Algorithms for Dynamic Memory Management (236780)

Lecture 5

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Topics last week

- Generational Garbage Collection.
- The Train Algorithm (for the old generation).

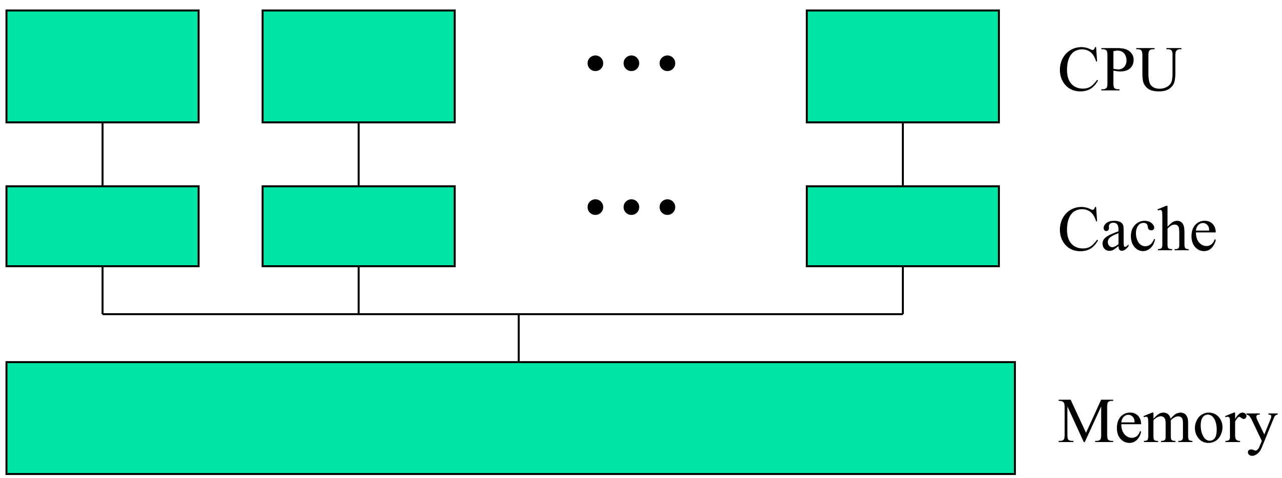
Today: Concurrent Garbage Collection

Plan (tentative)

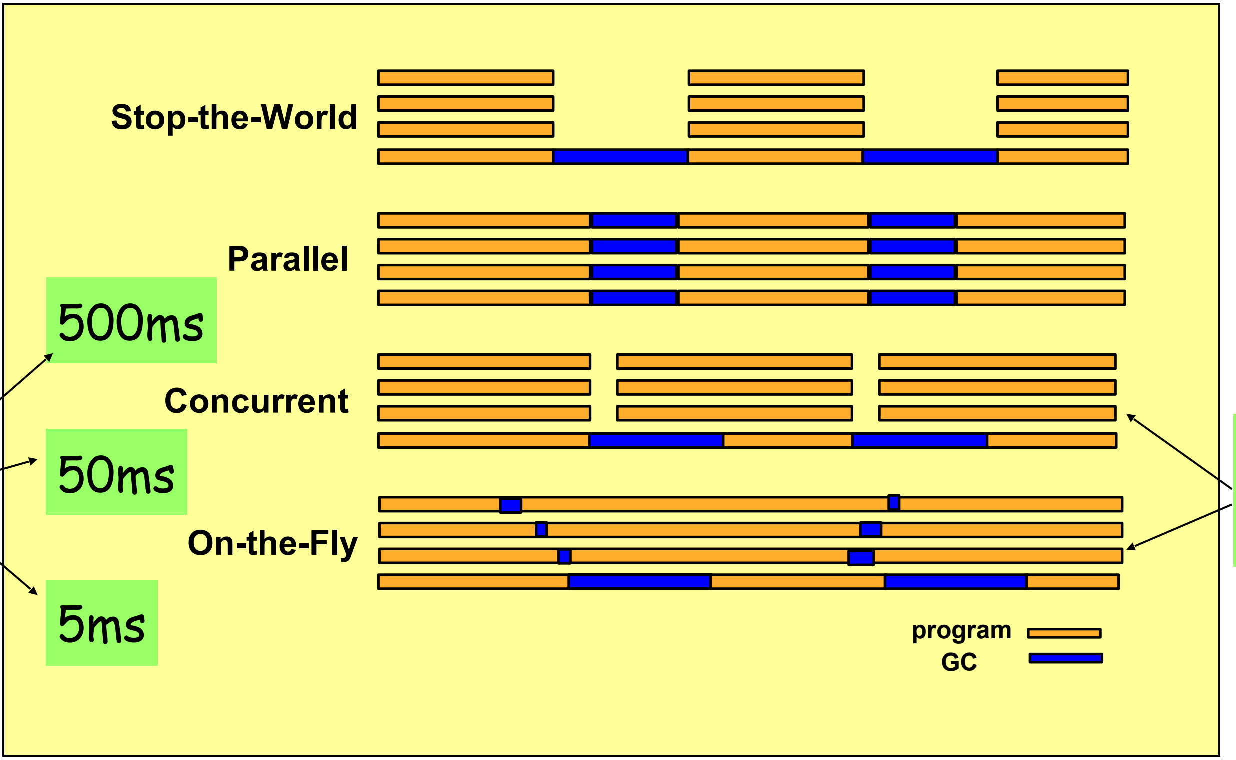
- On the fly collectors:
  - [Dijkstra-Lamport-Martin-Scholten-Steffens 1977]
  - [Doligez-Gonthier-Leroy 1993-94]
- Snapshot: the copy-on-write concurrent collector
  - [Demers-Weiser-Hayes-Boehm-Bobrow-Shenker 1990] + [Furusou-Matsuoka-Yonezawa 1991]
- Mostly concurrent collection
  - [Boehm-Demers-Shenker 1991] + [Printezis-Detlefs 2000] + [Ossia-Ben Yitzhak-Goft-Kolodner-Leikehman-Owshanko 2002] + [Barabash-Ossia-Petrank 2003]
- Sliding Views:
  - [Azatchi-Levanoni-Paz-Petrank 2003] following [Levanoni-Petrank 2001].

Platfrom in Mind

- Multiprocessors (SMP) and multicores. Yesterday's servers, today's desktops, laptops, and smartphones.
- Shared memory.



Terminology



Informal Pause times

500ms

50ms

5ms

Throughput Loss: 10%

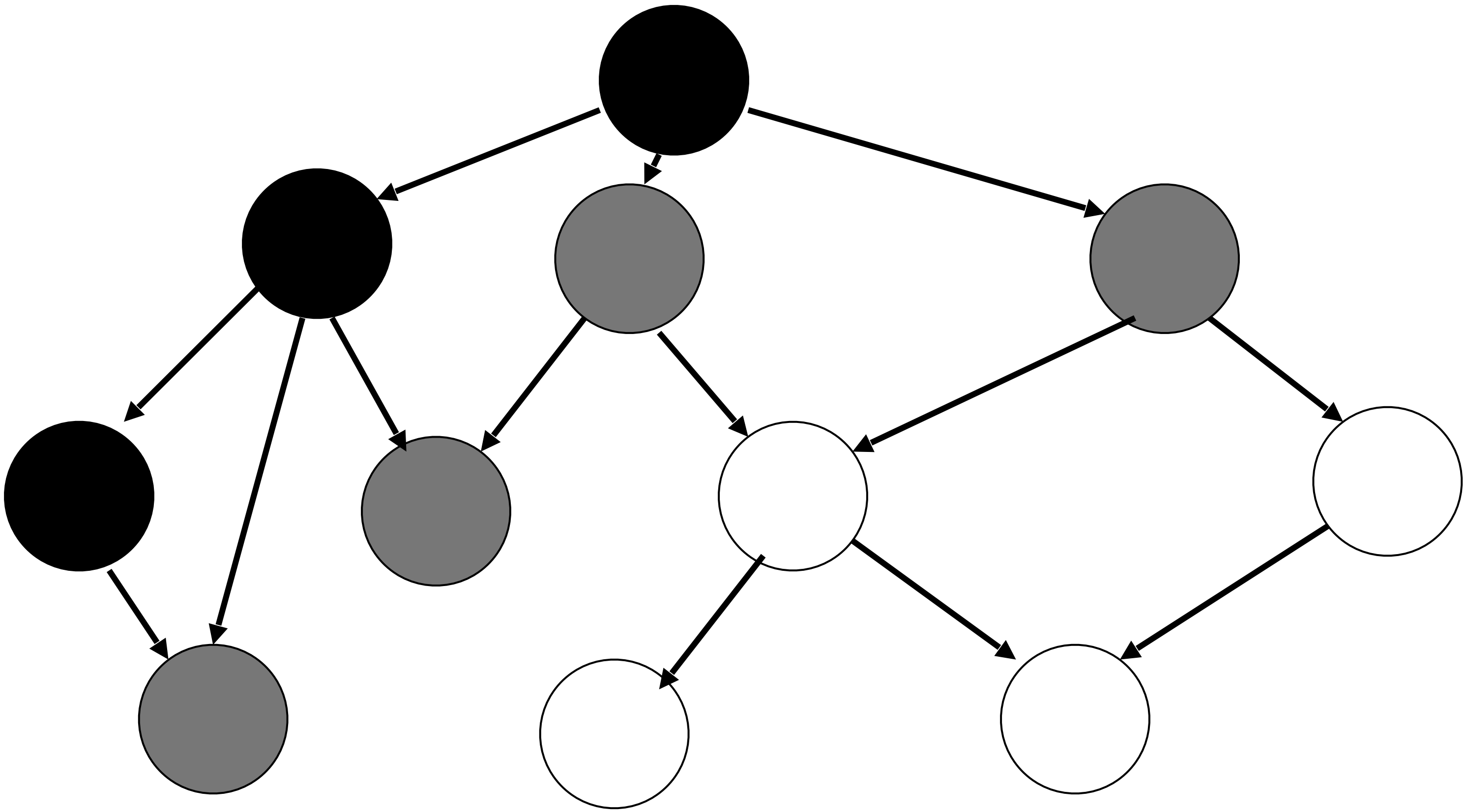
Part I: On-the-fly Garbage Collection

[Dijkstra-Martin-Steffens-Lamport-Scholten 1976]

Recall Three-Color Abstraction

- Black – objects that have been marked and their children have been marked as well.
- Gray – objects have been marked but their children have not been traced yet.
- White – objects that have not yet been marked.

