Abstract

Shape recognition deals with the study geometric structures. Modern surface processing methods can cope with non-rigidity - by measuring the lack of isometry, deal with similarity - by multiplying the Euclidean arc-length by the Gaussian curvature, and manage equi-affine transformations - by resorting to the special affine arc-length definition in classical affine geometry. Here, we propose a computational framework that is invariant to the affine group of transformations (similarity and equi-affine) and thus, by construction, can handle non-rigid shapes. Technically, we add the similarity invariant property to an equi-affine invariant one. Diffusion geometry encapsulates the resulting measure to robustly provide signatures and computational tools for affine invariant surface matching and comparison.

1. Introduction

Differential invariants for planar shape matching and recognition were introduced to computer vision in the 80’s [49] and studied in the early 90’s [13, 12, 20, 14, 17, 11], where global invariants were computed in a local manner to overcome numerical sensitivity of the differential forms. Scale space entered
the game as a stabilizing mechanism, for example in [15], where locality was tuned by a scalar indicating how far one should depart from the point of interest. Along a different path, using a point matching oracle to reduce the number of derivatives, semi-differential signatures were proposed in [46, 47, 33, 39, 18]. Non-local signatures, which are more sensitive to occlusions, were shown to perform favorably in holistic paradigms [36, 10, 35, 28]. At another end, image simplification through geometric invariant heat processes were introduced and experimented with during the late 90’s, [44, 2, 27]. In the beginning of this century, scale space theories gave birth to the celebrated scale invariant feature transform (SIFT) [30] and the affine-scale invariant feature transform ASIFT [34], that are used to successfully locate repeatable informative (invariant) features in images.

Matching surfaces while accounting for deformations was performed with conformal mappings [29], embedding to finite dimensional Euclidean spaces [24] and infinite ones [4, 43], topological graphs [26, 48], and exploiting the Gromov-Hausdorff distance [31, 7, 19]. Which are just a subset of the numerous methods used in this exploding field. Another example, relevant to this paper is diffusion geometry, introduced in [21] for manifold learning and first applied for shape analysis in [8]. This geometry can be constructed by the eigen-structure of the Laplace-Beltrami operator. The same decomposition was recently used in [45, 37, 38, 8] to construct surface descriptors for shape retrieval and matching.

In this paper, following the adoption of metric/differential geometry tools to image-analysis, we introduce a new geometry for affine invariant surface analysis. In [42, 3], an equi-affine invariant metric for surfaces with effective Gaussian curvature were presented, while a scale invariant metric was the main theme of [1, 9]. Here, we introduce a framework that handles both, that is, affine transformations in its most general form including similarity, that is scaling and isometry.

The paper is organized as follows. In Section 2 we introduce the affine invariant metric. Section 3 briefly reviews the main concepts of diffusion geometry. Section 4 is dedicated to experimental results, and Section 5 concludes the paper.

2. Affine metric construction

We model a surface \((S, g)\) as a compact two dimensional Riemannian manifold \(S\) with a metric tensor \(g\). We further assume that \(S\) is embedded in \(\mathbb{R}^3\) by a regular map \(S : U \subset \mathbb{R}^2 \to \mathbb{R}^3\). The construction of a Euclidean metric tensor can be obtained from the re-parameterization invariant arc-length \(ds\) of a parametrized curve \(C(s)\) on \(S\), as the simplest Euclidean invariant is length, we search for the parameterization \(s\) that would satisfy \(|C_s| = 1\), or explicitly,

\[
1 = \langle C_s, C_s \rangle = \langle S_s, S_s \rangle = \left\langle \frac{\partial S}{\partial u} du + \frac{\partial S}{\partial v} dv, \frac{\partial S}{\partial u} du + \frac{\partial S}{\partial v} dv \right\rangle = ds^2 \left( g_{11} du^2 + 2g_{12} dudv + g_{22} dv^2 \right),
\]

where

\[
g_{ij} = \langle S_i, S_j \rangle,
\]

using the short hand notation \(S_1 = \partial S/\partial u, S_2 = \partial S/\partial v\), and \(u\) and \(v\) are the coordinates of \(U\). An infinitesimal displacement \(ds\) on the surface is thereby given by

\[
ds^2 = g_{11} du^2 + 2g_{12} dudv + g_{22} dv^2.
\]
The metric coefficients translate the surface parametrization into a Euclidean invariant distance measure on the surface.

