)$-approximation

Our Results

- APX-hard for bounded degree graphs
- MCM as submodular maximization (David Adjashvili)
  - $1/2$-approximation
- $(1/2 - \varepsilon)$-approximation
- Group Carpool (not submodular)
  - $(1/2 - \varepsilon)$-approximation
- $\frac{1}{2} + \frac{1}{2|\text{🚚}|} - \varepsilon$ (unweighted)

Local Search

Unweighted, $|\text{🚚}| = O(1)$

Local Search - Unweighted, $|$🚚$| = O(1)$

- For a fixed K

Local Search - Unweighted, |🚚| $= O(1)$

- For a fixed K
- Start with any solution

Local Search - Unweighted, |🚚| $= O(1)$

- For a fixed K
- Start with any solution
- while possible

Local Search - Unweighted, $|🚚| = O(1)$

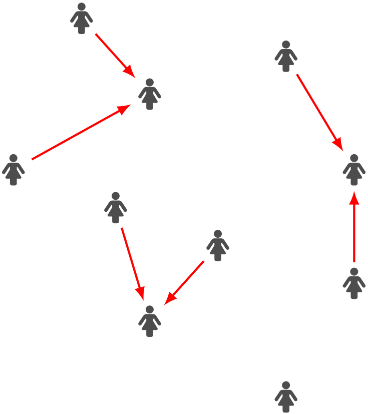
- For a fixed K
- Start with any solution
- while possible
  - Remove $k \leq K$ arcs

Local Search - Unweighted, $|🚚| = O(1)$

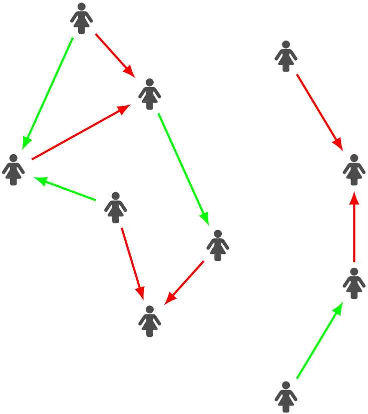
- For a fixed K
- Start with any solution
- while possible
  - Remove $k \leq K$ arcs
  - Add at least $k + 1$

Local Search - Analysis (Star Graph)

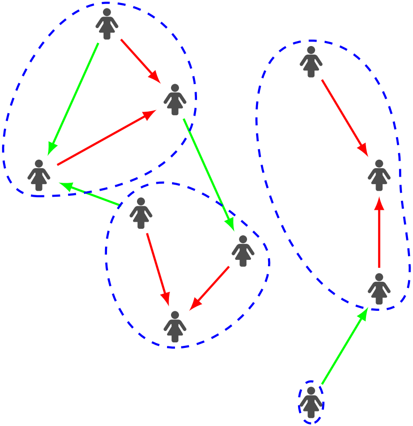
Local Search - Analysis (Star Graph)



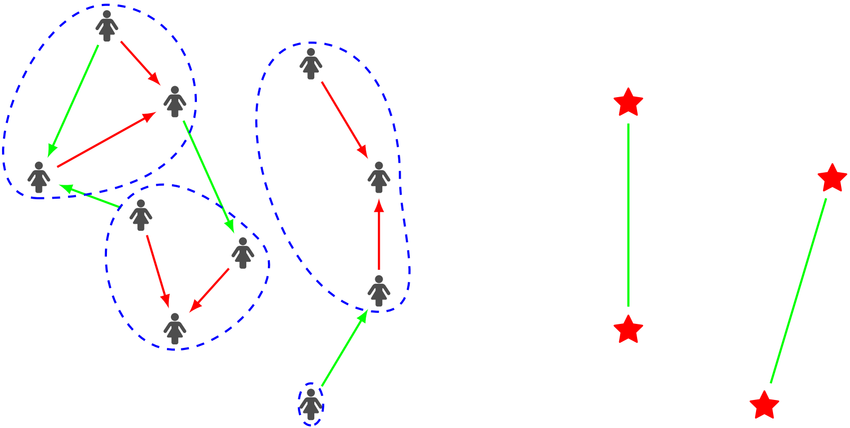
Local Search - Analysis (Star Graph)