An Intermediate Tracing View



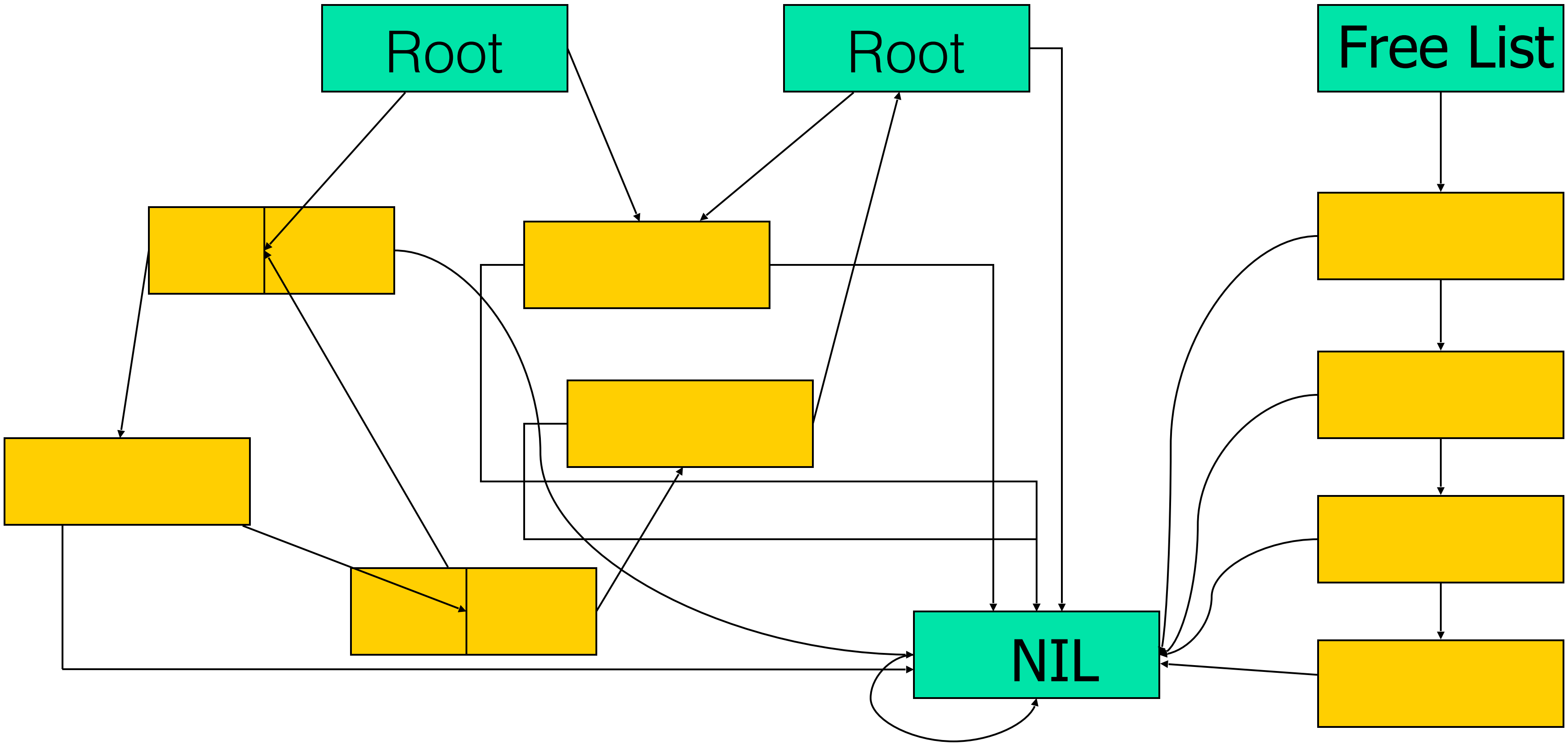
Simplified Setting: A Graph

- Assume all objects are of fixed size and are represented as nodes in a directed graph.
- Possible manipulations: add a new edge, delete an edge, redirect an edge
- Simplifying assumptions:
  - Free-list holds all nodes ready for allocation.
  - A special nil node. All null edges point to the NIL node.
- Thus, no adds & deletes, only:
  - redirect edge &
  - get/return object to the free-list.

Reachable Nodes

- Several nodes in graph are defined roots.
- Other nodes are reachable (or not) from the roots.
- Note: program redirects an edge only between reachable nodes (source & target)
- Final abstraction: define a special root for the free-list, and NIL as a root.
- Now all possible program operations are redirecting an outgoing edge of one reachable node to point to another reachable node

Abstract Graph



Goals

- Find unreachable nodes concurrently with program run.
- Fine-grained synchronization between program and collector (short pauses).
- Low mutator overhead (low penalty on throughput).
- Correctness:
  - Do not reclaim reachable objects.
  - Some floating garbage allowed.

Terminology

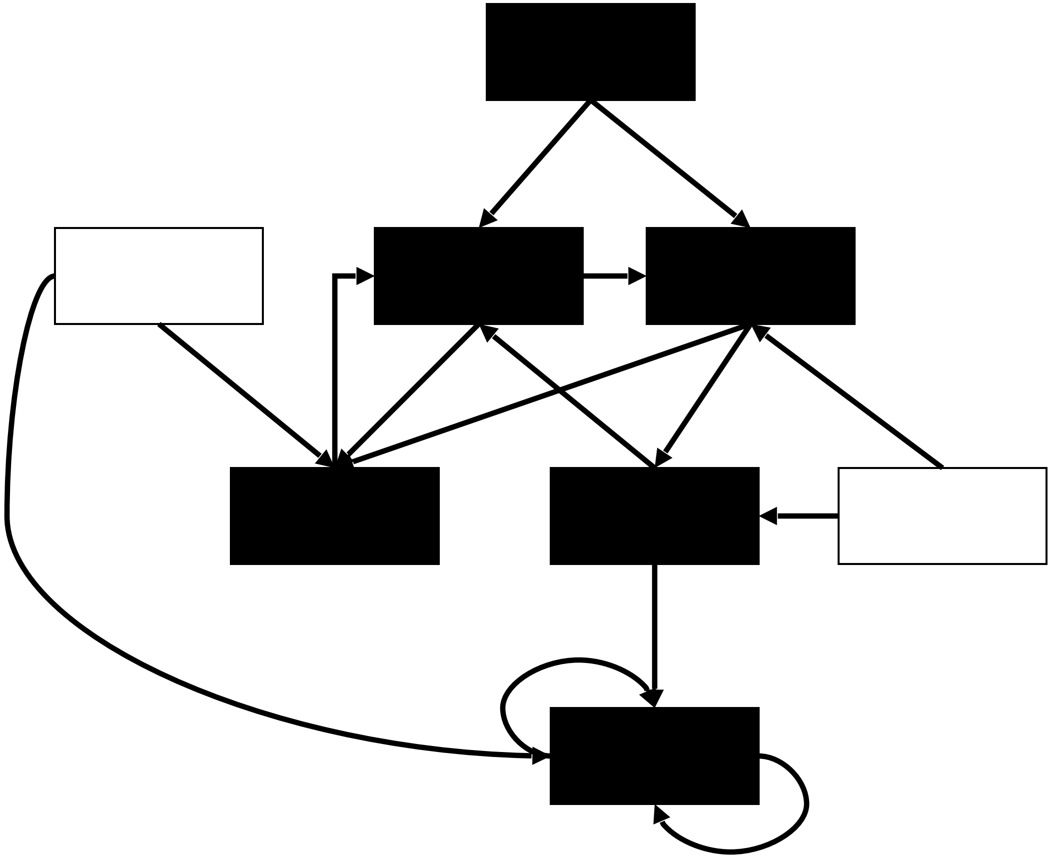
- **Mutator** – program thread.
- **Collector** – the thread running the garbage collection.
- **Shading** – Changing white node to gray (no change for black or gray nodes).
- **Floating Garbage** - unreachable objects that were not reclaimed.

- Dijkstra et al.:
  - Only one mutator is allowed.
  - The collector employs a mark & sweep algorithm.

Basic Collector Algorithm

- Mark Phase
  - Mark the roots gray.
  - For every gray node: shade its successors, and then mark it black.
  - Quit when there are no gray nodes.
- Sweep phase: Visit each node once:
    - If it is white – append it to the free-list.
    - If it is black – color it white.
- Mutator cooperation:
  - When changing an edge, shade the new target.

