Vector Quantization

Vector quantization
- Mapping a set of values (vectors, colors, etc.) into a finite [smaller] set of values (representatives)
  - Without losing too much information!

Example: rounding a set of doubles into integers

Applications:
- Image and voice compression
- Voice recognition
- Color display
- Geometric compression

Quantization error
- Quantization error:
  - Sum of differences between original value and mapped value
    - Colors – difference in color
    - Coordinates – distance between points
  \[ \text{dist}(p, q) = \sqrt{(p_x - q_x)^2 + (p_y - q_y)^2} \]
  \[ \text{quantization error} = \sum_{s \in \text{pixels}} \text{dist}(s, \text{rep}(s)) \]
- Objective – minimization of quantization error

Example: color quantization
- Used for limited dynamic-range displays
  - 8 bit display can display only 256 colors
- Reducing number of colors
  - Choose set of representative colors (colormap or palette)
  - Map rest of colors to them
  - Usually use 256 colors

Quantization to 4 colors
Representatives

- How to choose representative colors?
  - Fixed representatives, image independent - fast
  - Image content dependent – slow

- Which image colors mapped to which representatives ?
  - Nearest representative - slow
  - By space partitioning - fast

Color quantization examples

256 colors
64 colors
16 colors
8 colors
4 colors
2 colors

Uniform quantization

- Partition quantization space into equal sized regions (e.g. grid)

  Good:
  - Fixed representatives
  - Input independent
  - Fast

  Bad:
  - Wasted representatives
  - Large error for non uniform distribution

  Common for 24 → 8 bit color quantization
  - Retain 3+3+2 most significant bits of R, G, B components.

Non-uniform quantization

- Quantization space partitioned according to input data

  Goal: choose “best” representatives
  - Locations and assignments
  - Minimal distance error (if “distance” is defined)
Examples

- Uniform quantization to 4 colors (large quantization error)
- Image-dependent quantization to 4 colors (small quantization error)

Quantization & lossy coding

- Quantization used as lossy coding method when there is a notion of distance between symbols to be coded
- Coordinates
- Colors
- Not good for characters

Median cut quantization

- Median cut algorithm
  - Heuristic approximation to optimal VQ solution
- While (num_of_cells<k) do
  - Split cells at median of longest axis

Choose the representatives for each cell:
- Geometric cell center
- Centroid of sites in cell (better results)
Uniform vs. median cut

Original - 256 colors

Uniform - 8 colors
Median cut

Uniform geometry quantization

- Coordinates can be considered integers in finite range after quantization
- Quantization done on data bounding box/cube intervals
- Geometry quantization to $n$ bits:
  - All integer values in $[0, 2^n-1]$ can be used
  - Scale/transform coordinates to be maximal over given range
  - Quantize each coordinate (rounding to nearest integer)

Quantization effects (uniform)

6 bits / vertex | 8 bits / vertex | 12 bits / vertex