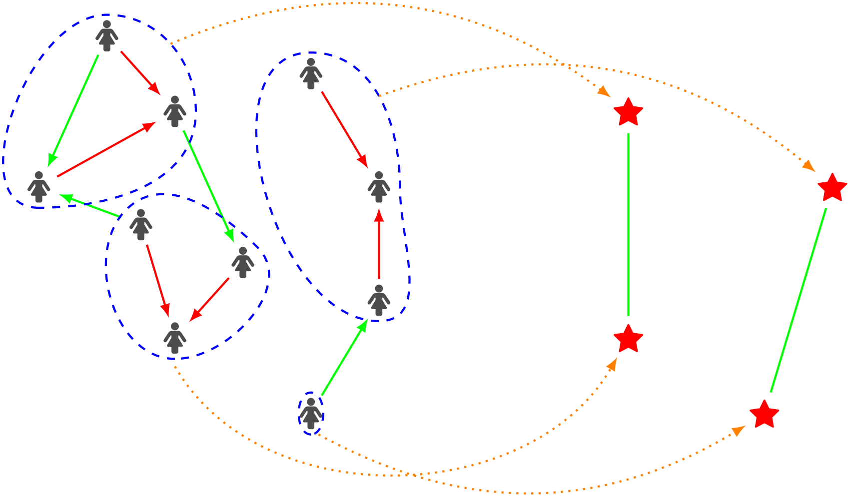
Local Search - Analysis (Star Graph)



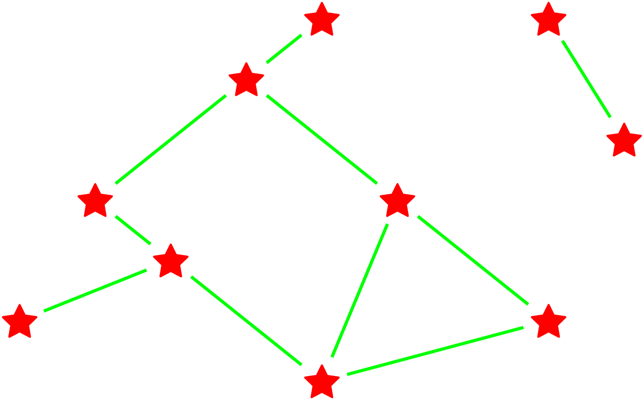
Local Search - Analysis (Star Graph)



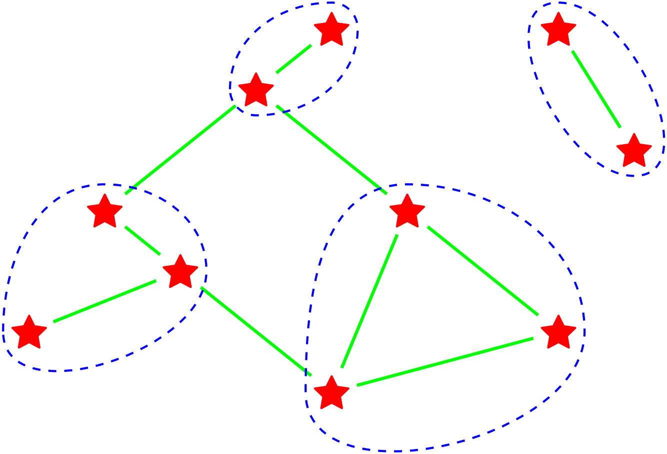
Local Search - Analysis (Star Graph)