Basic Collector Algorithm - Example



Basic Marking Algorithm

- Termination by monotonicity:
  - Nodes only grow darker during marking phase.
- Correctness invariant:
  - No black to white edge during marking phase.
- Problem: invariant assumes that pointer modification operation is atomic:
  - Change an edge and shade the new target.

Basic Marking Algorithm - Problem

- We do not want to use a critical section in the write barrier, so we must break it into two atomic writes. Either:
  - Option 1: Change and then shade, or
  - Option 2: Shade and then change.
- But:
  - Change and then shade violates the invariant.
  - Shade and then change: a concurrent sweep can erase the shade…

The Plan

- Use “change and then shade”, but employ a better analysis.
- The “no black to white” invariant is not preserved.
- But a finer invariant does, actually, two invariants.

The Invariants

- Two invariants (P_1 <u>and</u> P_2) hold simultaneously during the marking phase (after marking the roots):

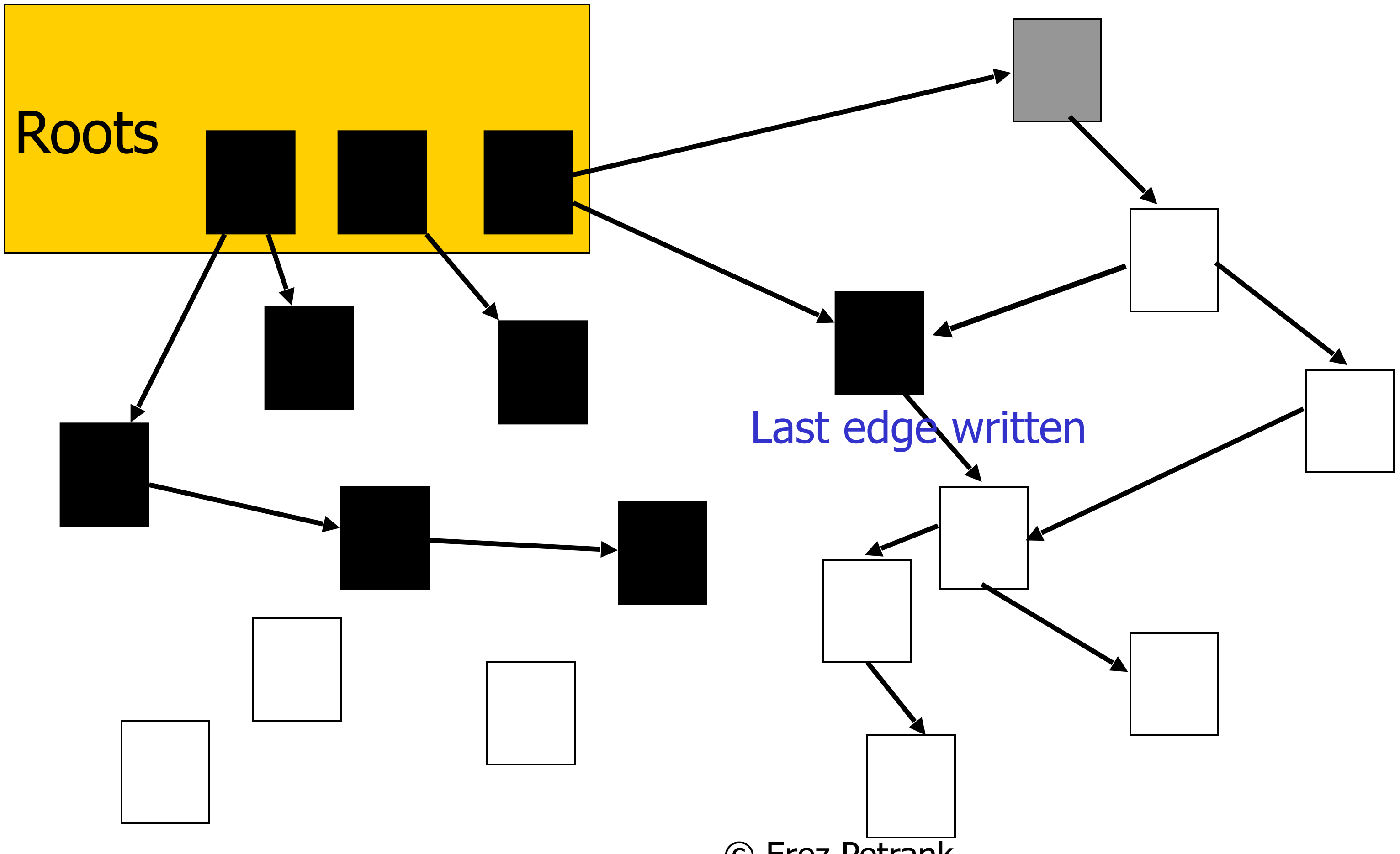
- **P_1** : For any white reachable node v, there exists a "propagation path" starting at a gray node, continuing through white nodes and ending in v.

- **P_2** : A black to white edge is allowed once in the graph: the edge most recently written by the mutator.

Observation

- If there are no black to white edges, after marking the roots, the two invariants trivially hold.
  1. P_1 – For any white reachable node v, there exists a "propagation path" starting at a gray node, continuing through white nodes and ending in v.
  2. P_2 – A black to white edge is allowed once in the graph: the edge most recently written by the mutator.

The invariants depicted



Remarks:

- The statement of P2 assumes that there is only one mutator. (There exists the last modification operation.)
- P1 suffices for the proof.
  - Implies that at the end of marking, when there are no gray nodes, all white nodes are unreachable.
- But P_1 is too weak to be shown as invariant --> P_2 is needed for the induction.

Proof Base

- After root scanning P1 and P2 hold.
  - No black to white edge
  - Everything reachable is reachable from a gray node by a path of white nodes.
- Next we show that each action preserves P1 & P2.
- For simplicity and w.l.o.g. actions do not happen simultaneously.

Mutator Actions

- Mutator actions
  - Mutator actions sequence look like:

    Change edge e_1

    Shade new target of edge e_1

    Change edge e_2

    Shade new target of edge e_2

    Change edge e_3

    Shade new target of edge e_3

  - The interesting points in the sequence are after change and before shade.
    At other times, there are no black-white edges.

Mutator Preserves Invariants

- We need to show that mutator's changing an edge preserves the invariant "P_1 and P_2".
- Preserving P_2 is easy.
  - After "change e_i", e_i may become a black to white edge. It is the recently modified edge.
  - After "shade target" e_i is not a black-white edge anymore. At this time there are no black-white edges at all.
  - By induction, P_2 is preserved.

Mutator Preserves P_1

- We show that P_1 holds, given that P_2 holds.
- Shading a vertex cannot destroy any propagation path.
- Thus, we concentrate on the “change” operation.
- We separate the analysis into two cases:
  - Source of modified edge is black
  - Source of modified edge is white or gray.

Preserving P_1 (black source)

- Suppose source of the modified edge is black.

  Preserving propagation paths: the edge that "disappeared" could not have been part of a propagation path since the source is black. Thus, all white reachable vertices must still have propagation paths.

- Note that for P1 we do not care about the new link, we only care about destroyed links.

Preserving P_1 (non-black source)

- Suppose source of the edge was white or gray:
  By P2, after the modification there are no black to white edges. (The recently modified edge has a gray/white source.) Thus, P1 holds trivially!

- (This is enough for the proof but:) what happens if (the last) propagation path is eliminated to the “previous” child?
- Answer: if there are no other propagation paths than this child is not reachable anymore.

Preserving P_1 (non-black source)

- Conclusion: Mutator actions do not break the invariant relation.

Collector Preserves Invariant

- To check that the collector preserves the invariants we need to take a look into the algorithmic details.

Marking phase: (M nodes, in array)

```
for each root j do "shade the root j";
while (gray nodes exist)
{
        choose a gray node N;
        shade N's children and make N black
}
```

Collector Actions (Makring phase)

- The collector may
  - C1: Shade a single root
  - C2: Check the color of a node
  - C3: Shade the successors of a node and make the node black.
- Run atomically, none of c1, c2, c3 can break (P_1 and P_2): Propagation paths cannot be eliminated, black-to-white edge cannot be created.

Collector Actions (Makring phase)

- Problem: C3 is a compound operation!
  - C3: Shade the successors of a node and make the node black.
- Details:
  1. For each child of N
     1. C = next child of N
     2. shade C
  2. make N black

Collector Actions (Makring phase)

- Details:
  1. For each child of N
     1. C = next child of N
     2. shade C
  2. make N black
- 1-1 and 1-2 clearly preserve P1 and P2. Shading a node cannot destroy a propagation path and cannot create a black-to-white edge.
- Let's check Operation 2…

Operation 2
(Making the node black after shading its children.)

- If mutator does not interfere all along:
  - All is well: Blackened node is not on a propagation path (P1) and black-to-white edge is not created (P2).
- Otherwise, mutator has modified one of node's pointers.
- If mutator pointer modification completed,
  - then all children are gray - no problem.
- If mutator concurrently redirects an edge from father to a new son, but hasn't shaded it yet:
  - Black-to-white edge may be created, legitimately.
  - A propagation path cannot be eliminated since previous descendant is gray.

Conclusion of proof

- P1 and P2 are preserved.
- By P1: collection is safe.

- Does this algorithm work with multithreading?