The equi-affine transformation, defined by the linear operator $A S + b$, where $\det(A) = 1$, is a bit more tricky to deal with, see [5, 16]. Consider the curve $C \in S$ parametrized by $w$. The equi-affine transformation is volume preserving, and thus, its invariant metric could be constructed by restricting the volume defined by $S_u, S_v$, and $C_{ww}$ to one. That is,

$$1 = \det(S_u, S_v, C_{ww}) = \det(S_u, S_v, S_{uw})$$

$$= \det\left( S_u, S_v, S_{uw} \frac{du^2}{dw^2} + 2S_{uw} \frac{du}{dw} \frac{dv}{dw} + S_{v} \frac{dv^2}{dw^2} + S_u \frac{d^2 u}{dw^2} + S_v \frac{d^2 v}{dw^2} \right)$$

$$= dw^{-2} \det\left( S_u, S_v, S_{uw} du^2 + 2S_{uw} dudv + S_{v} dv^2 \right)$$

$$= dw^{-2} (r_{11} du^2 + 2r_{12} dudv + r_{22} dv^2), \quad (4)$$

where now the metric elements are given by $r_{ij} = \det(S_1, S_2, S_{ij})$, and we extended the short hand notation to second order derivatives by which $S_{11} = \frac{\partial^2 S}{\partial u^2}$, $S_{22} = \frac{\partial^2 S}{\partial v^2}$, and $S_{12} = \frac{\partial^2 S}{\partial u \partial v}$. Note that the second fundamental form in the Euclidean case is given by $b_{ij} = \sqrt{g} r_{ij}$ where $g = \det(g_{ij}) = g_{11} g_{22} - g_{12}^2$. The equi-affine re-parametrization invariant metric [16, 5] reads

$$q_{ij} = |r|^{-\frac{1}{4}} r_{ij}, \quad (5)$$

where $r = \det(r_{ij}) = r_{11} r_{22} - r_{12}^2$.

In [42] others and us modified the equi-affine metric to accommodate for surfaces with effective Gaussian curvature. There, the idea was to project the metric tensor matrix $q_{ij}$ onto the class of positive definite subspaces. Practically, we decompose the tensor into its eigen-structure, take the absolute value of its eigenvalues, and compose the tensor back into a valid invariant metric.

Next, we resort to the similarity (scale and isometry) invariant metric proposed in [1], according to which, scale invariance is obtained by multiplying the metric by the Gaussian curvature. The Gaussian curvature is defined by the ratio between the determinants of the second and the first fundamental forms and is denoted by $K$. We propose to compute the Gaussian curvature of the equi-affine invariant metric, and construct a new metric by multiplying the metric elements by $|K|$. Specifically, consider the surface $(S, q)$, where $q_{ij}$ is the equi-affine invariant metric, and compute the Gaussian curvature $K^q(S, q)$ at each point. The affine invariant metric is defined by

$$h_{ij} = |K^q| q_{ij}. \quad (6)$$

Let us justify the above construction of an affine invariant metric for surfaces.

### 2.1. Proof of invariance

In order to prove the affine invariance of our metric construction let us first justify the scale invariant metric constructed by multiplication of a given Euclidean metric by the Gaussian curvature. Assume that the surface $S$ is scaled by a scalar $\alpha > 0$, such that

$$\tilde{S}(u, v) = \alpha S(u, v). \quad (7)$$

In what follows, we omit the parameters $u, v$ for brevity, and denote the quantities $q$ computed for the scaled surface by $\tilde{q}$. The first and second fundamental forms are scaled by $\alpha^2$ and $\alpha$ respectively,

$$\tilde{g}_{ij} = \langle \alpha S_i, \alpha S_j \rangle = \alpha^2 \langle S_i, S_j \rangle = \alpha^2 g_{ij},$$
\[ \tilde{b}_{ij} = \langle \alpha S_{ij}, N \rangle = \alpha \langle S_{ij}, N \rangle = \alpha b_{ij}, \]

which yields
\[ \det(\tilde{g}) = \alpha^4 \det(g) \]
\[ \det(\tilde{b}) = \alpha^2 \det(b). \]

Since the Gaussian curvature is the ratio between the determinants of the second and first fundamental forms, we readily have that
\[ \tilde{K} \equiv \frac{\det(\tilde{b})}{\det(\tilde{g})} = \frac{\alpha^2 \det(b)}{\alpha^4 \det(g)} = \frac{1}{\alpha^2 K}, \]

from which we conclude that multiplying the Euclidean metric by the magnitude of its Gaussian curvature indeed provides a scale invariant metric. That is,
\[ \left| \tilde{K} \right| \tilde{g}_{ij} = \left| \frac{1}{\alpha^2 K} \right| \alpha^2 g_{ij} = |K| g_{ij}. \]

