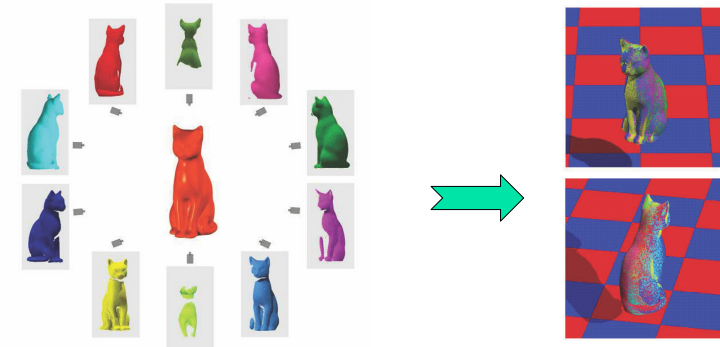


## Iterative Closest Point

## Motivation

- Align partially overlapping meshes

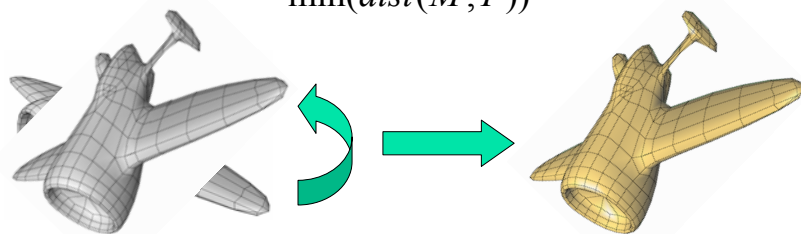


Images from: "Geometry and convergence analysis of algorithms for registration of 3D shapes" by Pottman

## The Problem

- Input: Meshes  $M, P$
- Output: Rotation  $R$ , translation  $T$ , s.t.

$$\tilde{M} = R * M + T$$
$$\min(\text{dist}(\tilde{M}, P))$$

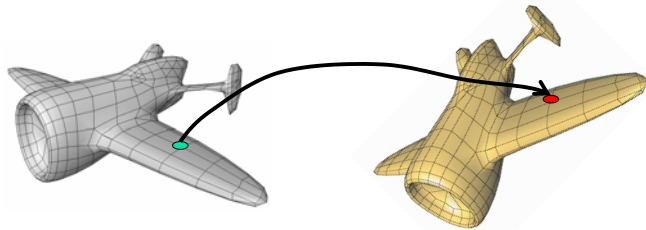


## The Challenges

- Should support partial matching
- Should be robust to noise
- Should be efficient

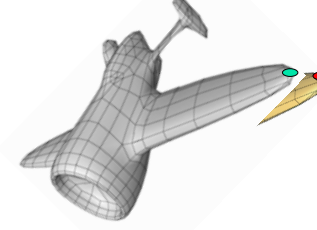
## ICP Insight 1

- If correspondance is known, easy to find transformation



## ICP Insight 2

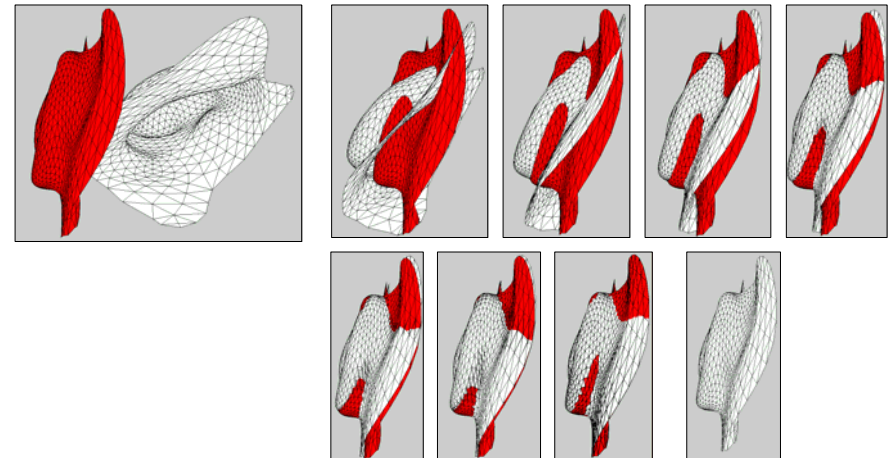
- If transformation is known, easy to find correspondance (closest point)



## ICP Algorithm

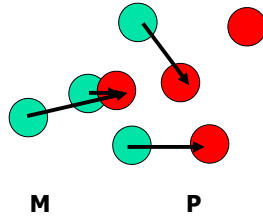
- Start from initial guess
- Iterate
  - For each point on  $M$ , find closest point on  $P$
  - Find best transform for this correspondance
  - Transform  $M$

