The Convex Hull of Freeform Surfaces

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Abstract

We present an algorithm for computing the convex hull of freeform rational surfaces. The convex hull problem is reformulated as one of finding the zero-sets of polynomial equations; using these zero-sets we characterize developable surface patches and planar patches that belong to the boundary of the convex hull.

Key Words: Convex hull, common tangent, zero-set finding, freeform rational surface, B-spline, symbolic computation

1 Introduction

Computing the convex hull of a freeform surface is a challenging task in geometric modeling. Because of the difficulty in computing the exact convex hull of a spline surface, the convex hull of its control points is usually used as a simple, yet rough, approximation to the convex hull. When a tighter bound is needed for the convex hull, we can subdivide the surface into smaller pieces and then union the convex hulls of the control points of these small pieces. This simple approach will require a large number of subdivisions until the approximating convex hull converges to the exact convex hull within a certain reasonable bound. In this paper, we present an algorithm that computes the convex hull of a rational surface without resorting to a polygonal approximation of the given surface or the subdivisions.

The convex hull computation has applications in many important geometric problems such as interference checking in motion planning and object culling in graphics rendering. Interference between two convex objects is considerably easier to test than between two non-convex objects [13]. Consequently, convex hulls of general non-convex objects (or their rough approximations such as spheres and axis-aligned bounding boxes) are often used in a simple first test for checking the interference between the two original objects. (If there is no interference between the convex hulls, it is guaranteed that the original objects have no interference.) These applications motivated the development of many efficient convex hull algorithms in computational geometry [15]. The previous work, however, has been mostly limited to computing the convex hull of discrete points, polygons, and polyhedra [9, 12, 15].

There are a few previous algorithms that can compute the convex hulls of rational curves in the plane. Some of these algorithms are quite theoretical [2, 3, 16] in the sense that there are certain delicate issues that must be resolved for the efficiency and robustness of these algorithms to hold in actual implementation. More practical approaches were taken in two recent results reported by Elber et al. [7] and Johnstone [10]. The convex hull algorithm we present in this paper is based on extending our recent result [7] to the three-dimensional case. In the special case for a set of ellipsoids, Geismann et al. [8] presented an algorithm, which, using dualization, reduces the convex hull problem to that of computing a cell in an arrangement of quadrics.

Given a point $S(u, v)$ on a rational surface, let $T(u, v)$ denote the tangent plane of the surface at the point $S(u, v)$. For the point $S(u, v)$ to be on the boundary of the convex hull, the surface S should be completely contained in one side of the tangent plane $T(u, v)$. Thus the Gaussian curvature of $S(u, v)$ must be non-negative. For the sake of simplicity of presentation, we assume that the Gaussian curvature is positive on this surface point $S(u, v)$ and the tangent plane $T(u, v)$ intersects the surface S at no other point $S(s, t)$, for $(s, t) \neq (u, v)$.

Let D denote the region in the uv-domain where the surface patch $S(u, v)$ belongs to the boundary of the convex hull. In this paper, we show that the boundary of this region D can be computed in terms of the zero-set of three equations in four variables u, v, s, t, and sometimes in terms of the zero-set of two equations in three variables u, v, t. The boundary curves of the surface S may also contribute to the convex hull of S. Let I denote the parameter interval for a boundary curve segment that appears on the boundary of the convex hull of the surface S. The end points of the interval I can be computed by solving three equations in three variables, and sometimes by solving two equations in two variables. The boundary of the convex hull may contain some triangles, each of which is obtained from a tangent plane touching at three different points of the surface. These planes can be computed by solving a system of six equations in six variables.

The rest of this paper is organized as follows. In Section 2, we reduce the convex hull problem to computing the zero-sets of polynomial equations. The convex hull of rational space curves is needed to deal with the boundary curves of a rational surface. This is an important problem by itself; Section 3 thus presents an algorithm for computing the convex hull of a rational space curve. Section 4 computes tritangent planes by solving a system of six equations in six variables. Section 5 discusses how to combine all these components to construct the boundary of the convex hull. Finally, in Section 6, we conclude this paper.

