Assignment no. 3

Given: December 31, 2015
Due: January 14, 2016
Submission in singletons

Question 1.
Let $L$ be a set of $n$ lines in the plane. Give an $O(n \log n)$-time algorithm to compute an axis-parallel rectangle that contains all the vertices of $A(L)$ in its interior.

Question 2.
1. The convex hull of a point set $S = \{p_i\}$, $\text{CH}(S)$, is defined as the intersection of all convex sets containing $S$. The convex hull can also be defined as the set of all convex combinations of $S$, i.e., $x \in \text{CH}(S)$ if $\exists a_i, 0 \leq a_i \leq 1$, such that $x = \sum_i a_ip_i$ and $\sum_i a_i = 1$. Prove that these definitions are equivalent.
2. Show that the convex-hull of a point set $S$ is the convex set with the smallest perimeter (amongst all convex set which contain $S$). (The perimeter of a polygon is the length of its boundary.)

Question 3.
1. Let $S = \{p_1, \ldots, p_n\}$ (for $n \geq 3$) be the vertices of a regular convex polygon, and let $C$ be its center. Let $P = S \cup C$. Prove that in the Voronoi diagram of $P$, the Voronoi cell of $C$ contains $n$ vertices. (A regular polygon is a polygon that is equiangular (all angles are equal in measure) and equilateral (all sides have the same length).
2. Assuming general position, prove that for a Voronoi diagram of $n$ points, where $n$ is large enough, the average number of vertices of a cell is 6.

Question 4.
GG($S$), the Gabriel Graph of a point set $S$ in the plane, is defined as follows: Two points $p, q \in S$ are connected by an edge of the graph if the circle with diameter $pq$ does not contain any other point of $S$ in its interior.
1. Prove that DT($S$) (Delaunay Triangulation of $S$) contains the Gabriel graph of $S$.
2. Prove that $p$ and $q$ are adjacent in GG($S$) iff the Delaunay edge that connects between them intersects its dual Voronoi edge.
3. Give an $O(n \log n)$-time algorithm to compute the Gabriel graph of a set of $n$ points.

Question 5.
Let $S$ be a set of $n$ points in the plane, and let $t$ be the number of lines that pass through exactly $\sqrt{n}$ points of $S$. Prove that $t = O(\sqrt{n})$. 