Photon Mapping

Part II – Participating Media

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Participating Media

- Rendering algorithms so far assumed that light reflects and disperses only on surface.
- … All scenes are rendered in vacuum.
- Outdoor scenes, or scenes with a non-solid media (e.g., dust, water, clouds) require new light transport techniques.

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Light Scattering in Media

- When a photon interacts with a particle in a media, it can either be absorbed or reflected.
- Let σ_a be the absorption coefficient and σ_s be the scattering coefficient, then the change of radiance due to out-scattering (loss) is:

$$(\vec{\omega}\bullet\nabla)L(x,\vec{\omega}) = -(\sigma_s + \sigma_a)L(x,\vec{\omega})$$

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Light Scattering – Cont’d

- There is also a gain (in-scattering), which is integrated from all directions.
- The gain from in-scattering:

$$(\vec{\omega}\bullet\nabla)L(x,\vec{\omega}) = \sigma_s(x)\int_{\Omega} p(x,\vec{\omega}',\vec{\omega})L_i(x,\vec{\omega}')d\vec{\omega}'$$

p is the distribution function of the scattered light, called ***phase function***.

- The gain from emission (in cases like flames):

$$(\vec{\omega}\bullet\nabla)L(x,\vec{\omega}) = \sigma_a L_e(x,\vec{\omega})$$

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The Volume Rendering Equation

- By Integrating the equation (somehow..) on both sides of a segment *s* ,we get the *volume rendering equation*:

$$L(x,\vec{\omega}) = \int_0^s e^{-\tau(x,x')}\sigma_a(x')L_e(x',\vec{\omega})dx' +$$

$$\int_0^s e^{-\tau(x,x')}\sigma_s(x')\int_\Omega p(x',\vec{\omega}',\vec{\omega})L_i(x',\vec{\omega}')d\vec{\omega}'dx' +$$

$$e^{-\tau(x,x+s\vec{\omega})}L(x+s\vec{\omega},\vec{\omega})$$

- $\tau(x,x') = \int_x^{x'} \sigma_t(x'')dx''$ is called the *optical depth.*

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The Phase Function

- Describes the distribution of the scattered light.
- Unlike BRDF, it is normalized in the sphere.
  - The actual amount of absorption is controlled by an external coefficient.

$$\int_\Omega p(x,\vec{\omega},\vec{\omega}')d\vec{\omega}' = 1$$

- Often, the function only depends on the angle θ between the incoming ray and the scattered ray, and we use p(θ).
  - Θ=0 means forward scattering, and θ=π means backward scattering.

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The Phase Function – Cont'd

- Isotropic Scattering:

$$p(\theta) = \frac{1}{4\pi}$$

  - A photon will scatter in a random direction

- The Henyey-Greenstein phase function:

$$p(\theta) = \frac{1-g^2}{4\pi(1+g^2-2g\cos\theta)^{1.5}}, g \in (-1,1)$$

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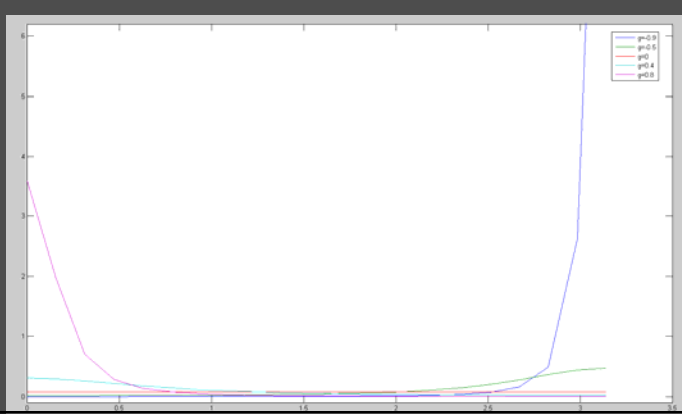
HG Phase Function

- *g* controls the scattering distribution.
  - g=0 – isotropic
  - g>0 – more forward scattering
  - g<0 – more backward scattering



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Phase Functions – cont'd

- The HG phase function has been found to model well scattering in skin, ocean, stone and etc.
- It is possible to use a linear combination of weighted HG functions for better realistic results.
- The Schlick phase function approximates HG, without the costly 1.5 power:

$$p(\theta) = \frac{1-g^2}{4\pi(1+k\cos\theta)^2}$$

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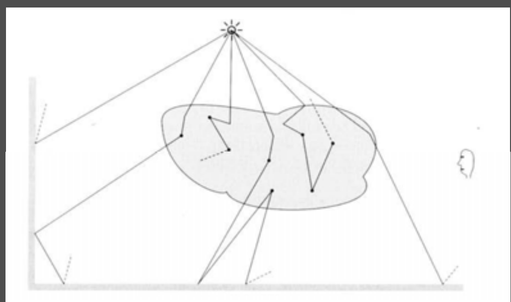
Photon mapping in Volumes

- The algorithm is similar, with a few key adaptations.
- The photon mapping algorithm:
  - Photon tracing
  - Volume radiance estimation
  - Rendering



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Creating The Photon Map

- When the photon enters the media (volume), it has an average distance before it forms an intersection:

$$d = \frac{1}{\sigma_t}$$

- Photon scattering is done according to Russian roulette:

$$x \in [0,1] \rightarrow \begin{cases} x \le \frac{\sigma_s}{\sigma_t} \Rightarrow scattered \\ x > \frac{\sigma_s}{\sigma_t} \Rightarrow absorbed \end{cases}$$

  - Color bands are scaled according to the regular algorithm.