Next, we prove that multiplying the equi-affine metric by the Gaussian curvature computed for the equi-affine metric provides an affine invariant metric. Using Brioschi formula [25] we can evaluate the Gaussian curvature directly from the metric and its first and second derivatives. Specifically, given the metric tensor \( q_{ij} \), we have
\[ K \equiv \frac{\beta - \gamma}{\det^2(q)}, \]

where
\[ \beta = \det \left( \begin{array}{ccc} \frac{1}{2} q_{11,v} + q_{12,u} - \frac{1}{2} q_{22,uu} & \frac{1}{2} q_{11,u} & q_{12,a} - \frac{1}{2} q_{11,v} \\ q_{12,v} - \frac{1}{2} q_{22,u} & q_{11} & q_{12} \\ \frac{1}{2} q_{22,v} & q_{12} & q_{22} \end{array} \right), \]
\[ \gamma = \det \left( \begin{array}{ccc} 0 & \frac{1}{2} q_{11,v} & \frac{1}{2} q_{12,u} \\ \frac{1}{2} q_{11,u} & q_{11} & q_{12} \\ \frac{1}{2} q_{22,u} & q_{12} & q_{22} \end{array} \right), \]

here \( q_{ij,u} \) denotes the derivation of \( q_{ij} \) with respect to \( u \), and in a similar manner \( q_{ij,uv} \) is the second derivative w.r.t. \( u \) and \( v \). Same notations follow for \( q_{ij,v}, q_{ij,uv}, \) and \( q_{ij,uu} \). Scaling the surface \( S \) by \( \alpha \), the corresponding equi-affine invariant components are
\[ \tilde{r}_{ij} = \det(\alpha S_1, \alpha S_2, \alpha S_{ij}) = \alpha^3 r_{ij} \]
\[ \tilde{r}_{ij,m} = \alpha^3 r_{ij,m} \]
\[ \tilde{r}_{ij,mn} = \alpha^3 r_{ij,mn} \]
\[ \det(\tilde{r}) = \alpha^6 \det(r), \]

that leads to
\[ \tilde{q}_{ij} = \frac{\tilde{r}_{ij}}{(\det(\tilde{r}))^{\frac{1}{4}}} = \frac{\alpha^3 r_{ij}}{(\alpha^6 \det(r))^{\frac{1}{4}}} = \alpha^3 q_{ij}, \]
that yields \( \det(\tilde{q}) = \left(\alpha^3\right)^2 \det(q) = \alpha^3 \det(q) \). Denote the Gaussian curvature constructed from the equi-affine invariant metric of the scaled surface \( \tilde{S} = \alpha S \) by \( \tilde{K}^q \). We have that

\[
\tilde{\beta} = \left(\alpha^3\right)^3 \beta, \\
\tilde{\gamma} = \left(\alpha^3\right)^3 \gamma, \\
\tilde{K}^q = \frac{\tilde{\beta} - \tilde{\gamma}}{\det^2(\tilde{q})} = \left(\alpha^3\right)^3 \left(\beta - \gamma\right) = \alpha^{-3} K^q.
\]

It immediately follows that

\[
\left| \tilde{K}^q \right| q_{ij} = \left| K^q \right| q_{ij} = \alpha^{-3} K^q |q_{ij}|
\]

which concludes the proof.

2.2. Implementation considerations

Given a triangulated surface we use the Gaussian curvature approximation proposed in [32] while operating on the equi-affine metric tensor. The Gaussian curvature for smooth surfaces can be defined using the Global Gauss-Bonnet Theorem, see [22]. Polthier and Schmies used that connection in order to approximate the Gaussian curvature of triangulated surfaces in [40]. Given a vertex in a triangulated mesh that is shared by \( p \) triangles, such that the angle of each triangle at that vertex is given by \( \theta_i \), where \( i \in 1, \ldots, p \), the Gaussian curvature \( K \) at that vertex can be approximated by

\[
K \approx \frac{1}{\frac{2}{3} \sum_{i=1}^{p} A_i} \left( 2\pi - \sum_{i=1}^{p} \theta_i \right), 
\]

where \( A_i \) is the area of the \( i \)-th triangle, and \( \theta_i \) is the corresponding angle.

