Chapter 1

On Scope Dominance With Monotone Quantifiers

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ABSTRACT. We characterize pairs of monotone generalized quantifiers Q_1 and Q_2 that give rise to an entailment relation between their two relative scope construals. This result is used for identifying entailment relations between the two scopal interpretations of simple sentences of the form NP_1-V-NP_2. The general characterization that we give turns out to cover more examples of such entailments besides the familiar type where the NPs are headed by *some* and *every*.

1.1 Introduction

Scope ambiguity in simple transitive sentences of the form NP_1-V-NP_2 is one of the well-studied areas in natural language semantics. It has been often observed that whether this kind of ambiguity is manifested in natural language may depend on entailment relations between the readings of such sentences. For instance, Zimmermann (1993) characterizes the class of *scopeless* ("name like") noun phrases – the class of NP_2s for which the two readings of the sentence NP_1-V-NP_2 are equivalent for any noun phrase NP_1 and transitive verb V. A more general notion, first addressed by Westerståhl (1986), involves uni-directional entailment between the two readings, which is referred to here as *scope dominance*. A sentence NP_1-V-NP_2 exhibits scope dominance if one of its readings entails the other. A familiar case is when the subject (or object) denotes an existential quantifier (e.g., *some student*) and the object (or subject, respectively) denotes a universal quantifier (e.g., *every teacher*). Westerståhl shows that in the class of non-trivial upward monotone (simple) quantifiers over finite domains, scope dominance appears if and only if the subject or object are existential or universal.

Altman et al. (2002) generalize Westerståhl's result, and show a full characterization of scope dominance with *arbitrary* upward monotone quantifiers over *countable* domains. In this paper we generalize Westerståhl's result in another way,

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and characterize scope dominance between simple upward *or downward* monotone quantifiers over finite domains. This result is based on the numerical presentation of quantifiers over finite domains as recently proposed by Väänänen and Westerståhl (2001). It leads to a general characterization of entailments over finite domains between readings of sentences with (potential) scope ambiguity as in the following cases, where both subject and object are monotone.

(1) Less than five referees read at least one of the abstracts.

(2) Less than five referees read each of the abstracts.

In sentence (1), the object narrow scope reading entails the object wide scope reading. In (2) the entailment between the two readings is in the opposite direction. Note that the definite noun phrase *the abstracts* leads in both sentences to the presupposition that abstracts exist, which is crucial for the respective entailments to hold. Similarly to Westerståhl's result about upward monotone quantifiers, in both examples scope dominance is created by the presence of an existential or universal quantifier. However, as we shall see, our extension of Westerståhl's characterization also reveals cases of scope dominance with monotone quantifiers other than *every* or *some*.

1.2 Background

This section briefly reviews some notions from generalized quantifier theory, which will be used in our characterization of scope dominance. A (*generalized*) *quantifier* over a domain E is a set $Q \subseteq \wp(E)$. A quantifier Q over E is *upward (downward) monotone* iff whenever $A \in Q$ and $A \subseteq A'$ ($A' \subseteq A$), then $A' \in Q$. In the sequel, we sometimes use the abbreviations "$\mathrm{MON}\uparrow$" and "$\mathrm{MON}\downarrow$" for "upward/downward monotone". A quantifier Q is called *trivial* iff either $Q = \emptyset$ or $Q = \wp(E)$.

Given a binary relation $R \subseteq E^2$ and $x \in E$ we write $R_x \stackrel{def}{=} \{y \in E : R(x,y)\}$ and $R^y \stackrel{def}{=} \{x \in E : R(x,y)\}$. The *Object Narrow Scope* (ONS) reading of a simple transitive sentence is naturally interpreted in a domain E as the proposition Q_1Q_2R as defined below, where Q_1 and Q_2 are the subject and object quantifiers (respectively) over E, and the relation $R \subseteq E^2$ is the denotation of the verb.

(3) $Q_1Q_2R \stackrel{def}{\Leftrightarrow} \{x \in E : R_x \in Q_2\} \in Q_1$.

Similarly, the *Object Wide Scope* (OWS) reading is interpreted as $Q_2Q_1R^{-1}$, which by (3) is equivalent to the requirement $\{y \in E : R^y \in Q_1\} \in Q_2$.

Given two quantifiers Q_1 and Q_2 we say that Q_1 is *scopally dominant* over Q_2 iff for every $R \subseteq E^2$: $Q_1 Q_2 R \Rightarrow Q_2 Q_1 R^{-1}$.

