Adaptive Deterministic lower bound

$$\text{OPT}_{\text{adaptive}}(c) \geq \log |C|$$

where $C$ is the set of all possible actions. Let $C_1$ be a subset of $C$ such that

$$C_1 \cup C_2 = C$$

and $C_1$ is such that

$$|C_1| = \frac{|C|}{2}$$

Let $f: C \rightarrow \{0, 1\}$ be a function such that

$$\{x \in C_1' \mid f(x) = 1\}$$

is empty and

$$\{x \in C_1' \mid f(x) = 0\}$$

contains exactly $\frac{|C_1'|}{2}$ elements.
Randomized Adaptive Lower Bound

\[ \text{OPT}(c) = \frac{\ln(\log 16)}{16} \]

\[ \delta \geq \frac{1}{2(\ln 2)} \]

\[ \mathbb{P}_s[A(s, \text{opt}, s) \neq f] \geq 1 - \delta \]

\[ X_f(s) = \left\{ \begin{array}{ll}
1 & A(s, \text{opt}, s) \neq f \\
0 & \text{otherwise}
\end{array} \right. \]

\[ \mathbb{E}_s[X_f(s)] = \mathbb{P}_s[A(s, \text{opt}, s) \neq f] \leq \delta \]

\[ \mathbb{E}_f \mathbb{E}_s[X_f(s)] \leq \delta \]

\[ \mathbb{E}_s \left[ \mathbb{E}_f[X_f(s)] \right] = \mathbb{E}_f \left[ \mathbb{E}_s[X_f(s)] \right] \leq \delta \]
\[ \text{Markov's Inequality} \]

\[ \Pr \left[ E_f \left[ X_f(s) \right] \geq 2\delta \right] \leq \frac{1}{2} \]

\[ \text{for} \quad s \in \left\{ \mathbb{I}^T - N \right\} \quad \text{and} \quad 2^T = |S| \]

\[ E_f \left[ X_f(s) \right] \geq 2\delta \]

\[ E_f \left[ X_f(s) \right] \leq 2\delta \]

\[ \Pr \left[ A(\delta, M_{Q_f}, s) = \mathbb{F} \right] < 2\delta \]

\[ 0 < \delta \leq \frac{1}{2} \]

\[ A(\delta, M_{Q_f}, s) = \mathbb{F} \quad \text{for} \quad s_0 \in S \quad \text{and} \quad \text{for the entire set} \quad C \subseteq \mathbb{I}^\ast \]

\[ |C_{s_0}| \geq (1 - 2\delta) |C| \]

\[ \text{Consider} \quad A(\delta, M_{Q_f}, s_0) \leq \exists \delta \text{ such that} \quad \text{for} \quad \delta \text{ satisfying} \quad C \subseteq C_{s_0} \]

\[ A(\delta, M_{Q_f}, s_0) \subseteq \exists \delta \text{ such that} \quad \text{for} \quad \delta \text{ satisfying} \quad C \subseteq C_{s_0} \]

\[ \text{see by (1 - 2\delta) + by 1c} \]