Consider the triangle \( ABC \) which is one face of a triangulated surface \( S \), defines by its vertices \( A, B, \) and \( C \), as can be seen in Figure 2. Without loss of generality, we could define the surface tangent vectors at vertex \( A \) to be \( S_u = B - A \) and \( S_v = C - A \), where we have chosen the arbitrary local parametrization \( u, v \) to align with the corresponding edges \( AB \) and \( AC \). Translating from the vertex \( A \) to an arbitrary point in the triangle could be measured in terms of \( du \) along the \( S_u \) direction and \( dv \) along the \( S_v \) direction. We can thereby write the displacement \( ds \) by

\[
ds^2 = |S_u du + S_v dv|^2 = |S_u|^2 du^2 + 2\langle S_u, S_v \rangle du dv + |S_v|^2 dv^2 = g_{11} du^2 + 2g_{12} du dv + g_{22} dv^2. \]

We could also express the angle \( \theta_A \) as a function of \( S_u \) and \( S_v \),

\[
\cos \theta_A = \frac{\langle S_u, S_v \rangle}{|S_u||S_v|} = \frac{g_{12}}{\sqrt{g_{11}g_{22}}},
\]

and the area of the triangle

\[
A_{ABC} = \frac{1}{2} |S_u \times S_v| = \frac{1}{2} \sqrt{|S_u|^2 |S_v|^2 - |\langle S_u, S_v \rangle|^2} = \frac{1}{2} \sqrt{g_{11}g_{22} - g_{12}^2} = \frac{1}{2} \sqrt{\det(g_{ij})}. \]
In order to construct the eigenfunctions of the Laplace-Beltrami operator w.r.t. the affine metric using a finite elements method (FEM), the Gaussian curvature needs to be interpolated within each triangle for which we use a linear interpolation. Finally, following [42], we use the finite elements method (FEM) presented in [23] to compute the spectral decomposition of the affine invariant Laplace-Beltrami operator constructed from the metric in Eq. (6). The whole framework provides us with an affine invariant metric, invariant eigenvectors and corresponding invariant eigenvalues.

3. Invariant diffusion geometry

Diffusion Geometry, see e.g. [21], deals with geometric analysis of metric spaces where usual distances are replaced by the way heat propagates in a given space. The heat equation

$$\left( \frac{\partial}{\partial t} + \Delta_h \right) f(x, t) = 0,$$

(22)

describes the propagation of heat, where $f(x, t)$ is the heat distribution at a point $x$ in time $t$, with initial conditions at $f(x, 0)$, and $\Delta_h$ is the Laplace Beltrami Operator for our metric $h$. The fundamental solution of (22) is called the heat kernel, and using spectral decomposition it can be represented as

$$k_t(x, x') = \sum_{i \geq 0} e^{-\lambda_i t} \phi_i(x) \phi_i(x'),$$

(23)

where $\phi_i$ and $\lambda_i$ are, respectively, the eigenfunctions and eigenvalues of the Laplace-Beltrami operator satisfying $\Delta_h \phi_i = \lambda_i \phi_i$. As the Laplace-Beltrami operator is an intrinsic geometric quantity, it can be expressed in terms of the metric of $\mathcal{S}$.

The value of the heat kernel $k_t(x, x')$ can be interpreted as the transition probability density of a random walk of length $t$ from the point $x$ to the point $x'$. This allows to construct a family of intrinsic metrics known as diffusion metrics,

$$d^2_t(x, x') = \int (k_t(x, \cdot) - k_t(x', \cdot))^2 \, da$$

$$= \sum_{i > 0} e^{-2\lambda_i t} (\phi_i(x) - \phi_i(x'))^2,$$

(24)

that define the diffusion distance between two points $x$ and $x'$ for a given time $t$. A special attention was given to the diagonal of the kernel $k_t(x, x)$, that was proposed as robust local descriptor, referred to as heat kernel signatures (HKS), by Sun et al. [45].
4. Experimental results

The first experiment demonstrates resulting eigenfunctions of the Laplace Beltrami operator using different metrics and different deformations. In Figure 3 we present the 9'th eigenfunction textured mapped on the surface using an Euclidean metric, scale invariant metric, equi-affine metric and the proposed affine invariant metric. In each row a different deformation of the surface is presented. On the second row we applied local scaling, on the third row volume preserving stretching (equi-affine), and on the last row affine transformation (including local scale). At the top are plotted the accumulated histogram values of the eigenfunction. Blue color is used for the original shape, red for the locally scaled shape, green for the equi-affine transformed one, and black for the affine transformation.