The *dual* of a quantifier Q over E is the quantifier $Q^d = \{X \subseteq E : E \setminus X \notin Q\}$. The following fact summarizes some simple properties of quantifier duality.

Fact 1 *For any quantifiers Q, Q_1, Q_2 over E:*

1. $(Q^d)^d = Q$
2. Q_1 *is scopally dominant over* Q_2 *iff* Q_2^d *is scopally dominant over* Q_1^d
3. $Q = \emptyset \Leftrightarrow Q^d = \wp(E)$
4. Q *is* MON↑ *(*MON↓*) iff* Q^d *is* MON↑ *(*MON↓*).*

A *determiner* over a domain E is a function D that assigns to every $A \subseteq E$ a quantifier $D(A)$. In this paper we concentrate on *simple* quantifiers Q: quantifiers that satisfy $Q = D(A)$, for some $A \subseteq E$ and a *conservative* and *permutation invariant* determiner D. Standardly, by saying that a determiner D over E is *conservative* we mean that for all $A, B \subseteq E$: $B \in D(A) \Leftrightarrow B \cap A \in D(A)$. Also standardly, a determiner D over E is called *permutation invariant* iff for every permutation π on E, and for all $A, B \subseteq E$: $B \in D(A) \Leftrightarrow \pi B \in D(\pi A)$, where for a set $X \subseteq E$, $\pi X = \{\pi(x) : x \in X\}$. In the sequel, whenever a quantifier Q can be interpreted as $D(A)$ for such A and D, we say that Q is *CPI-based*.

As pointed out by Väänänen and Westerståhl (2001), every monotone CPI-based quantifier Q over a finite domain E can be represented as follows, for some $A \subseteq E$ and $n \geq 0$.

(4) a. $Q = \{X : |A \cap X| \geq n\}$, if Q is MON↑
b. $Q = \{X : |A \cap X| < n\}$, if Q is MON↓

The duals of such CPI-based quantifiers can be represented as follows, respectively.[1]

(5) a. $Q^d = \{X : |A \cap X| \geq |A| - n + 1\}$
b. $Q^d = \{X : |A \cap X| < |A| - n + 1\}$

In table 1.1 we give some examples of monotone CPI-based quantifiers $D(A)$ over a finite domain E for various determiners D and arbitrary sets $A \subseteq E$, together with their presentation according to the scheme in (4). In these examples, for any real number r, the notations $\lfloor r \rfloor$ and $\lceil r \rceil$ standardly stand for the integer value closest to r from below and from above, respectively.

[1] Provably, a dual of a CPI-based quantifier is also CPI-based.

$\mathbf{every}'(A)$	=	$\{X \subseteq E : \vert A \cap X\vert \geq \vert A\vert\}$
$\mathbf{not\_every}'(A)$	=	$\{X \subseteq E : \vert A \cap X\vert < \vert A\vert\}$
$\mathbf{some}'(A)$	=	$\{X \subseteq E : \vert A \cap X\vert \geq 1\}$
$\mathbf{no}'(A)$	=	$\{X \subseteq E : \vert A \cap X\vert < 1\}$
$\mathbf{more\_than\_half}'(A)$	=	$\{X \subseteq E : \vert A \cap X\vert \geq \lfloor \frac{\vert A\vert}{2} \rfloor + 1\}$
$\mathbf{at\_least\_half}'(A)$	=	$\{X \subseteq E : \vert A \cap X\vert \geq \lceil \frac{\vert A\vert}{2} \rceil\}$
$\mathbf{less\_than\_half}'(A)$	=	$\{X \subseteq E : \vert A \cap X\vert < \lceil \frac{\vert A\vert}{2} \rceil\}$

Table 1.1: CPI-based Quantifiers

1.3 Scope dominance with monotone CPI-based quantifiers over finite domains

This section characterizes the pairs of CPI-based quantifiers Q_1 and Q_2 over finite domains, where Q_1 is scopally dominant over Q_2. Proposition 3 below first addresses the case where Q_1 is $\mathrm{MON}\uparrow$ and Q_2 is $\mathrm{MON}\downarrow$. Its proof uses the following simple combinatorial lemma, whose proof is given here for sake of completeness.