2 The Convex Hull of Freeform Surfaces

In this section, we consider the convex hull of freeform surfaces. Although we will use one non-convex surface in the following discussion for clarity, one may consider the second surface point $S(s, t)$ as a point on another surface.

Let $S(u, v)$ be a regular C^1-continuous rational surface. Consider the tangent plane of S at $S(u, v)$ as a moving plane while continuously touching the surface tangentially. Then, any surface point $S(u, v)$ such that the surface is completely contained in one side of the tangent plane is on the boundary of the convex hull of the surface S (see Figure 1(a)). On the other hand, if the tangent plane at $S(u, v)$ intersects the surface at any other surface point $S(s, t)$, then the surface point $S(u, v)$ cannot be on the boundary of the convex hull of the surface S (see Figure 1(b)). Note, however, that one must be careful of common bitangent planes because a surface point at which the tangent plane is also tangent to some other surface point could be on the boundary of the convex hull (see Figure 2).

The tangent plane of S at $S(u, v)$ contains another surface point $S(s, t)$ if and only if

$$\begin{aligned}\mathcal{F}(u, v, s, t) &= \langle S(u, v) - S(s, t), N(u, v)\rangle \\ &= |S(u, v) - S(s, t) \quad S_u(u, v) \quad S_v(u, v)| \\ &= 0,\end{aligned}$$

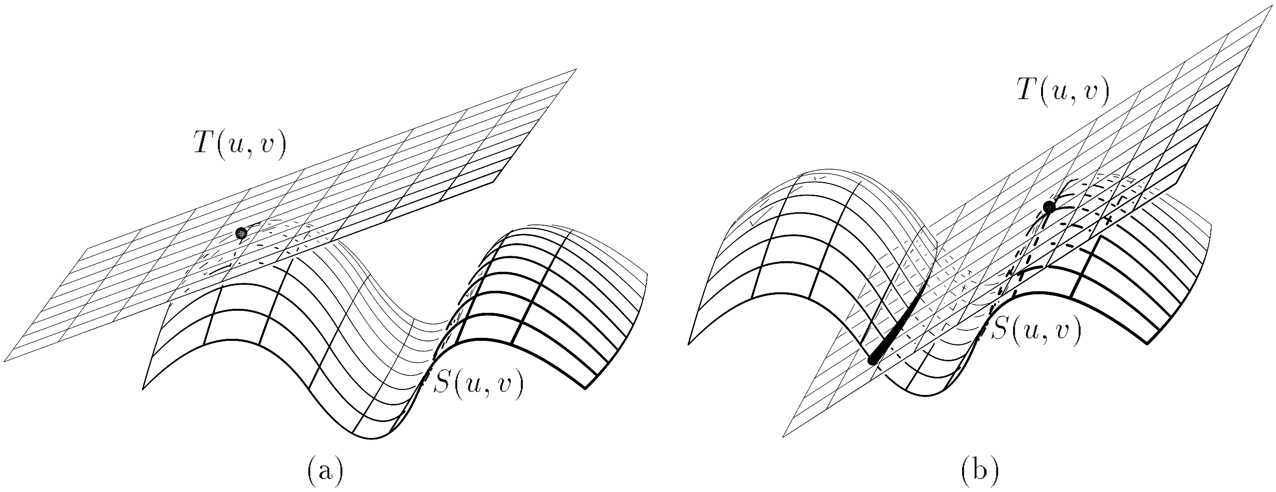


Figure 1: (a) S is completely in one side of $T(u,v)$; and (b) $T(u,v)$ intersects S transversally.

where $N(u,v) = S_u(u,v) \times S_v(u,v)$ is the normal vector field of the surface S and $|\cdot|$ denotes the determinant of a matrix consisting of three column vectors. Then, the set of surface points, for which the tangent plane intersects the surface S at no other points, is defined as follows:

$$\mathcal{CH}^o(S) = \{\ S(u,v)\ \mid \mathcal{F}(u,v,s,t) \neq 0,\ \forall (s,t) \neq (u,v)\}\,.$$

This set $\mathcal{CH}^o(S)$ is clearly a subset of the boundary of the convex hull of S and it is also a subset of the surface S itself:

$$\mathcal{CH}^o(S) \subset \mathcal{CH}(S) \cap S. \tag{1}$$

The difference $(\mathcal{CH}(S) \cap S) \setminus \mathcal{CH}^o(S)$ contains some extra points such as: (i) the boundary curve of each connected surface patch of $\mathcal{CH}^o(S)$ and, sometimes, (ii) the boundary curve of S itself if S is not a closed surface. (For now, we assume that each tangent plane of S is tangent to $S(u,v)$ at no more than two surface points; thus all isolated points and isolated curve segments of $\mathcal{CH}(S) \cap S$ must be on the boundary curve of S.)

