

## Exercise 1 - Due 17.11.2005

### Purpose

Become familiar with parameters of PCPs and their basic transformations (this will be used extensively later on).

### Questions

Assume  $L \subseteq \{0, 1\}^*$  is in the class

$\text{PCP}[\text{time} = t(n), \text{length} = l(n), \text{randomness} = r(n), \text{query} = q, \text{completeness} = 1, \text{soundness} = s > 0]$ ,

where  $q, s$  are constants (independent of  $n$ ). Prove the following statements and replace the "...” with the (best possible) missing parameters.

1. If  $t(n) = n^{O(1)}, l(n) = n^{O(1)}, r(n) = O(\log n), q = 2$ , then  $L$  is decidable in polynomial time (i.e.  $L \in \text{P}$ ).
2. There exists constant  $s' > 0$  such that

$$L \in \text{PCP}[\dots, \text{query} = 3, \text{completeness} = 1, \text{soundness} = s'].$$

What is  $s'$ ?

3. There exists constant  $s' > 0$  and finite alphabet  $\Sigma \supseteq \{0, 1\}$  such that

$$L \in \text{PCP}[\dots, \text{query} = 2, \text{completeness} = 1, \text{soundness} = s'].$$

(Each query is answered by a symbol of  $\Sigma$ ). What is  $|\Sigma|$ ? What is  $s'$ ?

4. For all  $s' < 1$ , there exists constant  $q'$  such that

$$L \in \text{PCP}[\dots, \text{query} = q', \text{completeness} = 1, \text{soundness} = s'].$$

What is  $q'$ ?