## Example



## Find Closest Point

- For each point in  $M$ 
  - Choose closest point (Euclidean) from  $P$



- Minimizes  $\frac{1}{|M|} \sum_{v \in M} \|v - \text{match}_p(v)\|_2^2$

## Find Best Transform

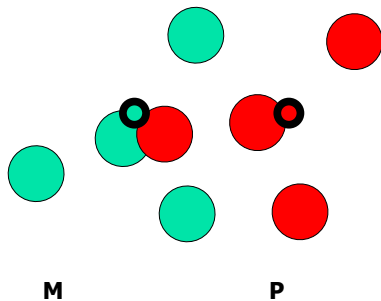
- Find R and T that minimize

$$\frac{1}{|M|} \sum_{v \in M} \|\text{match}_p(v) - (R * v + T)\|_2^2$$

- R – 3D rotation
- T – 3D translation

## Find Best Transform

- Translation part – from centroids



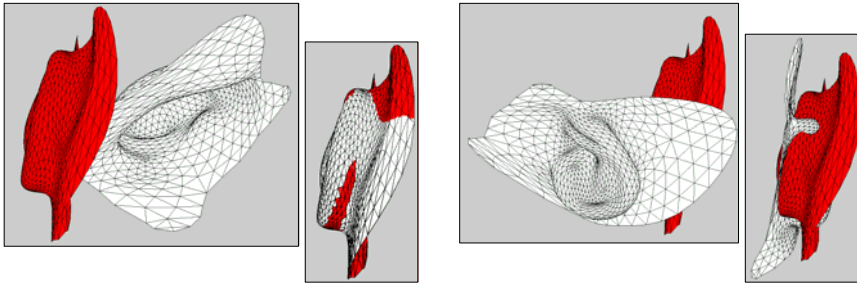
$$T = \text{avg}(P) - R * \text{avg}(M)$$

## Find Best Transform

- Rotation part
  - Closed form solution – solve a cubic equation
  - or
  - (Easier, but not equivalent)
  - Find best matrix Q
    - Linear least squares system
  - Find best approximating rotation
    - SVD

## Converges?

- Errors decrease monotonically
- Converges to local minimum
- Good initial guess → Converges to global minimum



## Extensions

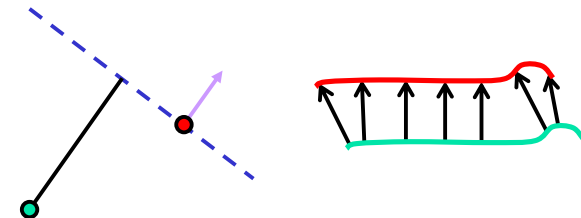
- Speed up correspondance
  - Use spatial subdivision
- Select only sample of points
- Different error metrics
- Change point matching
- Reject outliers

## Points Sampling

- All points
- Uniform sampling
- Random sampling
- Uniform normal distribution

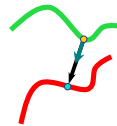
## Error Metrics

- Point-to-plane distance instead of point to point



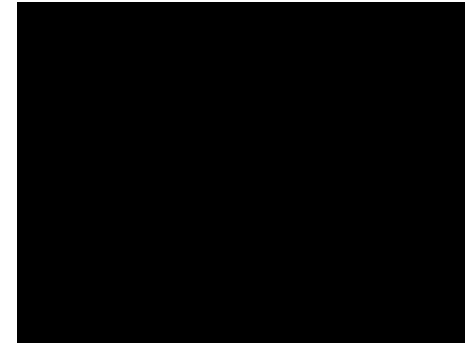
## Point Matching

- Standard – closest point
  - Slow
- Normal shooting
  - Bad for noisy meshes
- Consider only compatible points
  - Same curvature, normals, colors



## More Extensions

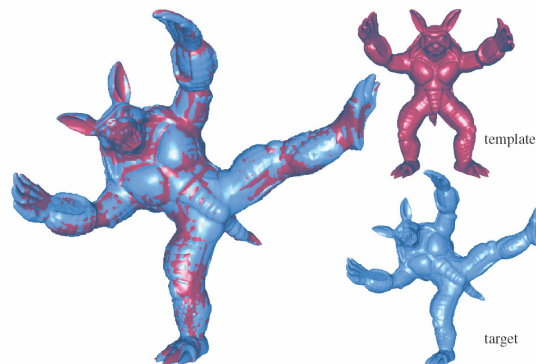
- Can be done in real time
  - Interactive scanning & registration



Movie from: "Efficient Variants of the ICP Algorithm" by Rusinkiewicz et al.

## More Extensions

- Non rigid deformations



Images from: "Generalized Surface Flows for Mesh Processing" by Eckstein et al