As mentioned above, the zero-set of $\mathcal{F}(u,v,s,t) = 0 \wedge (u,v) \neq (s,t)$ in the $uvst$-domain cannot contribute to the boundary of the convex hull of the surface $S(u,v)$. That is, if the point (u,v) in the parametric domain falls into the projection of this zero-set, then the corresponding surface point $S(u,v)$ cannot be on the boundary of the convex hull of S. The boundary of the 'uncovered' region of the uv-plane (under this projection) is characterized as the projection of the st-silhouette curves (along the st-direction) of the zero-set. This means that the s-partial derivative and the t-partial derivative must simultaneously vanish along the silhouette curve, which can be characterized as the intersection of the following three hypersurfaces in the $uvst$-space:

$$\mathcal{F}(u,v,s,t) \;=\; 0, \tag{2}$$
$$\mathcal{F}_s(u,v,s,t) \;=\; 0, \tag{3}$$
$$\mathcal{F}_t(u,v,s,t) \;=\; 0. \tag{4}$$

One should consider only the solutions satisfying $(u,v) \neq (s,t)$ from the above equations. Furthermore, if (u,v) and (s,t) are on the same surface, only $(u,v) > (s,t)$ should be considered, in lexicographic order.

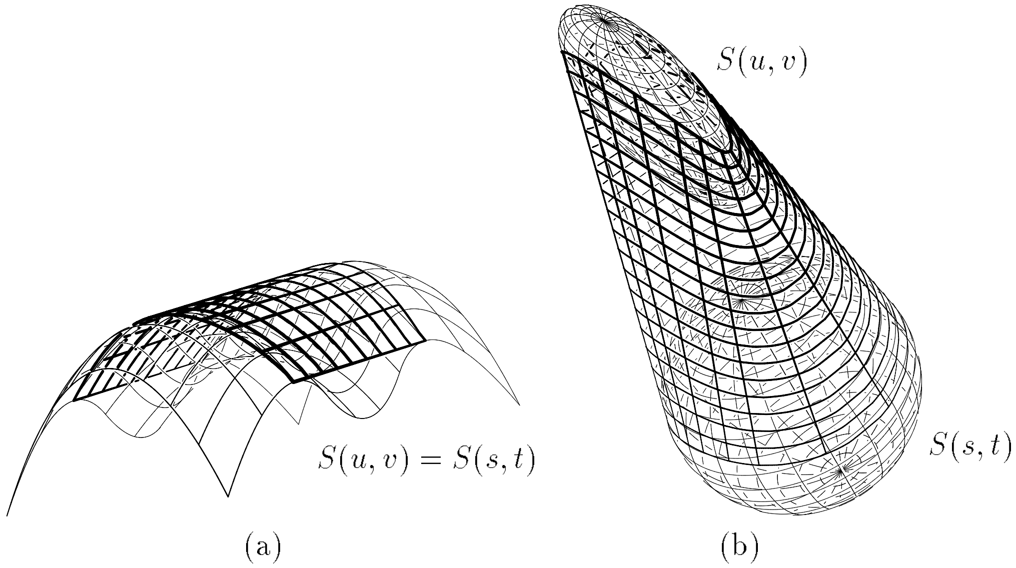


Figure 2: (a) A developable surface patch on the boundary of the convex hull of a rational surface $S(u, v)$ and (b) a similar patch that bounds two rational surfaces $S(u, v)$ and $S(s, t)$.

