# SEMI-LOCAL INVARIANTS

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## Abstract

*Geometric invariants are shape descriptors that remain unchanged under geometric transformations such as projection, or change of the viewpoint. In [2] we developed a new method of obtaining local projective and affine invariants for a general curve without any correspondences. Being local, the invariants are much less sensitive to occlusion than global invariants. The invariants computation is based on a canonical method. This consists of defining a canonical coordinate system using intrinsic properties of the shape, independently of the given coordinate system. Since this canonical system is independent of the original one, it is invariant and all quantities defined in it are invariant. Here we present a further development of the method to obtain local semi-invariants, that is local invariants for curves with known correspondences. Several configurations are treated: curves with known correspondences of one or two feature points or lines.*

## 1 Introduction

Geometric invariants are shape descriptors which remain invariant under geometrical transformations such as projection or viewpoint change. They are important in object recognition because they enable us to obtain a signature of an object which is independent of external factors such as the viewpoint. In this paper we treat projective (viewpoint) and affine invariants in various geometrical configurations.

In [2] we developed a new method of obtaining local projective and affine invariants for a general curve without any correspondences. The approach is based on transforming the shape to a canonical (intrinsic) system of coordinates, rather than obtaining closed form formulas for the invariants. Working with general plane curves without any correspondence information leads to fitting rather high order curves which may be sensitive to noise. This problem is discussed in [4] and it is shown that one way of overcoming it is using a wide window. In what follows we briefly describe another approach to increasing robustness by using some reference features, e.g. points or lines for which the correspondence is known. For example, a silhouette of an airplane can contain both curved parts and straight lines. We can use this information to eliminate some of the parameters of the projective or affine transformation, so there will be a need for fewer curve descriptors for the elimination of the remaining ones. Invariants involving both derivatives and reference points were found and discribed in [1, 3]. However, they still use a curve parameter $t$ which also has to be eliminated, and this reduces the robustness of their method.

The "parameterless" method described in [2] is perfectly suited for this situation, and again leads to saving in the number of data quantities needed from the image and to increased reliability. Here we use a canonical method similar to that used in the correspondenceless case ([2]) in order to find local invariants while avoiding the curve parameter. This makes the method more robust as there are fewer unknowns to eliminate.

In the first stage we fit a high order curve over some window around some $x_0, y_0$ and then translate and rotate until the origin is at $x_0, y_0$ and the $x$ axis is tangent to the curve. We obtain an auxiliary osculating curve that will help us find the canonical system. However, not as in [2], here we do not need the nodal cubic; the conic, with three parameters, will suffice in all cases:

$$f^* = c(x,y) = c_0x^2 + c_1y^2 + c_2xy + y = 0 \quad (1)$$

The exact process of finding the conic and canonizing differs in each case. However, the principles of invariance and locality must be maintained. In the following we will briefly describe the process for the different possible combinations. Each known feature point or line reduces the number of derivatives needed by two, because it eliminates two transformation factors.

- **A Curve and One Feature Point**:
We draw a line joining the given reference point $x_1, y_1$ with the curve point $x_0, y_0$. This is obviously a projectively invariant operation. We use this line as our new $y$ axis. As before we skew the system so that this line becomes perpendicular to $x$. We thus obtain an orthogonal system which we can scale and slant as before.

  To do this, we obtain an osculating conic to our fitted curve $f$. We need only fourth order contact, rather than sixth as before.

  After fitting the conic, our goal will be to go over to a canonical system in which this conic is a unit parabola $x^2 + y = 0$, and the distance between the curve point and the reference point is unity.

- **A Curve and One Feature Line**:
We convert to the previous case by finding the polar point of the given line with respect to the osculating conic. Polarity of a point and a line is an invariant relation. Given a point, we can draw from it two tangents to the conic, creating two points at which these tangents touch the conic. The line joining these two points is the polar line of the given point w.r.t. the conic.

  The conic is found in the same way as in the previous case, requiring osculation in the fourth derivatives. Having the polar point in a Euclidean canonical system, we are in the same situation as in the previous case, having a conic and a point, and we can proceed to find invariants as before.

- **A Curve and Two Feature Points**:
This case requires only the second derivative to determine the osculating conic, rather than the fourth as before. We first find the conic that osculates the fitted curve with second order contact, and also passes through the two reference points. This uniquely determines the conic. We then find the line that passes through the two reference points. This brings us to the same situation as before, namely a conic plus a line, but with two fewer derivatives.

- **A Curve and Two Feature Lines**:
This case requires only the second derivative to determine the osculating conic, rather than the fourth as before. We first find the conic that osculates the fitted curve with second order contact, and is also tangent to the two reference lines. We then find the intersection point of the reference lines. This brings us to the case of a conic plus a point that we dealt with before, but with two fewer derivatives.

- **A Curve, a Point and a Line**:
As before we require that the conic osculate the fitted curve up to second order contact. In addition we require that the reference line be polar to the reference point w.r.t. the conic. This provides sufficient conditions to determine the conic. Achieving this will bring us again to the situation of a conic plus a point, to be canonized as before, again with two fewer derivatives.

## 2 Summary and Conclusions

We have presented a method for finding semi-local projective and affine invariants. The method consists of defining a canonical coordinate system using intrinsic properties of the shape, independently of the given coordinate system. Since this canonical system is independent of the original one, it is invariant and all quantities defined in it are invariant. We have applied the method to find local invariants of a general curve with known correspondences of one or two feature points or lines.

## References

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