Lemma 2 *Let $\ell, m, k, n \in \mathbb{N}$ s.t. $\ell, k > 0$, $m \geq 0$ and $0 < n \leq k$. Let X be a set with $|X| = k$. Then 1 and 2 below are equivalent:*

1. *There are ℓ subsets of X: $X_1, \ldots, X_\ell$, s.t. $|X_i| = n$, $1 \leq i \leq \ell$, and every $x \in X$ is in at most m of the $X_i s$.*
2. *$\ell n \leq mk$.*

Proof. Let $X = \{x_1, \ldots x_k\}$. For every $X_1, \ldots, X_\ell \subseteq X$ let $m_i = |\{X_j : 1 \leq j \leq \ell \wedge x_i \in X_j\}|$.
(1) $\Rightarrow$ (2):
Let $X_1, \ldots, X_\ell \subseteq X$ such that for every j s.t. $1 \leq j \leq \ell$: $|X_j| = n$, and for every i s.t. $1 \leq i \leq k$: $m_i \leq m$. Thus,

$$\ell n = \sum_{i=1}^{k} m_i \leq mk$$

(2) $\Rightarrow$ (1):
Assume that $\ell n \leq mk$. Construct $X_1, \ldots, X_\ell \subseteq X$ as follows:

$$X_1 \quad = \quad \{x_1, \ldots, x_n\}$$

$$\begin{aligned} &\vdots \\ X_j &= \{x_{((j-1)n+1) \bmod k}, \ldots, x_{(jn) \bmod k}\} \\ &\vdots \\ X_\ell &= \{x_{((\ell-1)n+1) \bmod k}, \ldots, x_{(\ell n) \bmod k}\} \end{aligned}$$

It is not hard to verify that for all i, j s.t. $1 \leq i, j \leq k$: $m_j - 1 \leq m_i \leq m_j + 1$. Assume for contradiction that for some i s.t. $1 \leq i \leq k$: $m_i = m' > m$. Thus,

$$\ell n = \sum_{i=1}^{k} m_i \geq m' + (m' - 1)(k - 1) = (m' - 1)k + 1 > mk$$

in contradiction to the assumption that $\ell n \leq mk$. Hence, for all i s.t. $1 \leq i \leq k$: $m_i \leq m$. □

Proposition 3 *Let Q_1 and Q_2 be two CPI-based quantifiers over a finite domain E s.t. Q_1 is $\mathrm{MON}\uparrow$ and Q_2 is $\mathrm{MON}\downarrow$. According to the presentation in (4), assume that for some $A, B \subseteq E$ and $n, m \geq 0$: $Q_1 = \{X : |A \cap X| \geq n\}$ and $Q_2 = \{Y : |B \cap Y| < m\}$. Then Q_1 is scopally dominant over Q_2 iff one of the following holds:*

- *(i)* $|A| < n + \frac{n}{m}$ *and both* $0 < n \leq |A|$ *and* $0 < m \leq |B|$ *(both quantifiers are not trivial.)*
- *(ii)* $n > |A|$ *(*$Q_1 = \emptyset$*).*
- *(iii)* $m > |B|$ *(*$Q_2 = \wp(E)$*).*
- *(iv)* $n > 0$ *and* $m = 0$ *(*$Q_2 = \emptyset$ *and* $Q_1 \neq \wp(E)$*).*

Proof. It is easy to verify that if at least one of Q_1 and Q_2 is trivial, then Q_1 is scopally dominant over Q_2 iff one of the clauses (ii)-(iv) holds. Thus, we assume that both quantifiers are not trivial, i.e., $0 < n \leq |A|$ and $0 < m \leq |B|$. Now Q_1 is *not* scopally dominant over Q_2 iff the following condition holds:

C1. There exists $R \subseteq E^2$ such that $|\{x \in A : |R_x \cap B| < m\}| \geq n$ and $|\{y \in B : |R^y \cap A| \geq n\}| \geq m$.

We claim that C1 is equivalent to the following condition.

C2. There exist $T \subseteq E^2$ and $B' \subseteq B$ with $|B'| = m$ ($B' = \{b_1, \ldots, b_m\}$) such that $|A \setminus \bigcap_{i=1}^{m} T^{b_i}| \geq n$ and $\forall b \in B'$ $|T^b \cap A| = n$.