Now, consider Equations (3) and (4) differently:

$$\begin{aligned}\mathcal{F}_s(u, v, s, t) &= \frac{\partial}{\partial s} \langle S(u, v) - S(s, t), N(u, v)\rangle, \\ &= -\langle S_s(s, t), N(u, v)\rangle = 0, \\ \mathcal{F}_t(u, v, s, t) &= -\langle S_t(s, t), N(u, v)\rangle = 0.\end{aligned}$$

Therefore, Equations (3) and (4) characterize the condition that the tangent plane at the surface point $S(u, v)$ is also tangent to the surface at the other point $S(s, t)$.

Having three Equations (2), (3), and (4) in four variables, one gets a univariate curve as the simultaneous solution in the *uvst*-space. This solution curve can be parametrized by a variable α:

$$(u(\alpha), v(\alpha), s(\alpha), t(\alpha)).$$

Denote by T_α the common bitangent plane of S at $S(u(\alpha), v(\alpha))$ and $S(s(\alpha), t(\alpha))$. Then, the surface patch that bounds the convex hull of the surface S can be constructed by connecting the corresponding surface points $S(u(\alpha), v(\alpha))$ and $S(s(\alpha), t(\alpha))$ by line segments. All these line segments correspond to the common bitangent plane T_α. Consequently, the surface patch that bounds the convex hull of S is the envelope surface of the tangent planes T_α, which is a developable surface [1, 4, 11, 14]. We consider only solution points which satisfy $\langle N(u, v), N(s, t)\rangle > 0$. Redundant solutions (u, v, s, t), where $\langle N(u, v), N(s, t)\rangle < 0$, should be purged away. Figure 2(a) shows a developable surface patch that bounds the convex hull of a B-spline surface; Figure 2(b) shows a similar surface patch that bounds two convex surfaces.

In the above discussion, we assumed that both the surface points $S(u, v)$ and $S(s, t)$ are interior points of the surface S. When one of the two points is located on the boundary curve of S, the above characterization needs some refinements. We assume that the second point $S(s_0, t)$ is located on the boundary curve of S with a fixed parameter $s = s_0$. Then, the above characterization can be modified as follows

$$\begin{aligned} \mathcal{F}(u, v, s_0, t) &= 0, \\ \mathcal{F}_t(u, v, s_0, t) &= 0. \end{aligned} \tag{5}$$

The zero-set of Equation (5) generates a silhouette curve in the uvt-space, which can be parametrized by a variable α:

$$(u(\alpha), v(\alpha), t(\alpha)).$$

The bounding developable surface patch can be constructed by connecting the corresponding surface points $S(u(\alpha), v(\alpha))$ and $S(s_0, t(\alpha))$ by line segments.

The curve $(u(\alpha), v(\alpha))$ outlines the boundary on the surface patch $S(u, v)$ that belongs to the boundary of the convex hull of the surface S. The end points of the curve segment $S(s_0, t)$ that belong to the boundary of the convex hull are computed by solving the following three equations in three variables:

$$\begin{aligned} \mathcal{F}(u, v, s_0, t) &= 0, \\ \mathcal{F}_u(u, v, s_0, t) &= 0, \\ \mathcal{F}_v(u, v, s_0, t) &= 0. \end{aligned}$$

The second surface point may be a corner point $S(s_0, t_0)$ as well. In this case, one needs to solve the following bivariate equation:

$$\mathcal{F}(u, v, s_0, t_0) = 0,$$

which characterizes the tangent planes from $S(s_0, t_0)$ to the freeform surface $S(u, v)$. The zero-set generates a curve in the uv-plane, which can be parametrized by a variable α:

$$(u(\alpha), v(\alpha)).$$

The bounding developable surface patch is constructed as a conical surface that has its apex at the corner point $S(s_0, t_0)$ and is generated by the curve $S(u(\alpha), v(\alpha))$ on the surface S.

The convex hull of a freeform surface also contains the convex hull of its boundary space curves. The approach to computing the convex hull of a rational space curve is slightly different from the above three cases. There is no unique tangent plane at a point on a space curve. The tangent planes form a one-parameter family of planes, each of which contains the tangent line of the space curve. Representing all tangent planes at a curve point requires one additional parameter for the rotation about the tangent line. Moreover, computing the convex hull of a space curve is an interesting problem by itself. The next section is thus devoted to this special case.