Intuition of proof

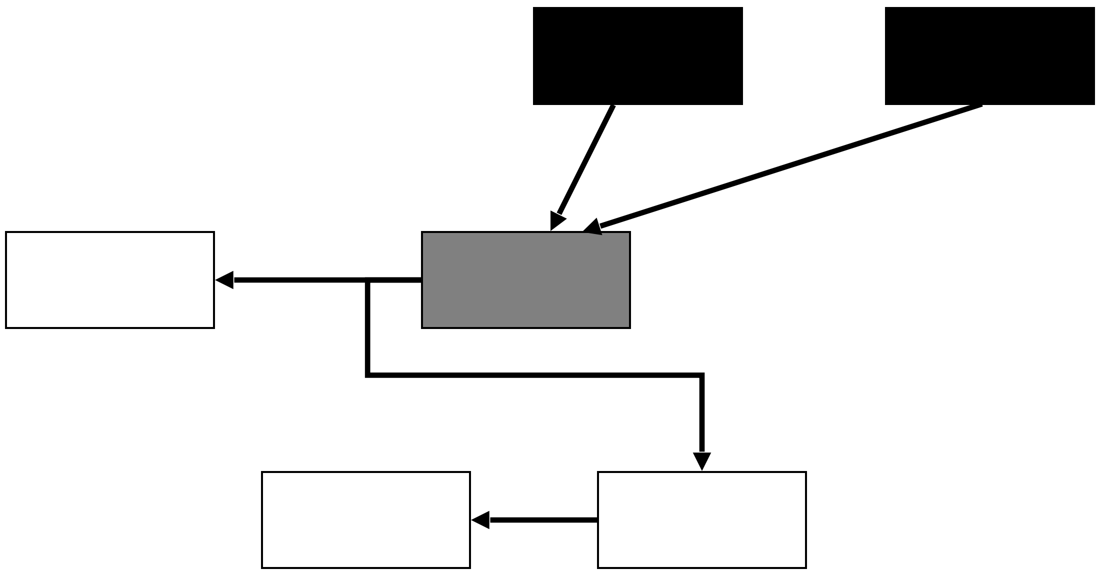
- If a black-white pointer is created, then the new white child must be reachable from somewhere else. That route must contain a propagation path because it has no black-to-white edges.

- This heavily relies on the fact that there is only one black-to-white edge and no other thread foils its propagation path.

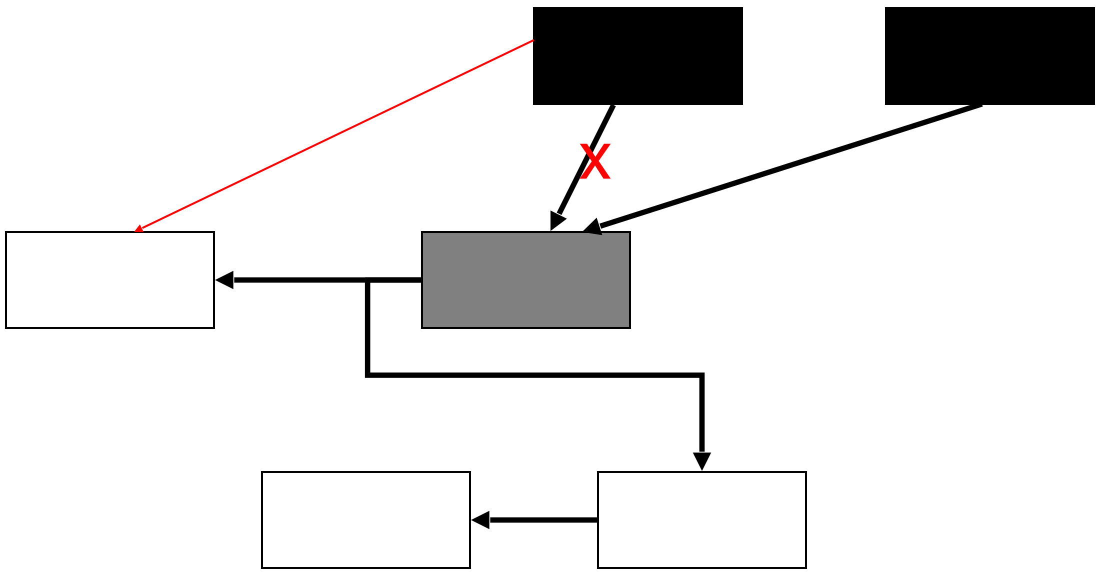
Multi-Threaded User Code

- The algorithm fails with multi-threaded programs (= many mutators)
- (P1 and P2) can't be kept invariant.
    - P2 is not even well defined.
- Let's look at a bad example.

Multi-Threaded Program – Example



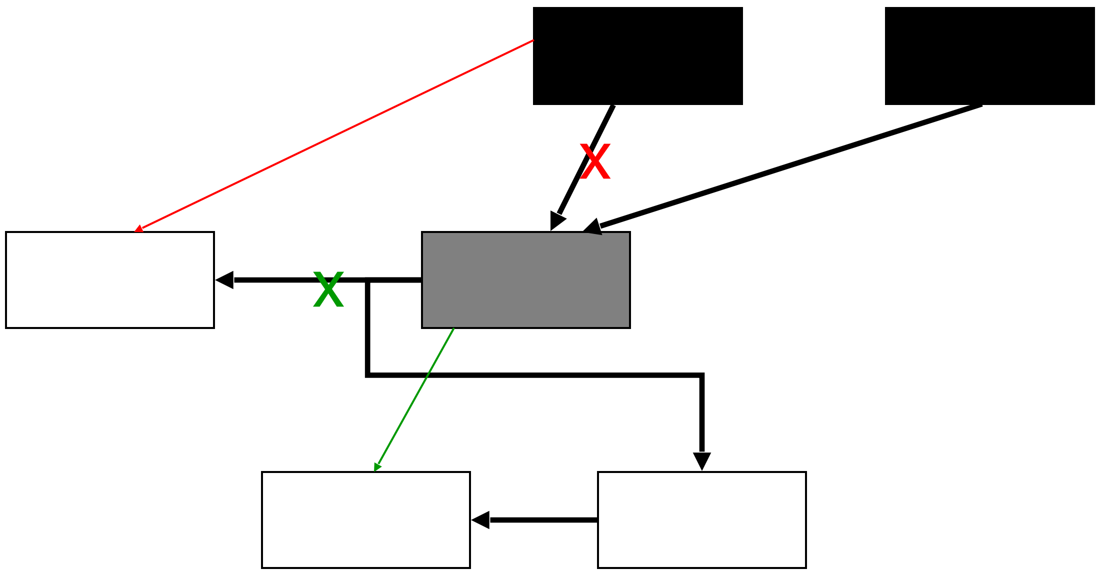
Multi-Threaded Program – Example



Mutator 1 in red

Mutator 2 in green

Multi-Threaded Program – Example



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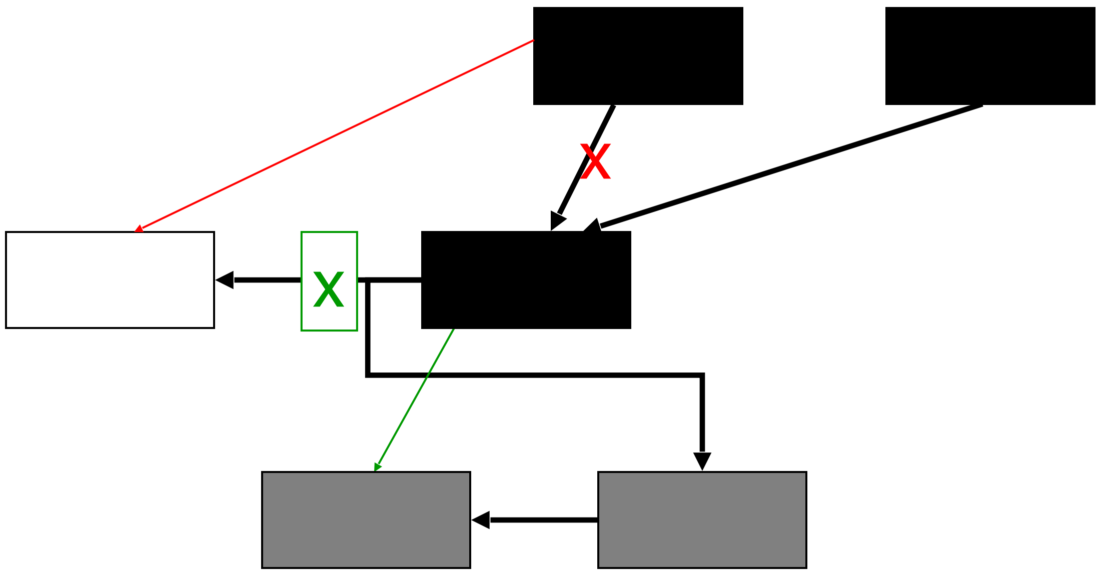