To see that, assume first that C1 holds, and consider $B' = \{b_1, \ldots, b_m\} \subseteq \{y \in B : |R^y \cap A| \geq n\}$. For each b_i, let $A_i \subseteq R^{b_i} \cap A$, $|A_i| = n$. Define $T = \bigcup_{i=1}^{m}(A_i \times \{b_i\})$, and observe that from the assumptions about the A_is it follows that $\{x \in A : |R_x \cap B| < m\} \subseteq A \setminus \bigcap_{i=1}^{m} A_i$.
As for the other direction, if C2 holds, define $R = T \cap (A \times B')$.

Now, C2 is equivalent to the requirement that there exist $m+1$ subsets of A: $A_1, \ldots, A_m, A_{m+1}$ such that $|A_i| = n$, $1 \leq i \leq m+1$, and $\bigcap_{i=1}^{m+1} A_i = \emptyset$. To see that, let A_i corresponds to $T^{b_i} \cap A$ for any i s.t. $1 \leq i \leq m$, and let A_{m+1} corresponds to $A \setminus \bigcap_{i=1}^{m} A_i$. By Lemma 2, this requirement holds iff $|A| \geq n + \frac{n}{m}$.
□

The dual of the kind of scope dominance that is characterized in Proposition 3 is the case in which Q_1 is $\mathrm{MON}\!\downarrow$ and Q_2 is $\mathrm{MON}\!\uparrow$. Using Fact 1 and the observation in (5), we get the following corollary of Proposition 3.

Corollary 4 *Let Q_1 and Q_2 be two CPI-based quantifiers over a finite domain E s.t. Q_1 is $\mathrm{MON}\!\downarrow$ and Q_2 is $\mathrm{MON}\!\uparrow$. According to the presentation in (4), assume that for some $A, B \subseteq E$ and $n, m \geq 0$: $Q_1 = \{X : |A \cap X| < n\}$ and $Q_2 = \{Y : |B \cap Y| \geq m\}$. Then Q_1 is scopally dominant over Q_2 iff one of the following holds:*

(i) $|B| > (m-1)(|A| - n + 2)$ *and both* $0 < n \leq |A|$ *and* $0 < m \leq |B|$ *(both quantifiers are not trivial.)*

(ii) $n = 0$ *(*$Q_1 = \emptyset$*).*

(iii) $m = 0$ *(*$Q_2 = \wp(E)$*).*

(iv) $n > |A|$ *and* $m \leq |B|$ *(*$Q_1 = \wp(E)$ *and* $Q_2 \neq \emptyset$*).*

Proposition 5 below covers the case in which both quantifiers are $\mathrm{MON}\!\downarrow$. The proof is similar to the proof of Proposition 3, and is omitted here.

Proposition 5 *Let Q_1 and Q_2 be two $\mathrm{MON}\!\downarrow$ CPI-based quantifiers over a finite domain E. According to the presentation in (4), assume that for some $A, B \subseteq E$ and $n, m \geq 0$: $Q_1 = \{X : |A \cap X| < n\}$ and $Q_2 = \{Y : |B \cap Y| < m\}$. Then Q_1 is scopally dominant over Q_2 iff one of the following holds:*

(i) $2 - \frac{|B|}{m} > \frac{n-1}{|A|-n+1}$ *and both* $0 < n \leq |A|$ *and* $0 < m \leq |B|$ *(both quantifiers are not trivial.)*

(ii) $n = 0$ *(*$Q_1 = \emptyset$*).*

(iii) $m > |B|$ *(*$Q_2 = \wp(E)$*).*

The same method that we use in the proof of Proposition 3, can also be used for the case in which the two quantifiers are $\mathrm{MON}\!\uparrow$, which is the case dealt with in Westerståhl (1986). This result is also mentioned here without proof.

Proposition 6 *Let Q_1 and Q_2 be two $\mathrm{MON}\!\uparrow$ CPI-based quantifiers over a finite domain E. According to the presentation in (4), assume that for some $A, B \subseteq E$ and $n, m \geq 0$: $Q_1 = \{X : |A \cap X| \geq n\}$ and $Q_2 = \{Y : |B \cap Y| \geq m\}$. Then Q_1 is scopally dominant over Q_2 iff one of the following holds:*

(i) $n = 1$ *or* $n > |A|$ *(*$Q_1 = \mathbf{some'}(A)$ *or* $Q_1 = \emptyset$*).*

(ii) $m = |B|$ *or* $m = 0$ *(*$Q_2 = \mathbf{every'}(A)$ *or* $Q_2 = \wp(E)$*).*

(iii) $n = 0$ *and* $m \leq |B|$ *(*$Q_1 = \wp(E)$ *and* $Q_2 \neq \emptyset$*).*

(iv) $n > 0$ *and* $m > |B|$ *(*$Q_2 = \emptyset$ *and* $Q_1 \neq \wp(E)$*).*

Examples: Let us consider some examples for scope dominance between CPI-based quantifiers over a finite domain E. For the representation of each quantifier, refer back to Table 1.1.