3 The Convex Hull of a Space Curve

In this section, we assume that the rational space curve $C(t)$ is non-planar; that is, its torsion is non-zero at all curve points except at finitely many inflection points. (The convex hull of a planar curve can be computed using the results of Elber et al. [7] and Johnstone [10].) The convex hull of a freeform space curve $C(t)$ is characterized by the planes that are tangent to the curve at two different locations.

Assume that there is a plane tangent to the curve C at two different locations $C(s)$ and $C(t)$. Then the three vectors of $C'(s)$, $C'(t)$, and $C(s) - C(t)$ are parallel to the tangent plane. Hence, the determinant of a matrix consisting of these three vectors must vanish:

$$\begin{aligned}\mathcal{G}(s,t) &= |C(s) - C(t) \;\; C'(s) \;\; C'(t)| \\ &= \langle C(s) - C(t), C'(s) \times C'(t) \rangle \\ &= 0,\end{aligned}$$

which is a necessary condition for the existence of a common tangent plane at the two curve points $C(s)$ and $C(t)$. Once again, we need to consider only the solutions satisfying $s \neq t$.

The zero-set of the above equation can be locally parametrized by t (or by s): $\mathcal{G}(s(t),t) = 0$ (or $\mathcal{G}(s,t(s)) = 0$). By connecting the corresponding curve points $C(s(t))$ and $C(t)$ by line segments, we can construct the surfaces that bound the convex hull of the space curve $C(t)$. Again, these surfaces are developable surfaces because all the line segments connecting the points $C(s(t))$ and $C(t)$ are from the common tangent planes and so the surfaces are envelope surfaces of these common tangent planes [14]. Figure 3(a) shows a space curve and Figure 3(b) shows its convex hull bounded by developable surface patches.

We now consider how to compute the interior points of the curve segment $C(t)$ that belong to the boundary of the convex hull. Let $T(t)$ denote the common tangent plane at two corresponding curve locations $C(s(t))$ and $C(t)$. This plane may not bound the convex hull of the curve C if the curve C intersects this plane transversally at a third point $C(u)$, where $u \neq s(t)$ and $u \neq t$. This condition can be formulated as follows

$$\begin{aligned}\mathcal{H}(u,s,t) &= |C(u) - C(t) \;\; C'(s) \;\; C'(t)| \\ &= \langle C(u) - C(t), C'(s) \times C'(t) \rangle \\ &= 0,\end{aligned}$$

but

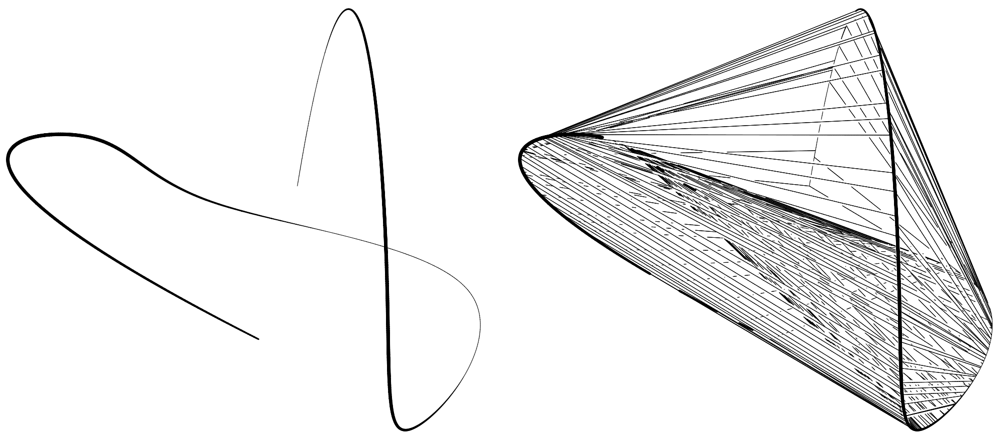
$$\begin{aligned}\mathcal{H}_u(u,s,t) &= |C'(u) \;\; C'(s) \;\; C'(t)| \\ &= \langle C'(u), C'(s) \times C'(t) \rangle \\ &\neq 0,\end{aligned}$$

where $\mathcal{H}_u$ is the u-partial of $\mathcal{H}$.