Mutator 1 in red

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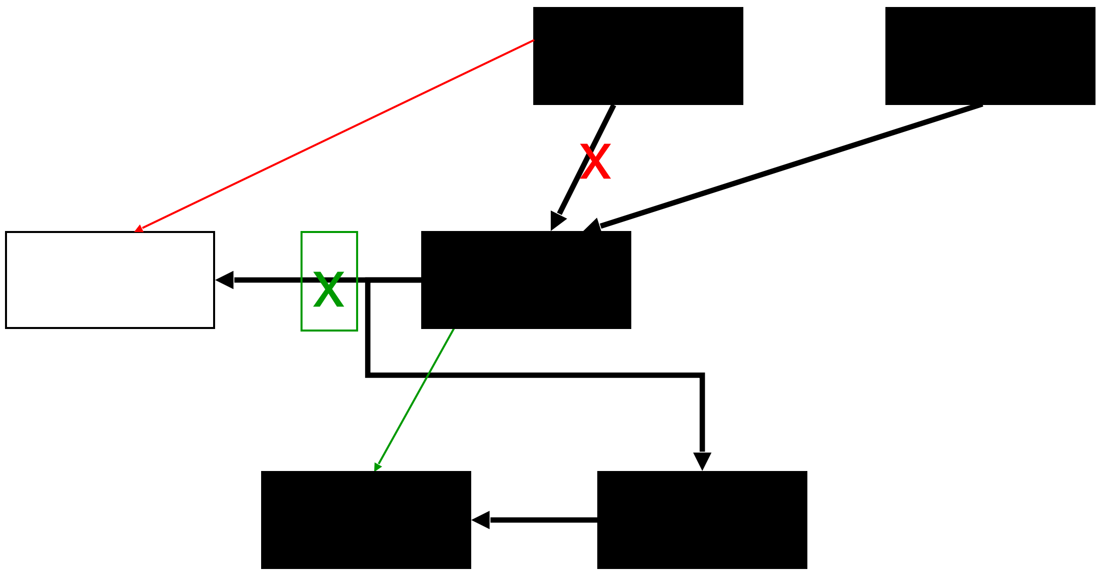
Multi-Threaded Program – Example



Mutator 1 in red

Mutator 2 in green

Properties of Dijkstra's Collector

- Concurrent with no synchronization.
- The algorithm is theoretical.
  - Marking the free list,
  - Write barrier on roots,
  - Unable to deal with multithreading.
- But, the ideas are innovative.
  - [Doligez-Leroy-Gonthier94] solved some of the issues.
  - [Domani et al. 00] made it practical and incorporated it into the IBM JVM.

Dijkstra, in Retrospect

- "Our exercise has not only been very instructive, but at times even humiliating, as we have fallen into nearly every logical trap possible."
- "It has been surprisingly hard to find the published solution and justification. It was only too easy to design what looked -- sometimes even for weeks and to many people -- like a perfectly valid solution, until the effort to prove it to be correct revealed a (sometimes deep) bug."

The DLG Algorithm

[Doligez-Leroy 1993] A concurrent, generational garbage collector for a multithreaded implementation of ML

[Doligez-Gonthier 1994] Portable, Unobtrusive Garbage Collection for Multiprocessor Systems

[Domani-Kolodner-Petrank 2000] Generational On-the-fly Garbage Collector for Java.

[Domani-Kolodner-Lewis-Salant-Barabash-Lahan-Petrank-Yanover-Levanoni 2000] Implementing an On-the-fly Garbage Collector for Java.

Original Paper

- Implementation for OCAML.
- Semi-generational, local heaps for immutable objects.
- We will concentrate on the main idea.
  - No local heaps, No semi-generations, No use of immutable objects
- We will not study the full algorithm and proof, but only point out some relevant problems and solutions.

Topics

- Eliminating free-list traversal.
- A race between allocation and sweep.
- Avoid repeated heap traversals.
- Using handshakes to accommodate “shade and then change”.
- Eliminating write-barrier on local variables.

Eliminating the free-list scan

- Dijkstra's algorithm traverses the free-list. This does not make sense in practice.
- DLG added a fourth color:
  - Blue = free list (neither traced, nor reclaimed)
  - White = unmarked
  - Gray = marked, children not visited
  - Black = marked and children visited.

The Color of New Objects

- If we allocate white – it may be reclaimed.
- During mark
  - black – it is true that they are reachable and their sons (none exist) have been traversed.
  - (gray is also OK, but no termination guarantee.)
- During sweep
  - Depending on object location.
  - white, if already swept
  - gray otherwise - to avoid reclamation

Race Allocation - Sweep

Sweep:

- If object(sweep_pointer) is black set to white
- If object(sweep_pointer) is white reclaim.

Allocation:

- If phase = marking then
  - Set object to black
- Else
  - If address(object) < sweep_pointer then
    - Set object to white
  - Else
    - Set object to black

Two problems:

- Perhaps *phase* value is outdated
- Perhaps *sweep_pointer* is outdated

The Solution

- It is always safe to set an object to gray
  - Allocation:

```
if phase = marking then
    set the object to black;
    if phase = sweeping then
        set the object to gray;
else
    if address(object) < sweep_pointer then
        set the object to white;
    else
        set the object to gray;
```

  - Sweep:
    - If object(sweep_pointer) is black: set to white
    - If object(sweep_pointer) is white: reclaim.

Avoid Repeated Heap Traversals

- Marking terminates when there are no gray objects in the heap.
- Saving heap traversals:
  - DLG went over the heap to find gray objects.
  - The practical Java implementations used a markstack
  - Parallel access to a markstack must be addressed in a modern implementation. We will elaborate in future lectures.

Recall Basic Write Barrier Problem

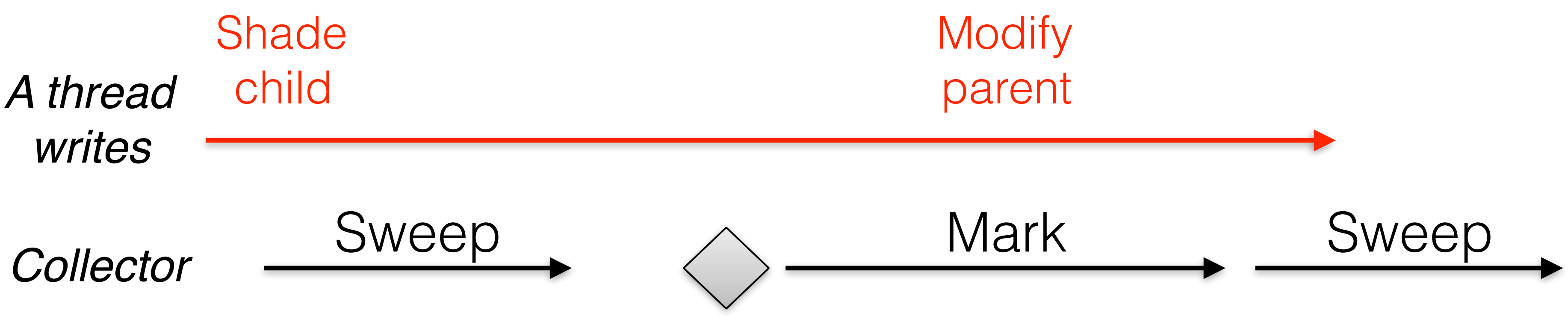
- Write barrier with atomic actions. Either:
  - Option 1: Change and then shade, or
  - Option 2: Shade and then change.
- But:
  - Change and then shade violates the invariant.
  - Shade and then change can violate that relation if there was sweeping phase in the middle.
- Dijkstra used option (1) with the extra P2. But P2 is not relevant for multithreading.

Write Barrier & Coordination

- DLG use the other option: “Shade and then change”
- Problem: shading may disappear before the change.
  - sweep terminates & marking starts again.

The Problem

- A long write extends between two collections.

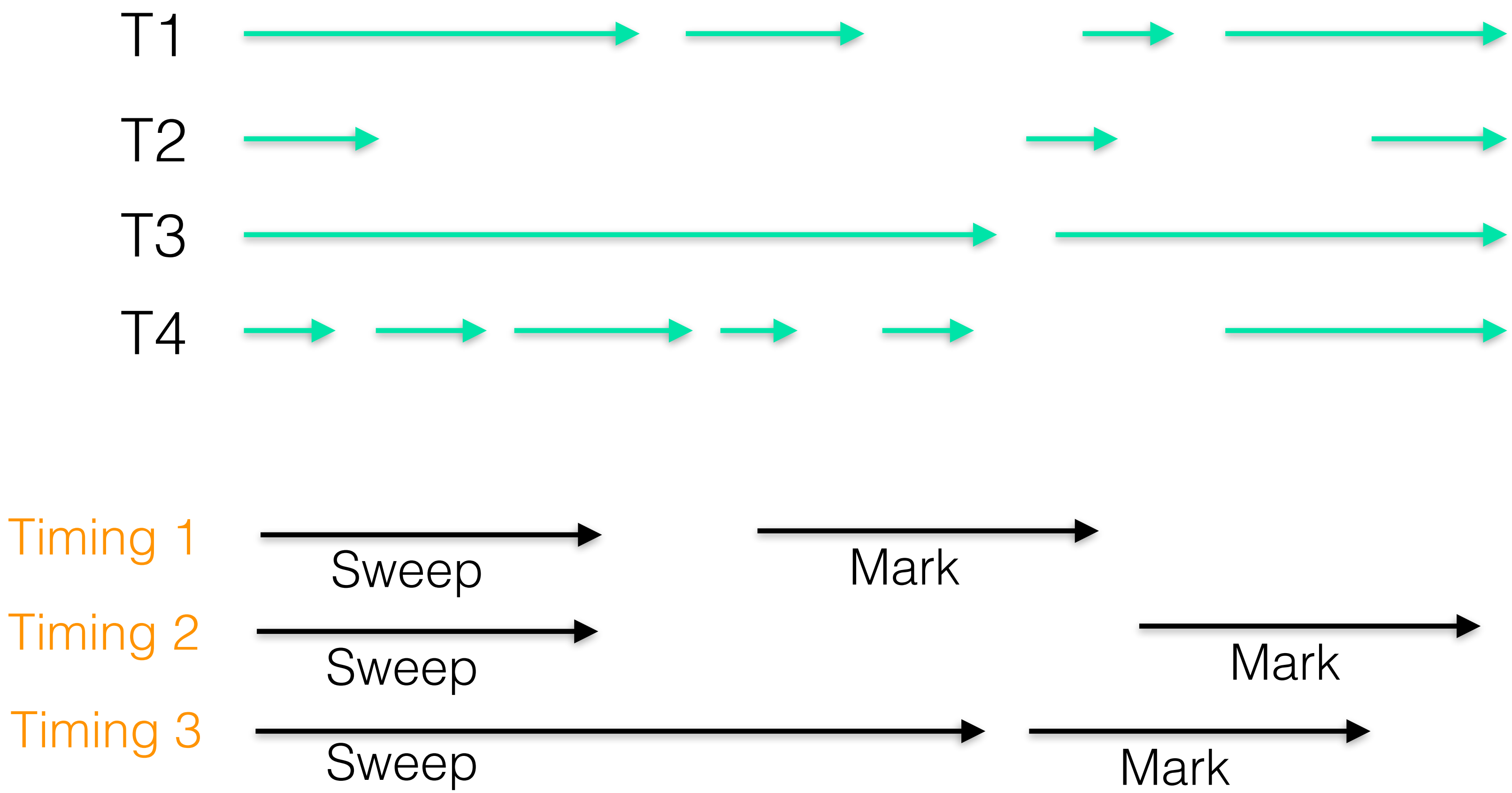


Solution: before starting a new mark, make sure each thread completes a write.