In the second experiment, Figure 4, we evaluate the Heat Kernel Signatures of the surface subject to local scaling, equi-affine and affine transformations, using the Euclidean and the affine metrics. In addition, we depict the accumulated values of the histograms, color coded as before.

In the third experiment, Figure 5 shows diffusion distances measured from the nose of a cat after anisotropic scaling and stretching as well as an almost isometric transformation.

Next, we compute Voronoi diagrams for ten points selected by the farthest point sampling strategy, as seen in Figures 6 and 1. Distances are measured with the global scale invariant commute time distances [41], and diffusion distances receptively, using a Euclidean metric and the proposed affine version. Again, the affine metric outperforms the Euclidean one.

In the next example we used the affine metric for finding the correspondence between two shapes. We used the GMDS framework [7] with diffusion distances with the same initialization for both experiments. Figure 7 displays the Voronoi cells of matching surface segments.

Finally, we evaluated the proposed metric on the SHREC 2010 dataset [6] using the shapeGoogle framework [37], after adding four new deformations; equi-affine, isometry and equi-affine, affine and isometry and affine. Table 1 shows that the new affine metric discriminative power is as good as the Euclidean one, performs well on scaling as the scale invariant metric, and similar to the the equi-affine one for volume preserving affine transformations. More over, the new metric is the only one capable to handle full affine deformations added to SHREC dataset. Note that shapes which were considered locally scaled in that database were in fact treated with an offset operation (morphological erosion) rather than scaling. This explains some of the degradation in performances in the local scaled examples.

Performances were evaluated using precision/recall characteristic. Precision $P(r)$ is defined as the percentage of relevant shapes in the first $r$ top-ranked retrieved shapes. Mean average precision (mAP), defined as $\text{mAP} = \sum_r P(r) \cdot \text{rel}(r)$, where $\text{rel}(r)$ is the relevance of a given rank, was used as a single measure of performance. Intuitively, mAP is interpreted as the area below the precision-recall curve. Ideal retrieval performance (mAP=100%) is achieved when all queries return relevant first matches. Performance results were broken down according to transformation class and strength.

5. Conclusions

We introduced a new metric that gracefully handles the affine group of transformations. Its differential structure allows us to cope with local and global non-uniform stretching and scaling of the surfaces. We demonstrated that the proposed geometry and resulting computational methods could be useful for shape analysis like the SIFT and ASIFT operations were useful for image analysis.
Figure 3: The 9’th LBO eigenfunction textured mapped on the surface using four different metrics, from left to right: Euclidean, scale-invariant, equi-affine and affine. Deformations from top to bottom: None, local scale, equi-affine and affine. At the top the accumulated (histogram) values of the eigenfunction are displayed. The blue graph is used for the original shape, red for a locally scaled one, green for equi-affine and black for an affine transformation.

References


Figure 4: Affined heat kernel signatures for the regular metric (left), and the invariant version (right). The blue circles represent the signatures for three points on the original surface, while the red plus signs are computed from the deformed version. Using a log-log axes we plot the scaled-HKS as a function of t.

Figure 5: Diffusion distances - Euclidean (left) and affine (right) - measured from the nose of the cat after non-uniform scaling and stretching. The accumulated (histogram) distances (at the bottom) are color coded as in Figure 3.

Figure 6: Voronoi diagrams using ten points selected by the farthest point sampling strategy. The commute
time distances were evaluated using the Euclidean metric (top) and the proposed affine one (bottom). We textured
mapped the result to the original mesh for comparison.

Figure 7: Correspondence search between two shapes using the GMDS \[7\] framework with diffusion distances.
The affine (right) metric clearly outperforms the Euclidean one (left) in the presence of non-uniform stretching.
Correlated surfaces segments have the same color.


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### Euclidean invariant metric

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<td>100.00</td>
<td>95.51</td>
</tr>
<tr>
<td>Equi-Affine</td>
<td>100.00</td>
<td>100.00</td>
<td>98.29</td>
</tr>
<tr>
<td>Iso+Equi-Affine</td>
<td>100.00</td>
<td>100.00</td>
<td>98.29</td>
</tr>
<tr>
<td>Affine</td>
<td>75.90</td>
<td>60.35</td>
<td>52.54</td>
</tr>
<tr>
<td>Isometry+Affine</td>
<td>75.46</td>
<td>59.59</td>
<td>52.08</td>
</tr>
</tbody>
</table>

Table 1: Performance of different metrics with shapeGoogle framework (mAP in %).