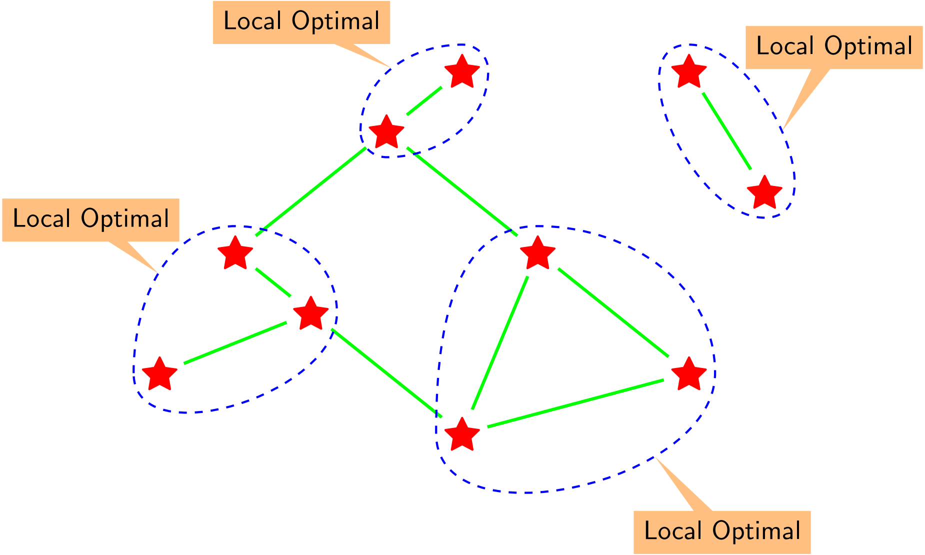
Local Search - Analysis cont.



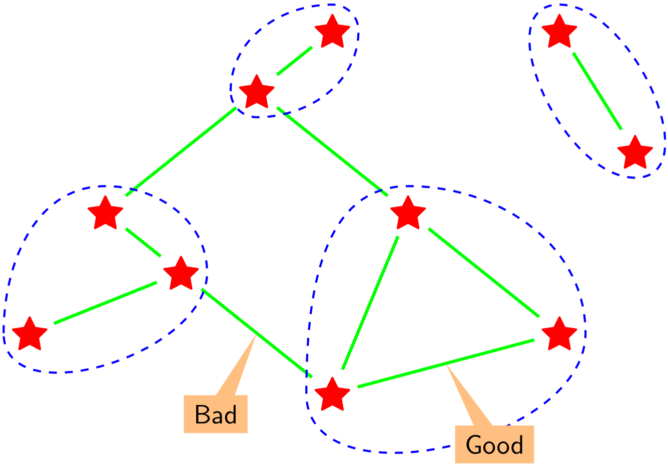
Local Search - Analysis cont.



Local Search - Analysis cont.

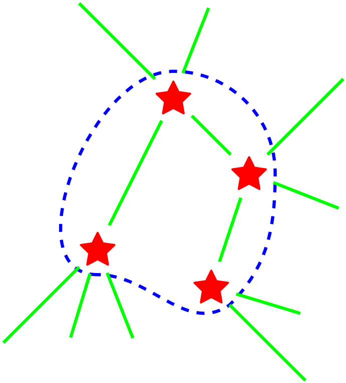


Local Search - Analysis cont.



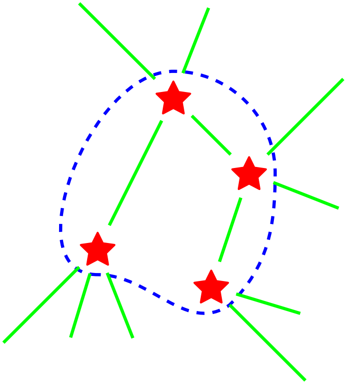
Local Search - Rough Analysis

- Let $r := \frac{\#\star}{\#\text{Good edges}}$



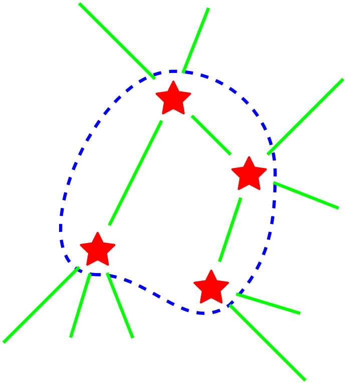
Local Search - Rough Analysis

- Let $r := \frac{\#★}{\#\text{Good edges}}$
- Each ★ intersects with (on averege):