Write Barrier & Coordination

- DLG use the other option: “Shade and then change”
- Problem: shading may disappear before the change.
  - sweep terminates & marking starts again.
- Such a scenario is prevented by coordinating a new collection with the mutators.
- Before starting a new collection the collector makes sure each mutator is not in the middle of a write.
- Simple option: stop all threads before starting the collection, make sure none is in the middle of a write, and start the collection.

No Thread is Continuously Writing



Handshakes

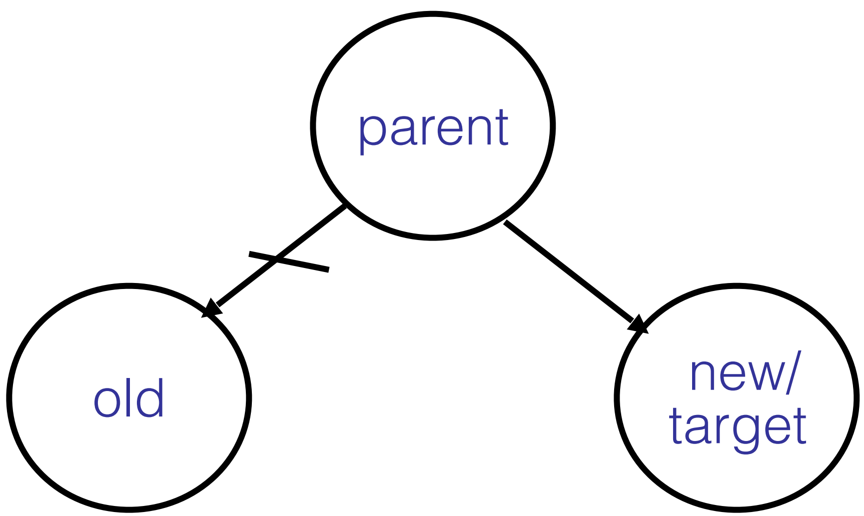
- Stopping all threads and making sure they are all “in a good state” is costly and unnecessary.
- We only need to know that each of them is not stuck in a write.
- We can check them one-by-one using a handshake.
- Handshake:
  - Collector tells mutators that it has started tracing by raising a flag.
  - Each mutator responds by raising a local flag.
  - Response does not happen during a write!
  - Handshake ends when all mutators respond.

Write-Barrier on Local Variables

- The majority of program updates are executed on the local variables (the roots).
- Modern collectors avoid a write-barrier on the roots.
- Can we avoid the write barrier and not miss live objects?

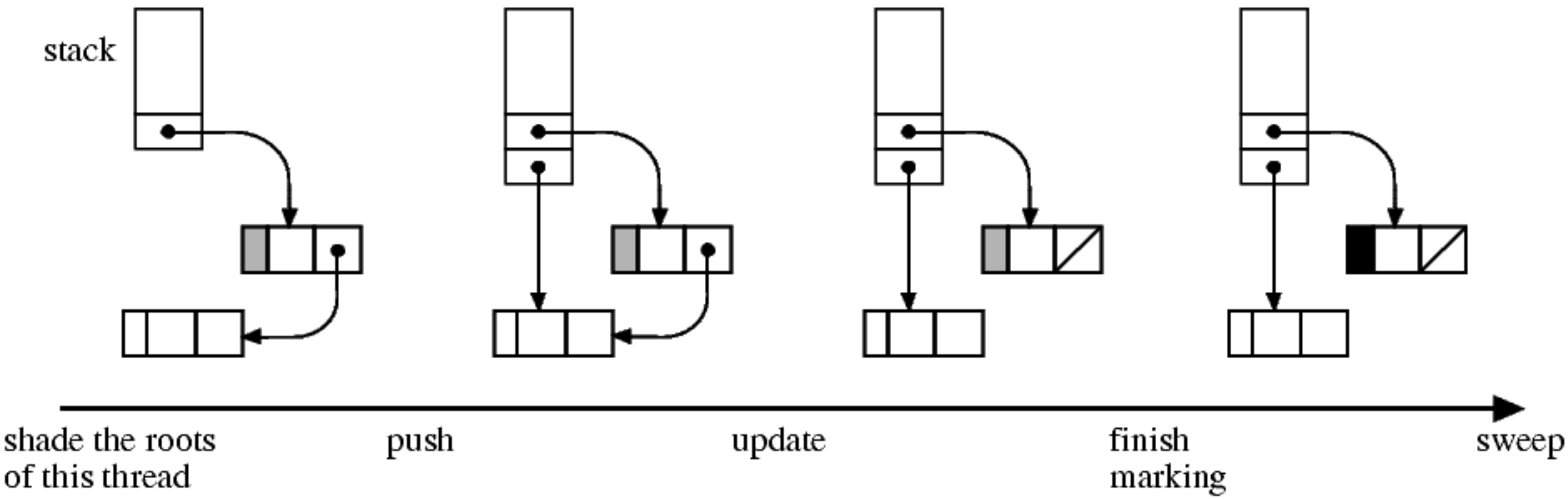
Eliminating stack write barrier

- With Dijkstra, target of modification was shaded.
- Can it work without a write barrier on the roots?
- Assume an atomic “shade and change” for this discussion. (Even that would not work…)



Concurrent modification (cont.)

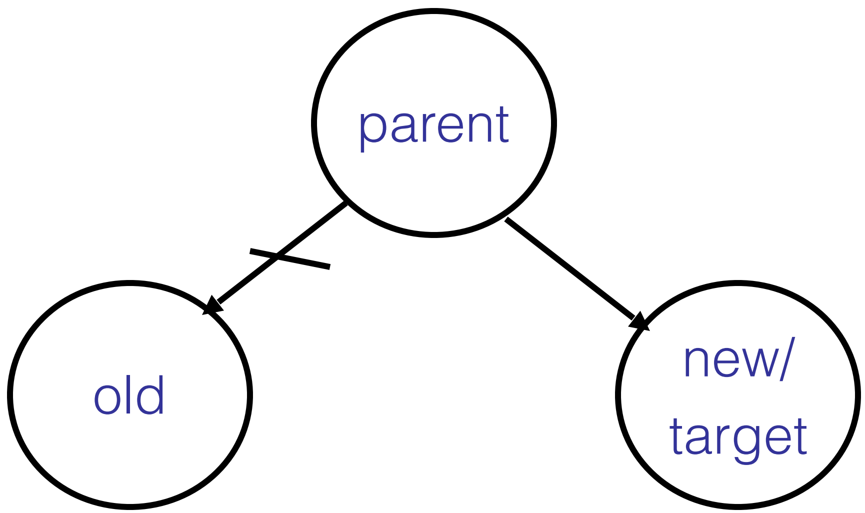
- Shading target values (only for heap pointers)



collector program program collector

A Second Try (no stack write barrier)

- Will it work better if we shaded the old value of the pointer (instead of the new target)?



