Straight Skeletons of Three-Dimensional Polyhedra

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ABSTRACT

In this video we present an algorithm for computing the *straight skeleton* of a polyhedron in three dimensions.

Categories and Subject Descriptors

I.3.5 [**Computer Graphics**]: Computational Geometry and Object Modeling—*Geometric algorithms, languages, and systems*

General Terms

Algorithms, Theory

Keywords

Straight skeleton, medial axis

1. INTRODUCTION

The notion of the *straight skeleton* of a two-dimensional polygon was introduced by Aichholzer et al. [1]. This geometric construction reduces the polygon into a 1-dimensional set of segments that approximates the shape of the original polygon. It is defined, and usually computed, in terms of an offset process in which edges of the polygon move inward, remaining straight and meeting at vertices. When a vertex collides with an opposite edge, the polygon is split into two polygons, and the process continues within each piece independently. The skeleton consists of the straight segments traced out by the vertices during this process.

The straight skeleton is more complex to compute than other types of skeleton [6, 9], but its piecewise-linear form offers many advantages. The most popular alternative, the medial axis [5], consists of both linear and quadratic curve segments. Thus, the straight skeleton has several applications, such as interpolation of surfaces in three dimensions from cross-sections of an unknown object [3], polygon decomposition [12], construction of offset curves [9], surface folding [8], etc.

The 3D medial axis of a polyhedron found applications, e.g., in surface reconstruction [4] and mesh generation [11]. However, unlike the 2D case, the 3D medial axis can be quite complex, both combinatorially and geometrically. Thus, one would like to generalize the notion of straight skeleton for polyhedra, analyze its shape and complexity, and develop efficient algorithms for computing it.

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Object	Vertices	Edges	Facets
(a)	12	20	10
(b)	20	30	12
(c)	28	42	16
(d)	20	30	12
(e)	20	18	9 (+1 hole)
(f)	12	18	10
(g)	16	24	11
(h)	16	24	11
(i)	16	24	10

	Skeleton				Time
Object	Vertices	Edges	Faces	Cells	(Sec.)
(a)	8	24	25	10	0.312
(b)	25	60	46	12	0.719
(c)	45	104	74	16	0.567
(d)	16	42	37	12	0.188
(e)	15	45	56	9	0.250
(f)	21	48	37	10	0.484
(g)	6	21	25	11	0.177
(h)	12	36	33	11	0.146
(i)	12	32	29	10	0.172

Table 1: Statistics

2. 3-D STRAIGHT SKELETON

For a convex polyhedron, the straight skeleton and the medial axis are identical. Held [10] showed that their complexity (for a convex polyhedron of complexity n) is $\Omega(n^2)$ in the worst case. In the nonconvex case, the only known bound on the complexity of the straight skeleton is $\Omega(n^2\alpha^2(n))$ [2].

Demaine et al. [7] provide the basic properties of three-dimensional straight skeletons. The only work, that we are aware of, which studies this skeleton in detail, is [2]. It begins with analyzing the structure of straight skeletons of orthogonal polyhedra and giving two algorithms for computing it, with running time proportional to either the volume of the polyhedron or the surface area of the skeleton. It then discusses the inherent ambiguity in defining the straight skeleton for a general polyhedron and suggests a consistent method for resolving it. Finally, it describes an algorithm for computing the skeleton in the general case. Figure 1 shows the straight skeletons of a few simple objects. Table 1 provides a brief statistics of these objects.