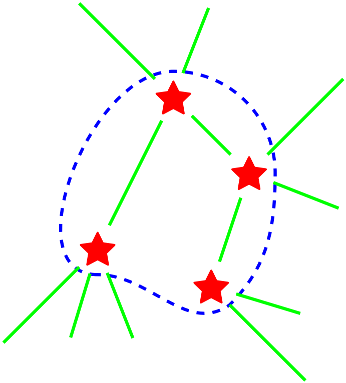
Local Search - Rough Analysis

- Let $r := \frac{\#\bigstar}{\#\text{Good edges}}$
- Each ★ intersects with (on averege):
  - r good edges



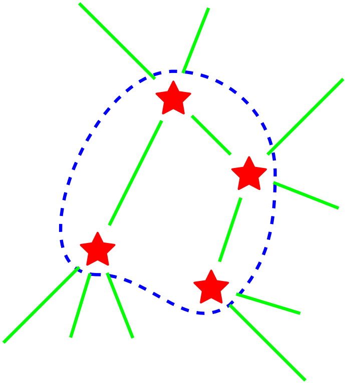
Local Search - Rough Analysis

- Let $r := \frac{\#★}{\#\text{Good edges}}$
- Each ★ intersects with (on averege):
  - r good edges
  - $\geq |★| - r$ bad edges



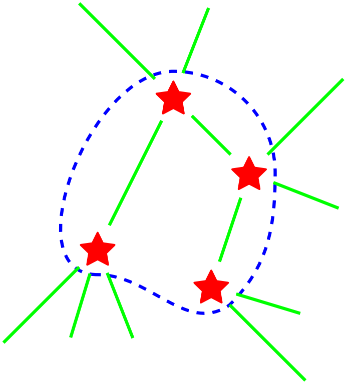
Local Search - Rough Analysis

- Let $r := \frac{\#★}{\#\text{Good edges}}$
- Each ★ intersects with (on averege):
  - r good edges
  - $\geq |★| - r$ bad edges
- Roughly, for large K, $r \geq 1$



Local Search - Rough Analysis

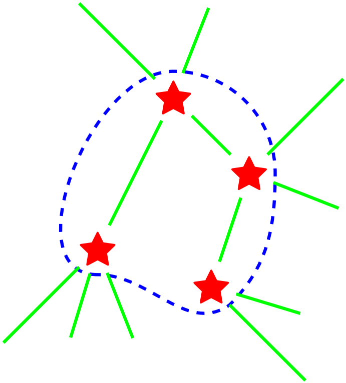
- Let $r := \frac{\#\bigstar}{\#\text{Good edges}}$
- Each $\bigstar$ intersects with (on averege):
  - r good edges
  - $\geq |\bigstar| - r$ bad edges
- Roughly, for large K, $r \geq 1$



$$\frac{r + 1/2(|\bigstar| - r)}{|\bigstar|}$$

Local Search - Rough Analysis

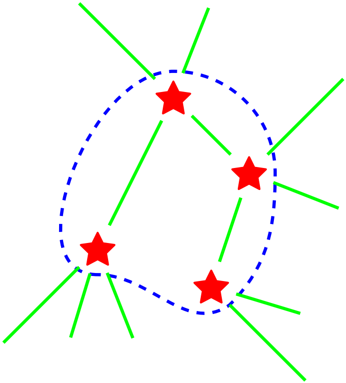
- Let $r := \frac{\#\bigstar}{\#\text{Good edges}}$
- Each $\bigstar$ intersects with (on averege):
  - r good edges
  - $\geq |\bigstar| - r$ bad edges
- Roughly, for large K, $r \geq 1$



$$\frac{r + 1/2(|\bigstar| - r)}{|\bigstar|} \geq$$

Local Search - Rough Analysis

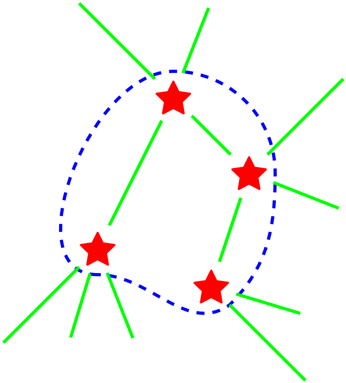
- Let $r := \frac{\#\bigstar}{\#\text{Good edges}}$
- Each $\bigstar$ intersects with (on averege):
  - r good edges
  - $\geq |\bigstar| - r$ bad edges
- Roughly, for large K, $r \geq 1$



$$\frac{r + 1/2(|\bigstar| - r)}{|\bigstar|} \geq \frac{1 + 1/2(|\bigstar| - 1)}{|\bigstar|}$$