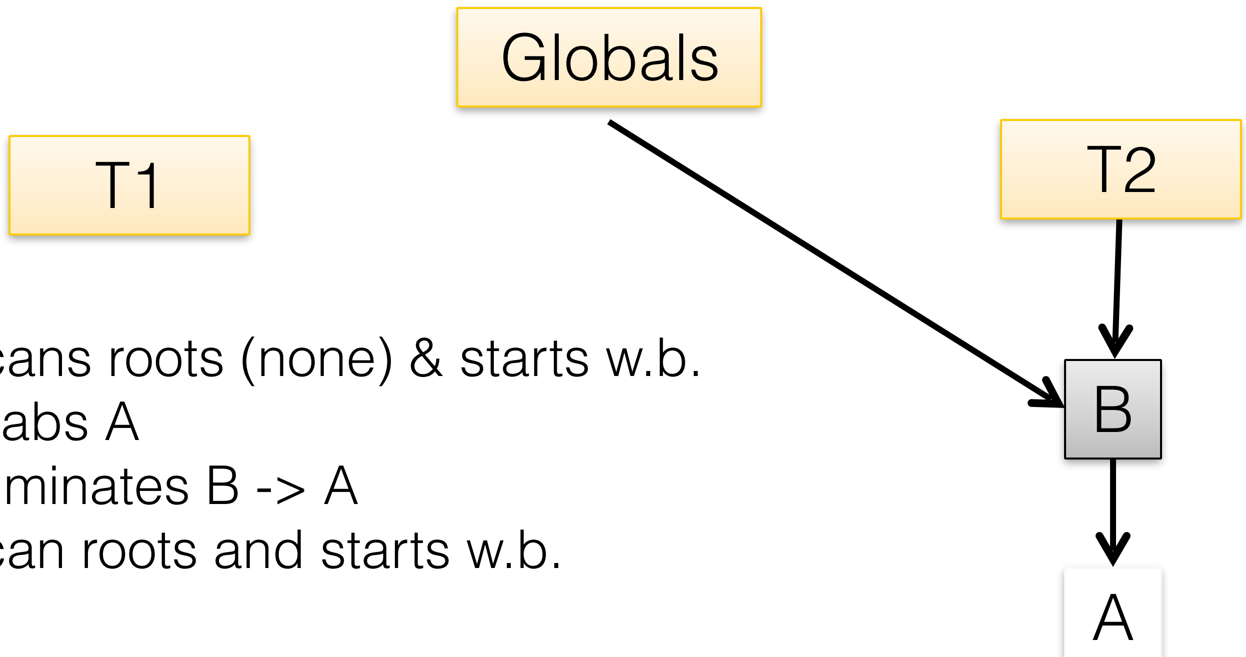
Yes and No

- **If** we
  - **stop all threads**
  - Scan all roots while threads stopped
  - Initiate write barrier (shading old for any heap pointer update)
  - Resume threads
- **Then all is OK**.
- Because anything reachable from the roots during the halt time must be blackened by the end of the concurrent trace.

Using Handshakes

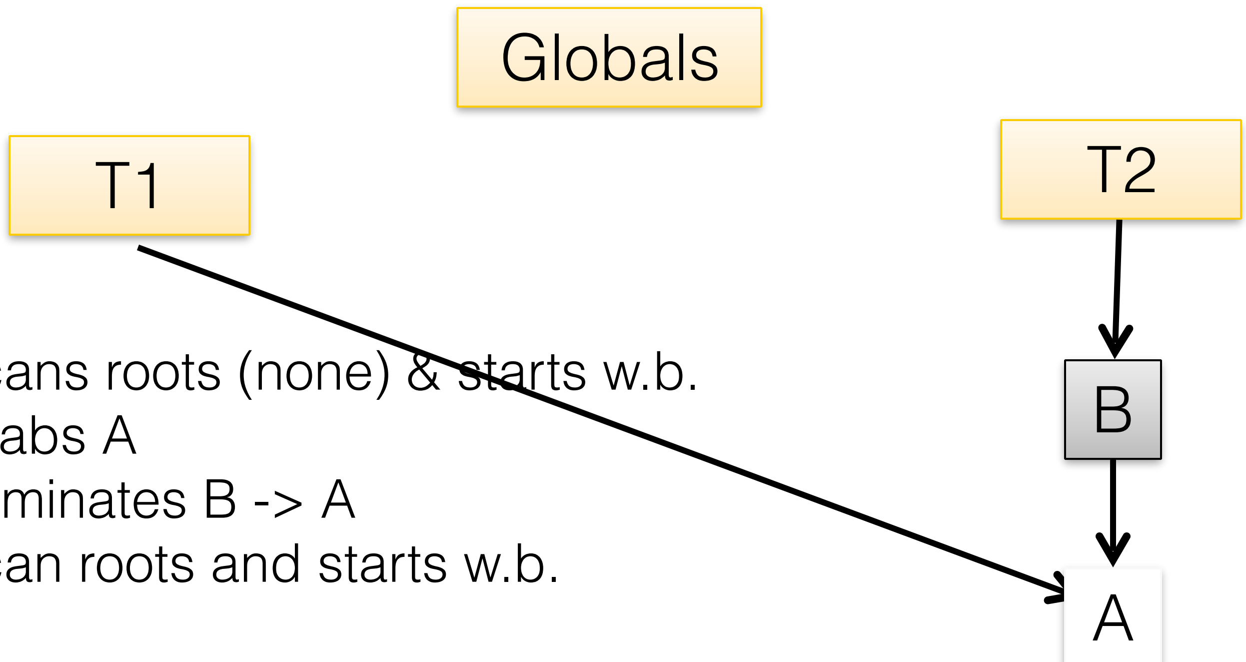
- If we
  - stop one thread at a time (via a handshake)
  - Scan its roots while halted
  - Initiate its write barrier (shading old for any heap pointer update)
  - Resume the thread
- Then it doesn’t work.

A Problem with Using a Handshake



- T1 scans roots (none) & starts w.b.
- T1 grabs A
- T2 eliminates B -> A
- T2 scan roots and starts w.b.

A Problem with Using a Handshake



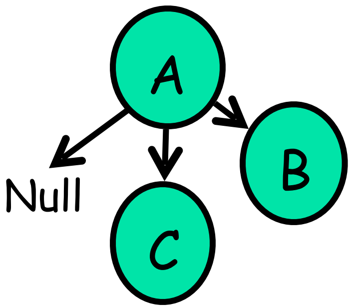
- T1 scans roots (none) & starts w.b.
- T1 grabs A
- T2 eliminates B -> A
- T2 scan roots and starts w.b.

Let's use Two handshakes; still problems exist…

A Problem with Graying Old & 2 HS

- Assume two handshakes, and shading old value.
- There is a time in which a thread can temporarily store objects in the heap without the GC noticing.
- Consider shade and change of a (heap) pointer p.
- In between the two, if a parallel thread assigns a different object to p, and then removes it, the collector does not know about it.
- We use this “temporary un-noticed assignment” to hide a pointer from the garbage collector.
- Other thread puts object in temp place before it reports its roots, and gets it back afterwards.

The specific scenario



- (T1 running w.b.).
- T1 now plans to write B into A.ptr
  - MarkGray A.ptr (null)

- T2 has a local pointer (lp) to C (has not reported roots yet)

- T2 writes C to A.ptr
  - MarkGray A.ptr (null)
  - A.ptr = C
- lp = null
- T2 reports roots
- lp <- A.ptr (so C is alive)

  - A.ptr = B

The collector is not aware of C being alive and reclaims it.

Another try: Shade New as Well

- The solution is to let T2 (and everybody else) also mark the object that it writes to the heap, in this case, C.
- Now C cannot be hidden at the time roots are reported.

- T2 has a local pointer (lp) to C (has not reported roots yet)
- T2 writes C to A.ptr
  - MarkGray A.ptr (null)
  - A.ptr = C
  - MarkGray C
- lp = null
- T2 reports roots
- lp = A.ptr (so C is alive)

Are we Done ?

- Not really. To obtain correctness, we must use three handshakes.

At What Interval should we Mark New?

- 2 handshakes are not enough !
- Suppose we start graying old and new after the first handshake.
- We give roots in the second handshake.
- A counterexample follows. The idea is that one thread will store locally before running the first handshake.
  The other thread will make a long write that will allow the temporary store.

The specific scenario

- T1 goes into a h.s. 1
- T1 now plans to write B into A.ptr
  - MarkGray A.ptr (null)
  - A.ptr [?] B
  - T1 runs h.s. 2 & reports its rots.

- T2 has a local pointer (lp) to *C*
- T2 writes lp to *A*.ptr (no write barrier yet)
- lp [?] null
- T2 runs h.s. 1
- T2 reports roots (at h.s. 2)
- lp <- *A*.ptr (so *C* is alive)

The collector is not aware of *C* being alive and reclaims it.

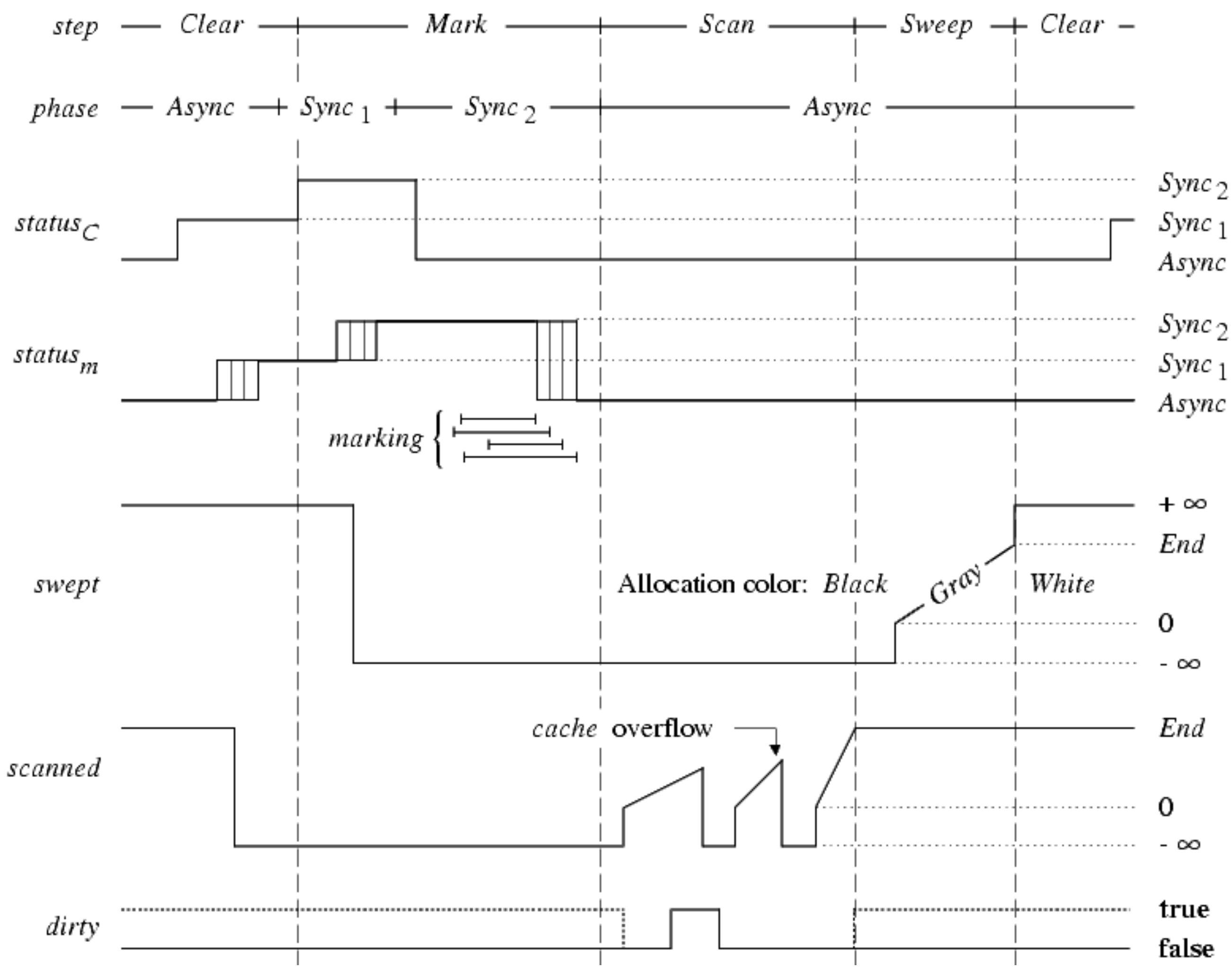
Solution: Add another h.s.