The redundancy of the solution (s,t) changes only through the configurations where three curve points $C(u), C(s), C(t)$ admit a common tangent plane. The condition is characterized by the following three equations:

$$\begin{aligned}\mathcal{G}(s,t) &= 0, \\ \mathcal{H}(u,s,t) &= 0, \\ \mathcal{H}_u(u,s,t) &= 0.\end{aligned}$$

In the above discussion, we assumed that both the curve points $C(s)$ and $C(t)$ are interior points of the curve C. When one of the two points is an end point $C(s_0)$ of the curve C, we need a modification to this approach. Let $T(t)$ denote the tangent plane from a fixed point $C(s_0)$ to



(a) (b)

Figure 3: $C(t)$ is a rational space curve with non-zero torsion: (a) shows a rational space curve $C(t)$; and (b) shows the developable surface patches that bound the convex hull of $C(t)$.

the curve $C(t)$. Assume that the curve C transversally intersects the tangent plane $T(t)$ at a third point $C(u)$, $u \neq t$. Then, we have the following condition:

$$\begin{aligned}
\mathcal{K}(u,t) &= |C(u) - C(s_0) \;\; C(t) - C(s_0) \;\; C'(t)| \\
&= \langle C(u) - C(s_0), (C(t) - C(s_0)) \times C'(t) \rangle \\
&= 0,
\end{aligned}$$

but

$$\begin{aligned}
\mathcal{K}_u(u,t) &= |C'(u) \;\; C(t) - C(s_0) \;\; C'(t)| \\
&= \langle C'(u), (C(t) - C(s_0)) \times C'(t) \rangle \\
&\neq 0.
\end{aligned}$$

The redundancy of the solution t changes only through the configurations where three curve points $C(s_0), C(u), C(t)$ admit a common tangent plane. The condition is characterized by the following two equations:

$$\begin{aligned}
\mathcal{K}(u,t) &= 0, \\
\mathcal{K}_u(u,t) &= 0.
\end{aligned}$$

Figure 4 shows a rational space curve $C(t)$ and its convex hull which is bounded by two conical developable surfaces, where each surface has its apex at an end point of the curve and is generated by the curve $C(t)$.

4 Tritangent Planes

In Section 2, we considered bitangent planes. In this section, we consider how to deal with tritangent planes. The bitangent condition prescribes a curve in the parametric domain. The case

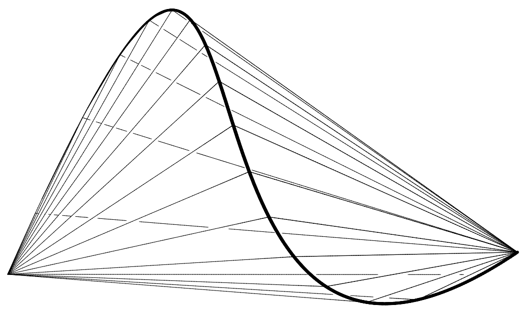


Figure 4: $C(t)$ is a rational space curve; its convex hull is bounded by two conical developable surface patches.

of tritangency results in a solution with zero dimension. To form the complete convex hull, one needs to combine these tritangent planes with the bitangent developable surfaces which bound the convex hull.

We can extend the three equations (2)–(4) from Section 2 to the tritangent condition. Let the three tangent points be $S(u,v), S(s,t)$ and $S(m,n)$. Then, the three equations (2)–(4) constrain the tangent plane at $S(u,v)$ to be tangent to surface $S(s,t)$. We now add three more equations to constrain this tangent plane to be tangent to the third surface point $S(m,n)$ as well. Consequently, we have six equations in six variables:

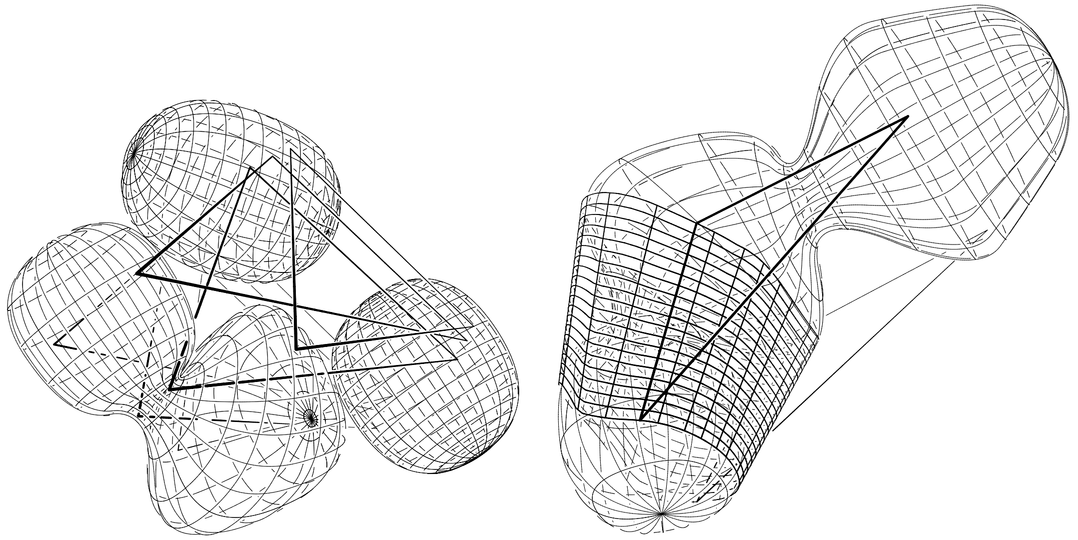
$$\begin{aligned} \mathcal{F}(u,v,s,t) &= \langle S(u,v) - S(s,t), N(u,v)\rangle = 0, & (6)\\ \mathcal{F}_s(u,v,s,t) &= -\langle S_s(s,t), N(u,v)\rangle = 0, & (7)\\ \mathcal{F}_t(u,v,s,t) &= -\langle S_t(s,t), N(u,v)\rangle = 0, & (8)\\ \mathcal{F}(u,v,m,n) &= \langle S(u,v) - S(m,n), N(u,v)\rangle = 0, & (9)\\ \mathcal{F}_m(u,v,m,n) &= -\langle S_m(m,n), N(u,v)\rangle = 0, & (10)\\ \mathcal{F}_n(u,v,m,n) &= -\langle S_n(m,n), N(u,v)\rangle = 0. & (11) \end{aligned}$$

These six equations (6)–(11) in six variables have a zero-dimensional solution or a finite set of points. Examples for common tritangent planes to three surfaces are shown in Figure 5.

5 Trimming and Combining the Convex Hull

As in the case of Section 2, the tritangent planes include some redundant solutions. The redundancy can be eliminated using the following three conditions:

$$\langle N(u,v), N(s,t)\rangle > 0 \quad (12)$$
$$\langle N(u,v), N(m,n)\rangle > 0 \quad (13)$$
$$\langle S(x,y) - S(u,v), N(u,v)\rangle \neq 0,\ \forall (x,y) \neq (u,v), (s,t), (m,n). \quad (14)$$



(a) (b)

Figure 5: (a) All the common tritangent planes to three surfaces. (b) One developable surface which bounds first two convex parts of the surface and the planes (triangles) which are common tangent to the surface.

The first two Equations (12) and (13) enforce the tritangent planes to have their normal vectors in the same direction. Then, Equation (14) constrains the planes not to pass through the interior of the convex hull.

To complete the convex hull construction, one needs to compute bitangency for each pair of surface parts: the bitangency of $S(u, v)$ and $S(s, t)$, the bitangency of $S(s, t)$ and $S(m, n)$, and the bitangency of $S(m, n)$ and $S(u, v)$. Not all the bitangent developable surfaces contribute to the boundary of the final convex hull. One needs to trim out certain parts of these developable surfaces according to the common tritangent planes to the three surface parts.

Because all the developable surface patches, which form the convex hull of two surface parts as mentioned in Section 2, bound the convex hull of each pair of surface parts, the boundary curves of these developable surfaces always intersect with tritangent planes at the corresponding tritangent points (see Figure 5(b)). In fact, we can also determine the trimming line of each developable surface as an edge of an adjacent tritangent plane. As all the developable surfaces are generated by a moving bitangent plane between two surfaces, the trimmed developable surface is continuously connected to a tritangent plane. Figures 6 and 7 show two examples of the convex hull.