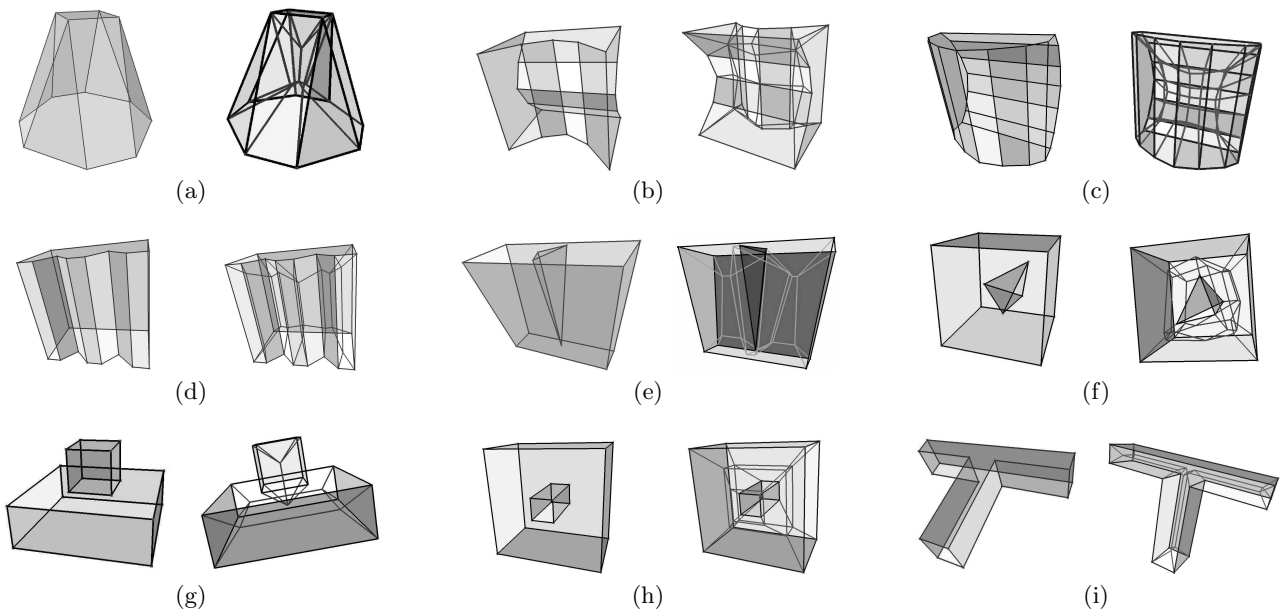

Figure 1: Sample objects

3. THE VIDEO

The video shows our implementation of the algorithm for computing the three-dimensional straight skeleton of a polyhedron, which was done in Visual C++ .NET2005 and experimented with on a 3GHz Athlon 64 processor PC with 1GB of RAM. We used the CGAL library to perform basic geometric operations. The source code consists of about 6,500 lines of code. The output was captured in real time and edited with the Camtasia(c) software.

In the video, we illustrate the construction of the three-dimensional straight skeleton of a few general polyhedra. We show the definition of the skeleton through the process of offsetting the boundary of the polyhedron. We demonstrate the inherent ambiguity of the skeleton, which arises in the propagation of vertices of initial degree greater than three, and show how our solution to this ambiguity allows a consistent propagation of the boundary of any polyhedron. We depict the events which occur in the process of creating the skeleton, and the resulting changes in the topology of the propagating boundary. Finally, we show examples of the straight skeletons of several polyhedra.

4. REFERENCES

[1] O. Aichholzer, F. Aurenhammer, D. Alberts, and B. Gärtner, A novel type of skeleton for polygons, *J. of Universal Computer Science*, 1 (1995), 752–761.

[2] G. Barequet, D. Eppstein, M.T. Goodrich, and A. Vaxman, Straight skeletons of three-dimensional polyhedra, *Proc. 16th Ann. European Symp. on Algorithms*, Karlsruhe, Germany, *LNCS*, 5193, Springer-Verlag, 148–160, 2008.

[3] G. Barequet, M.T. Goodrich, A. Levi-Steiner, and D. Steiner, Contour interpolation by straight skeletons, *Graphical Models*, 66 (2004), 245–260.

[4] E. Bittar, N. Tsingos, and M.-P. Gascuel, Automatic reconstruction of unstructured 3D data: Combining a medial axis and implicit surfaces, *Computer Graphics Forum*, 14 (1995), 457–468.

[5] H. Blum, A transformation for extracting new descriptors of shape, in: *Models for the Perception of Speech and Visual Form* (W. Wathen-Dunn, ed.), MIT Press, 362–380, 1967.

[6] S.-W. Cheng and A. Vigneron, Motorcycle graphs and straight skeletons, *Proc. 13th Ann. ACM-SIAM Symp. on Discrete Algorithms*, San Francisco, CA, 156–165, 2002.

[7] E.D. Demaine, M.L. Demaine, J.F. Lindy, and D.L. Souvaine, Hinged dissection of polypolyhedra, *Proc. 9th Workshop on Algorithms and Data Structures*, Waterloo, Ontario, Canada, *LNCS*, 3608, Springer-Verlag, 205–217, 2005.

[8] E.D. Demaine, M.L. Demaine, and A. Lubiw, Folding and cutting paper, in: *Revised Papers from the Japan Conf. on Discrete and Computational Geometry*, *Lecture Notes in Computer Science*, 1763, Springer-Verlag, 104–117, 1998.

[9] D. Eppstein and J. Erickson, Raising roofs, crashing cycles, and playing pool: Applications of a data structure for finding pairwise interactions, *Discrete & Comp. Geometry*, 22 (1999), 569–592.

[10] M. Held, On computing Voronoi diagrams of convex polyhedra by means of wavefront propagation, *Proc. 6th Canadian Conf. on Computational Geometry*, Saskatoon, Saskatchewan, Canada, 128–133, 1994.

[11] M.A. Price, C.G. Armstrong, and M.A. Sabin, Hexahedral mesh generation by medial surface subdivision: Part I. Solids with convex edges, *Int. J. for Numerical Methods in Engineering*, 38 (1995), 3335–3359.

[12] M. Tănase and R.C. Veltkamp, Polygon decomposition based on the straight line skeleton, *Proc. 19th Ann. ACM Symp. on Computational Geometry*, San Diego, CA, 58–67, 2003.