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Photon Map – Cont'd

- Photon Storing is done in a specified volume map, because radiance estimation is different in volume.
  - A good optimization is to save only photons that scattered at least once.
- Photon can be emitted from the volume with a spectrum based on local conditions.

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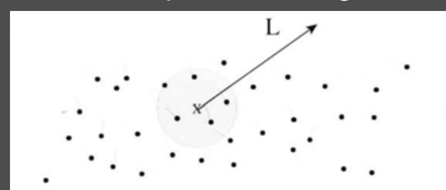
Radiance Estimate

- The two dimensional approximation does not hold in volumes.
- We estimate the required integral:

$$(\vec{\omega}\bullet\nabla)L_o(x,\vec{\omega}) = \sigma_s(x)\int_\Omega p(x,\vec{\omega}',\vec{\omega})L_i(x,\vec{\omega}')d\vec{\omega}' =$$

$$= \int_\Omega p(x,\vec{\omega}',\vec{\omega})\frac{d^2\Phi(x,\vec{\omega})}{dV} \approx \sum_{i=0}^{n} p(x,\vec{\omega}_p,\vec{\omega})\frac{\Phi_p(x,\vec{\omega}_p)}{\frac{4}{3}\pi r^3}$$

In a growing radius *r* of a sphere, to engulf *n* nearest photons.



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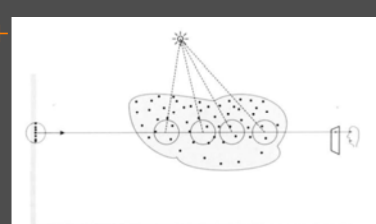
Rendering – Ray Marching

- Idea: Shoot ray through medium. Divide ray into small segments Δx, and accumulate radiance.
- Radiance is due to single scattering (direct illumination) and multiple scattering (using radiance estimate)
- In the next segment *n+1* we get:

$$L_{n+1}(x,\vec{\omega}) = \sum^{N} L_l(x,\vec{\omega})p(x,\vec{\omega}_l,\vec{\omega})\sigma_s(x)\Delta x +$$

$$\sum_{i=0}^{n} p(x,\vec{\omega}_p,\vec{\omega})\frac{\Phi_p(x,\vec{\omega}_p)}{\frac{4}{3}\pi r^3}\Delta x +$$

$$e^{-\sigma_t(x)\Delta x}L_n(x+\Delta x,\vec{\omega})$$



- The direct contribution of the light must be attenuated with the distance from the points.

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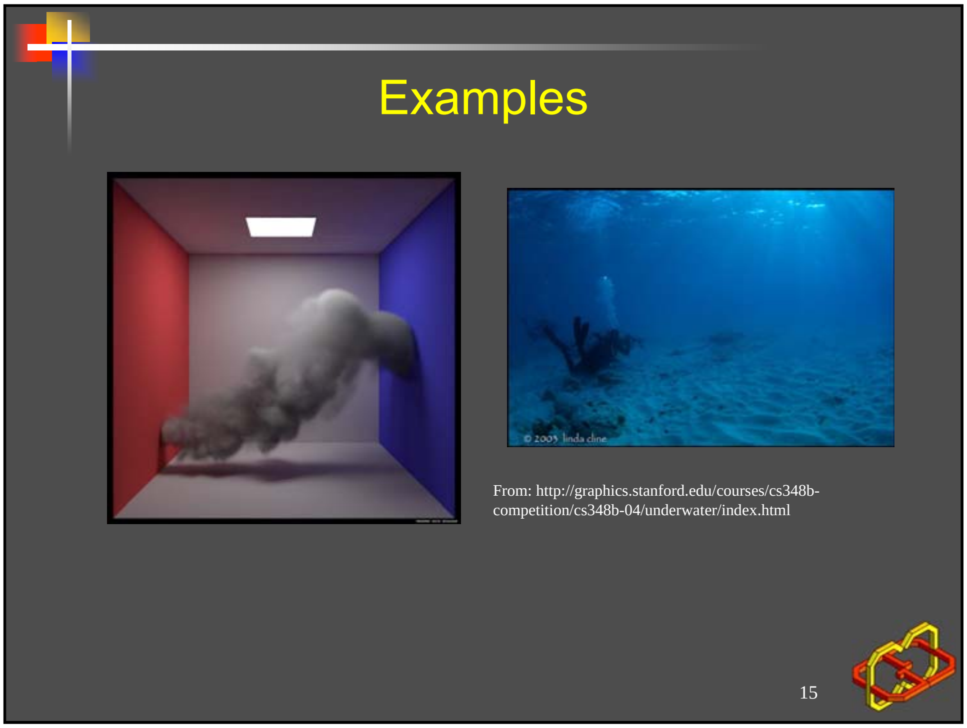
Examples
From: http://graphics.stanford.edu/courses/cs348b-competition/cs348b-04/underwater/index.html
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