6 Conclusion

In this paper, we have presented an algorithm for computing the convex hull of freeform surfaces. The problem was first reformulated as a zero-set finding problem; and it is then solved by computing the zero-set and projecting the solutions onto a proper subspace.

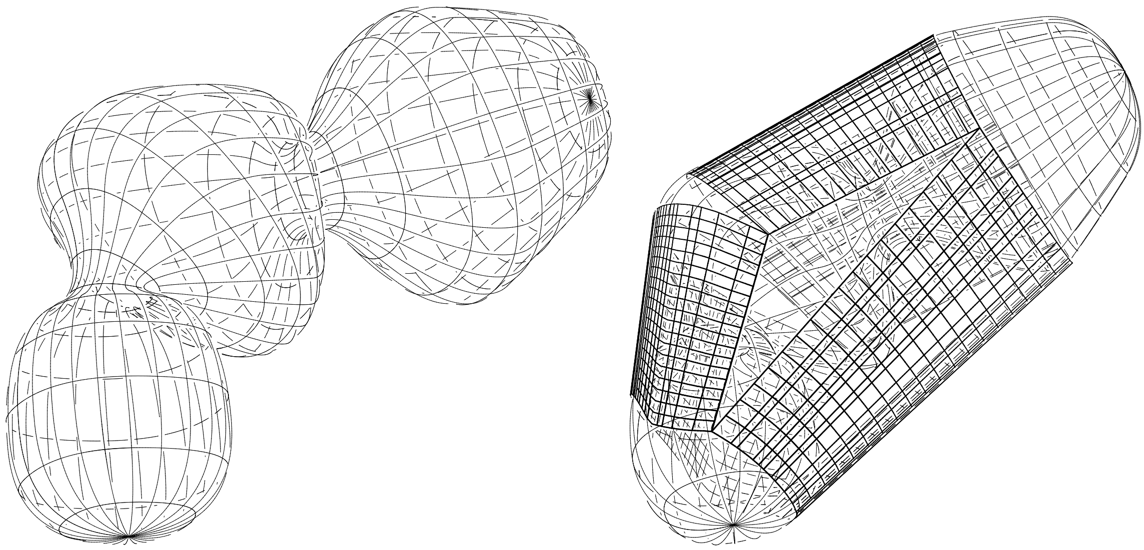


Figure 6: (a) A surface with three convex parts; and (b) its convex hull.

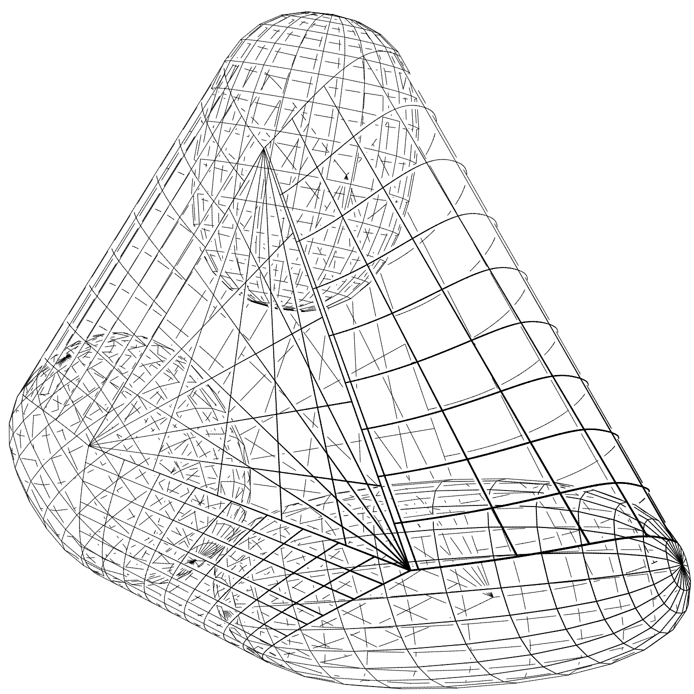


Figure 7: The convex hull of three ellipsoids.

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