- The solution is to have an additional h.s.
- This way, it is not possible for T2 to store the local pointer anymore.
- T2 needs to store the temp pointer before it runs first h.s. (before it runs the barrier), and it must leave it there until it finishes reporting the roots.
- But if T2 runs three h.s. in between, then T1 must be making at least one h.s., thus, T1 cannot have a long write (to cover the temp) for all this time.

Handshakes

- To summarize, 3 handshakes are used:
  - Tell mutators to start the write-barrier
  - Tell mutators that root marking is approaching
  - Tell mutators to mark their roots (marking started)
- After responding to the first handshake and before responding to the third, the mutator's write barrier marks "old" + "target". During the rest of the marking phase, only "old" is marked.
- No real proofs provided…

Timing diagram for global variables



No Proofs, No Algorithm

- We will not fully present or partially prove the DLG collector.
- We only discussed major issues in the design.
- The proof that appears in the paper cannot be taught in (this) class.
- Producing a simpler proof = a project!

An excerpt from the proof

$$
\begin{aligned}
xZy &\triangleq x \in ADDR \wedge heap[x] \in HEADER \\
&\quad \wedge y = heap[x].size + x + 1 \\
xYy &\triangleq \exists z, xZz \wedge x < y < z \\
xXy &\triangleq \exists z, xYz \wedge y = heap[z] \\
O &\triangleq Z^*(0) \cap [0, end) \\
W &\triangleq \{x \in O \mid heap[x].color = White\} \\
G &\triangleq \{x \in O \mid heap[x].color = Gray\} \\
B &\triangleq \{x \in O \mid heap[x].color = Black\} \\
C_m &\triangleq \{new_m \mid pc_m \in CREATE \setminus \{Split\}\} \\
N &\triangleq \{new_m \mid \exists m, pc_m \in CREATE \setminus \{Split, Fill\}\} \\
S_m &\triangleq \{old_m \mid pc_m = Split\} \\
F_m &\triangleq \{heap[y] \mid pc_m \in CREATE \wedge new_m \; Y \; y \notin fill_m\} \\
A_m &\triangleq \widehat{roots}_m \cup \widehat{mark}_m \cup F_m \\
&\quad \cup \{x \mid (x = new_m \vee x = old_m \vee xYfield_m) \\
&\qquad\qquad \wedge pc_m \in UPDATE\} \\
&\quad \cup \{x \in args_m \mid status_m \neq Quick\} \\
&\quad \cup \{x \in args_p \mid pc_m = Launch \wedge p = child_m\} \\
U &\triangleq X^* \left((G \cup B) \setminus \textstyle\bigcup_m C_m\right) \cup \textstyle\bigcup_m X^*(A_m) \\
V &\triangleq (W \cup G \cup B) \setminus \widehat{free} \\
J &\triangleq V \setminus \textstyle\bigcup_m (X^*(A_m) \cup C_m) \\
K &\triangleq \{field_m \mid \exists m, pc_m \in UPDATE \wedge status_m \neq Sync_1 \\
&\qquad\qquad \wedge (pc_m = Store \vee old_m \neq heap[field_m])\} \\
xTy &\triangleq \exists z \notin K, xYz \wedge y = heap[z] \notin B \\
R_C &\triangleq \{x \in G \mid step \neq Scan \vee x \geq ptr \vee reset \vee dirty\} \\
&\quad \cup blacken \cup \widehat{cache} \cup (heap[\widehat{fields} \setminus K] \cup trace) \setminus B \\
R_m &\triangleq (\widehat{mark}_m \cup \{x \in F_m \mid marking_m\}) \setminus B \\
M &\triangleq T^* \left(R_C \cup \textstyle\bigcup_m R_m\right) \cup (G \cup B) \setminus \textstyle\bigcup_m C_m
\end{aligned}
$$

(22) $U \oplus \bigoplus_m C_m \subseteq V \setminus \widehat{claim}$

(23) $free \oplus claim \subseteq W$

(24) $alloc \oplus \bigoplus_m (pool_m \oplus S_m) = O \setminus (W \cup G \cup B)$

(25) $\forall m, fill_m \subseteq \{x \in Y(new_m) \mid pc_m \in CREATE\}$

(26) $\forall m, pc_m \in UPDATE \Rightarrow field_m \in Y(O)$

(27) $whiten \subseteq V \wedge \widehat{fields} \subseteq Y(O) \wedge R_C \subseteq U \cup G$

(28) $\forall m, status_m \neq Async \vee step \in \{Mark, Scan\}$
$\qquad \Rightarrow C_m \subseteq W \cup B \wedge pc_m \neq GrayNew$

(29) $\forall m, pc_m = Split \Rightarrow heap[new_m].color = Black$

(30) $\{ptr, limit, rover\} \subseteq O \cup \{end\}$

(31) $ptr \leq limit \wedge sublimit \leq limit$

(32) $end \in Z^*(0)$

(33) $\forall m, pc_m = Split \Rightarrow old_m \; Y \; new_m$

(34) $\forall m, pc_m = Split \Rightarrow Z(old_m) = Z(new_m)$

(35) $step = Sweep \Rightarrow \left(U \cup \bigcup_m C_m\right) \cap W \subseteq [0, ptr)$

(36) $\forall m, pc_m = ClearNew \wedge step = Sweep \Rightarrow new_m < ptr$

(37) $step = Sweep \Rightarrow \widehat{free} \cap [ptr, sublimit) = \emptyset$

(38) $step = Sweep \Rightarrow B \subseteq whiten \cup N \cup [ptr, end)$

(39) $\forall m, pc_m = TestSweep \wedge step = Sweep \wedge old_m < swept$
$\qquad \Rightarrow new_m \notin [swept, ptr) \setminus (W \cup whiten)$

(40) $\forall x \in O, step = Sweep \wedge swept < x < ptr \Rightarrow x \in V$

(41) $step = Clear \Rightarrow B \subseteq whiten \cup N \cup [ptr, limit)$

(42) $step \in \{Mark, Scan\} \Rightarrow heap[K] \subseteq M = X^*(M)$

(43) $\widehat{cache} \subseteq G \cup B \wedge blacken \subseteq W \cup G$

(44) $\forall m, pc_m = Store \wedge status_m \neq Async \wedge \neg marking_m$
$\qquad \Rightarrow new_m \in B \cup G \cup R_m$

(45) $\forall m, pc_m \in \{TestScan, SetDirty\} \wedge step \in \{Mark, Scan\}$
$\qquad \Rightarrow old_m \in G \cup B$

(46) $\forall m, pc_m \in UPDATE \setminus \{Store\} \Rightarrow status_m = Async$

(47) $\forall m, status_m \in \{Dead, Free, Quick\} \Rightarrow A_m = pool_m = \emptyset$

(48) $\forall m, pc_m = Halt \Rightarrow args_m = \emptyset$

(49) $\forall m, marking_m \vee status_m = Async \wedge step \in \{Mark, Scan\}$
$\qquad \Rightarrow X^*(A_m) \subseteq M$

(50) $\forall m, marking_m \wedge pc_m = Fill \Rightarrow new_m \in W$

Properties

- On-the-fly.
- No write barrier on local variables.
- Adequate for multithreading.
- Handshakes & write-barriers overhead small.
- Made practical in a series of works.
- A modern version of the algorithm (adapted for Java) was/is used with the IBM production JVM on some IBM platforms.

Summary

- Dijkstra et al.:
  - Concurrent GC
  - Full proof (on abstraction)
  - But: theoretical, only one mutator.
- DLG:
  - On-the-fly
  - Full proof (less abstract)
  - Practical: several threads, no write barrier on roots, no scan of free-list.
  - Extension implemented on commercial products.