Local Search - Rough Analysis

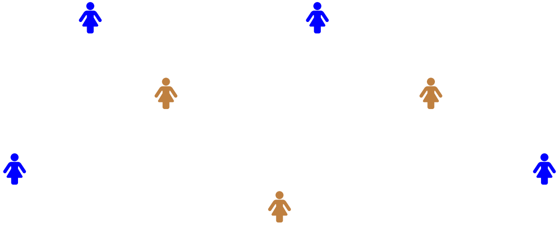
- Let $r := \frac{\#\star}{\#\text{Good edges}}$
- Each ★ intersects with (on averege):
  - r good edges
  - $\geq |\star| - r$ bad edges
- Roughly, for large K, $r \geq 1$



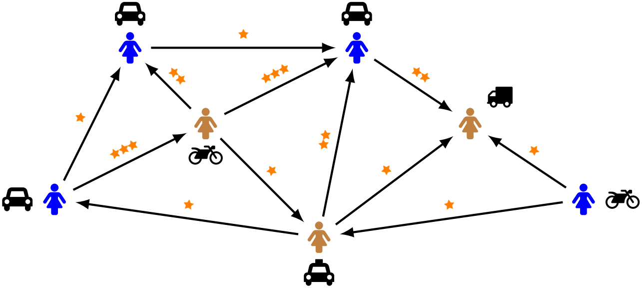
$$\frac{r + 1/2(|\star| - r)}{|\star|} \geq \frac{1 + 1/2(|\star| - 1)}{|\star|} = \frac{1}{2} + \frac{1}{2|\text{🚚}|}$$

Submodularity

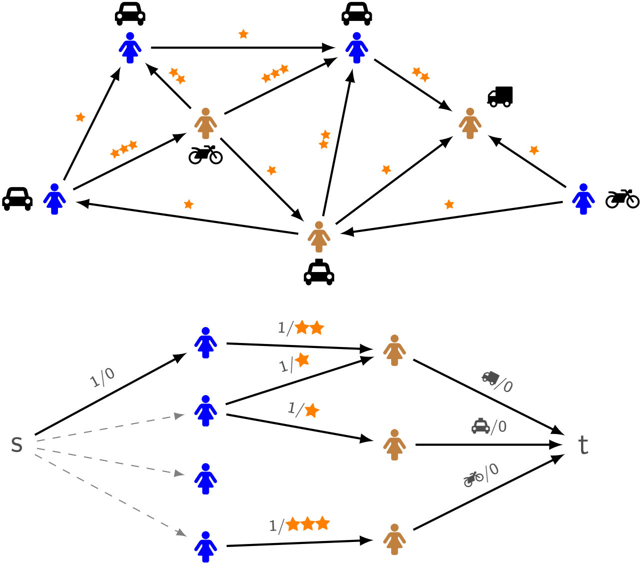
Fixed Matching



Fixed Matching



Fixed Matching



Submodular

- Let $D = \{ \text{👩}, \ldots, \text{👩} \} \subseteq V$

Submodular

- Let $D = \{$👩$, \ldots ,$ 👩$\} \subseteq V$
- Let $F(D) = OPT$ (w.r.t D)

Submodular

- Let $D = \{$$, \dots,$$\} \subseteq V$
- Let $F(D) = OPT$ (w.r.t D)

Theorem

F is submodular

Submodular

- Let $D = \{$🚺$, \ldots,$ 🚺$\} \subseteq V$
- Let $F(D) = OPT$ (w.r.t D)

Theorem

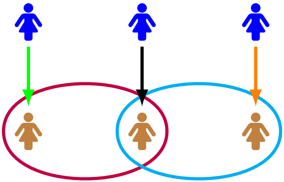
F is submodular

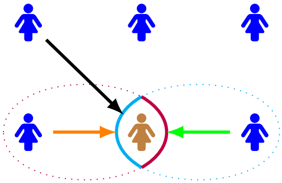
Corollary

MCM is $1/2$-approximable

Submodular cont.