First, note that by Corollary 4, for every non-empty $A \subseteq E$, every $\mathrm{MON}\!\downarrow$ CPI-based quantifier is scopally dominant over $\mathbf{some'}(A)$ (=$(\mathbf{every'}(A))^d$). This accounts for the fact that the ONS reading of sentence (1), paraphrased in (6a) below, entails its OWS reading, paraphrased in (6b). Both readings are paraphrased with a presupposition about the existence of abstracts.[2]

(6) a. $|\{x : \mathbf{referee'}(x) \wedge \exists y[\mathbf{abstract'}(y) \wedge \mathbf{read'}(x,y)]\}| < 5 \wedge \exists y[\mathbf{abstract'}(y)]$

b. $\exists y[\mathbf{abstract'}(y) \wedge |\{x : \mathbf{referee'}(x) \wedge \mathbf{read'}(x,y)\}| < 5] \wedge \exists y[\mathbf{abstract'}(y)]$

Analogously to this scope dominance with existential quantification, Proposition 3 entails that for every non-empty $A \subseteq E$, $\mathbf{every'}(A)$ is scopally dominant over every $\mathrm{MON}\!\downarrow$ CPI-based quantifier. This accounts for the entailment from the OWS reading of (2), with the *every–less-than-5* order of quantifiers, to its ONS reading, with the *less-than-5–every* order of quantifiers.

Such examples with existential and universal quantifiers do not exhaust the cases of scope dominance with monotone quantifiers. By Proposition 3, $\mathbf{more\_than\_half'}(A)$ is scopally dominant over $\mathbf{no'}(B)$ for all $A, B \subseteq E$. By Corollary 4, $\mathbf{not\_every'}(A)$ (=$(\mathbf{no'}(A))^d$) is scopally dominant over $\mathbf{at\_least\_half'}(B)$ (=$(\mathbf{more\_than\_half'}(B))^d$), for all $A, B \subseteq E$. Consider for instance the following sentences.

[2] Plausibly, plurality in sentence (1) leads to the presupposition that there are at least *two* abstracts. However, we do not use this presupposition here, since the relevant entailment also appears with the weaker presupposition that is assumed above.

(7) a. More than half of the referees read no abstract.

b. No abstract was read by more than half of the referees.

Our characterization accounts for the entailment from the ONS interpretation of (7a) to its OWS interpretation, and for the opposite relation in (7b). However, for many speakers both sentences are unambiguous, and have only an ONS reading. Under this unambiguous interpretation, our characterization accounts for the entailment from (the unambiguous) sentence (7a) to (the unambiguous) sentence (7b). Note that the *more than*/*at least half of* quantifiers that are involved in these examples are not first order definable, so these entailments cannot be derived by any axiom system of the first order Predicate Calculus.

As an example in which both quantifiers are $\mathrm{MON}\downarrow$, note that Proposition 5 entails that $\mathbf{less\_than\_half}'(A)$ is scopally dominant over $\mathbf{not\_every}'(B)$, for any $A \subseteq E$ and non-empty $B \subseteq E$.

1.4 Concluding remarks

In this paper we characterized scope dominance between upward/downward monotone CPI-based quantifiers over finite domains. This work is part of a wider project that aims to study ambiguity in natural language by way of characterizing entailments between readings of ambiguous sentences. This kind of entailments is a promising area for studying inference in natural language, where high expressibility requires strong restrictions on inferential structures. Moreover, with Van Deemteer (1998) we believe that a characterization of "semantically spurious" ambiguity may lead to improved underspecification methods, and to better techniques for reasoning with underspecified representations. This is of course a major task, and even the characterization of scope dominance that was presented in this paper still leaves some obvious questions open. Most notably, the behavior of non-CPI-based and non-monotone quantifiers, and of quantifiers over infinite domains needs to be further explored. These problems are currently under research.

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