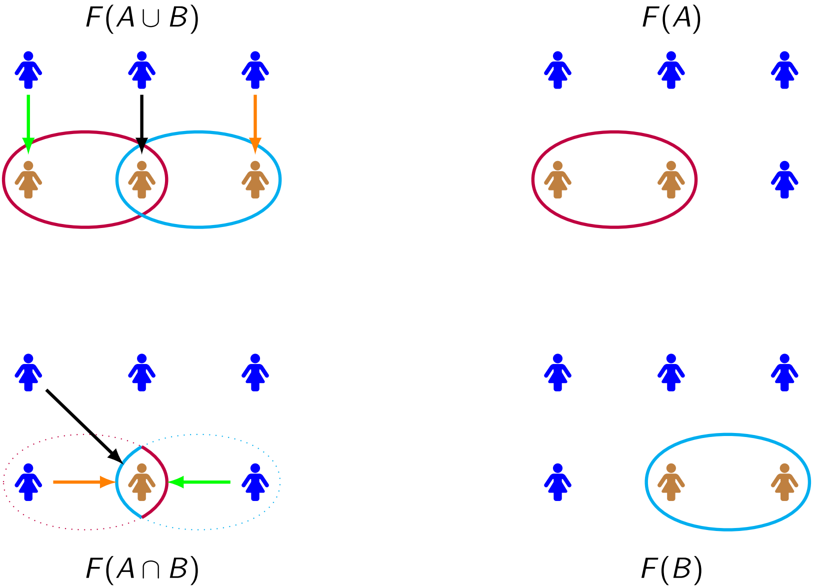
$F(A \cup B)$



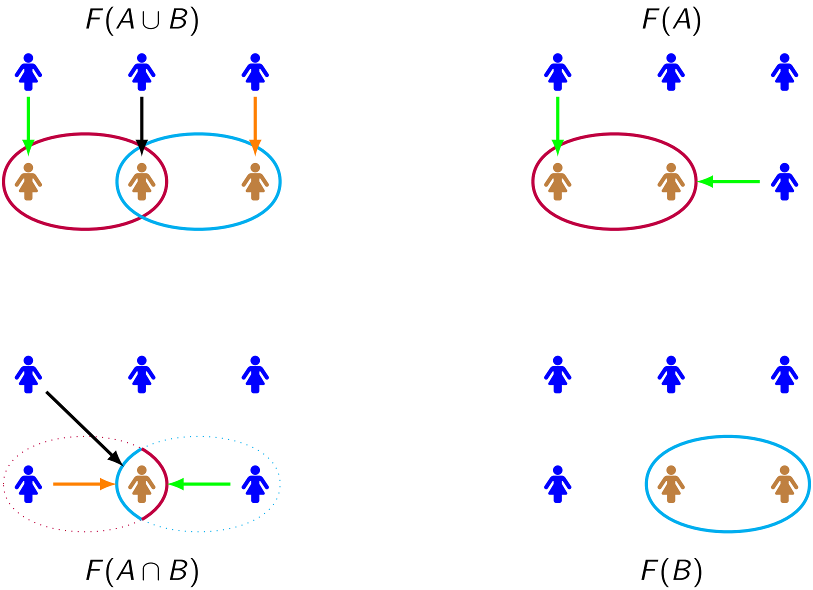


$F(A \cap B)$

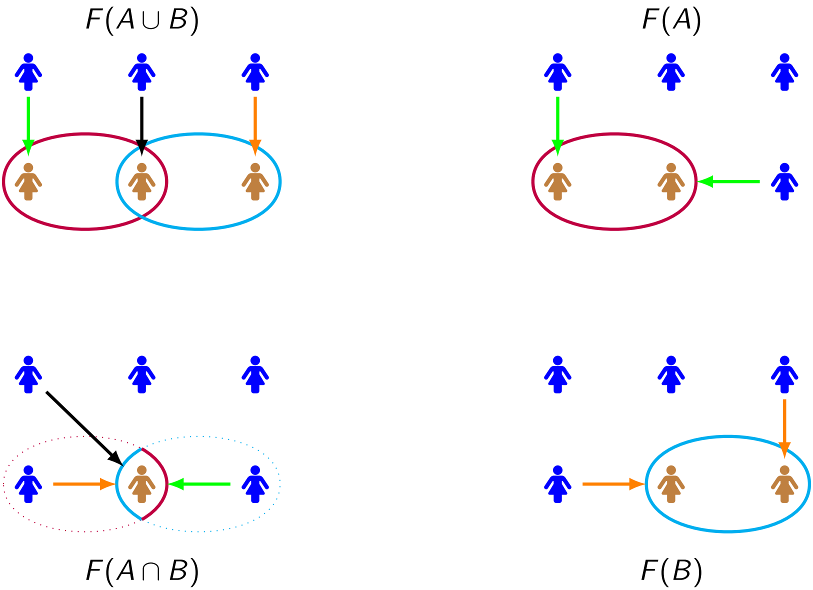
Submodular cont.



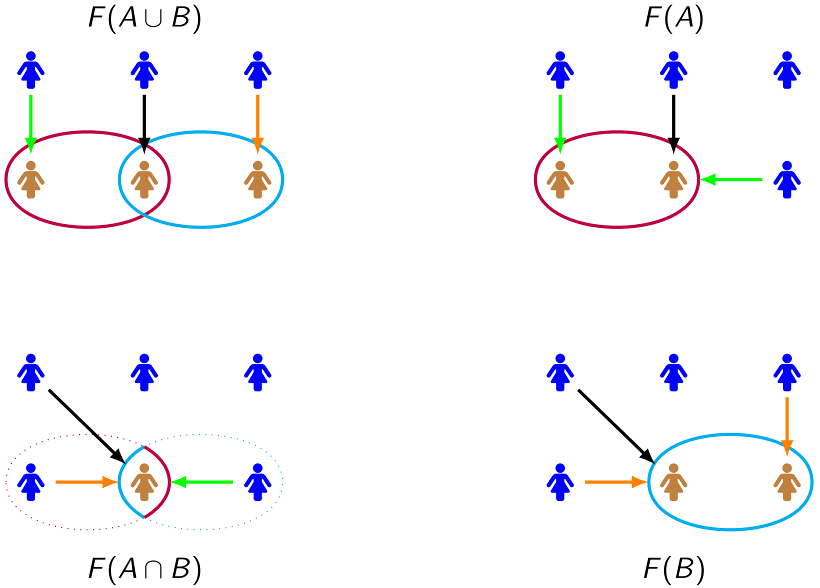
Submodular cont.



Submodular cont.



Submodular cont.



Summary

- APX-hardness for bounded degree graphs

Summary

- APX-hardness for bounded degree graphs
- MCM is submodular

Summary

- APX-hardness for bounded degree graphs
- MCM is submodular
  - 1/2-approximation

Summary

- APX-hardness for bounded degree graphs
- MCM is submodular
  - 1/2-approximation
- Group Carpool

Summary

- APX-hardness for bounded degree graphs
- MCM is submodular
  - 1/2-approximation
- Group Carpool
  - Not Submodular

Summary

- APX-hardness for bounded degree graphs
- MCM is submodular
  - 1/2-approximation
- Group Carpool
  - Not Submodular
  - $(1/2 - \varepsilon)$-approximation

Summary

- APX-hardness for bounded degree graphs
- MCM is submodular
  - 1/2-approximation
- Group Carpool
  - Not Submodular
  - $(1/2 - \varepsilon)$-approximation
- better approximation if unweighted

Summary

- APX-hardness for bounded degree graphs
- MCM is submodular
  - $1/2$-approximation
- Group Carpool
  - Not Submodular
  - $(1/2 - \varepsilon)$-approximation
- better approximation if unweighted
- Is $1/2$ the best we can hope for ?

Thank You.