SCALE INVARIANT GEOMETRY FOR NON-RIGID SHAPES

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ABSTRACT. Local scale variations within the same species are common in nature. The shape matching puzzle poses fascinating questions, like how should we measure the discrepancy between a small dog with large ears and a large one with small ears? Are there similar geometric structures that are common to an elephant and a giraffe? What is the morphometric similarity between a blue whale and a dolphin? Existing tools that attempt to quantify the resemblance between surfaces which are insensitive to deformations in size are limited to either scale invariant local descriptors, or global normalization methods. Here, we propose novel tools for shape exploration by introducing a scale invariant metric for surfaces. The geometric measures we consider can be used for non-rigid shape analysis, it could help in generating local invariant features, produce scale invariant geodesics, embed one surface into another while being robust to changes in local and global size, and assist in the computational study of intrinsic symmetries where size does not matter.

1. INTRODUCTION

The study of invariants in shape analysis started with differential invariant signatures for planar curves that were adopted from differential geometry and applied to contours that represent the boundaries of objects in images [BKLP92, BHN93, BN95, PMV∗95, COS∗98]. Although analytically elegant, differential signatures were sensitive to small perturbations due to their local nature. As a remedy, extending the support of the signature while preserving invariance, semi-differential signatures were suggested [VBB∗92, VM∗92, CLM94, MPVO95, CMM∗96, BRW97, BS98]. Extending further the support to the shape as a whole, while giving up robustness to occlusions, proved to be beneficial for planar shape recognition [Olv99, BBK05, Olv05, LJ05]. Efforts were also made to simplify images in an invariant manner using geometric diffusion of the level sets of a gray level image [Sap93, AGLM93, Kim96].
A milestone in the field of image analysis was Lowe’s introduction of the scale invariant feature transform (SIFT) [Low04] that had little to do with scale-invariance per-se. Lowe’s idea was to sample the blur space, thereby compensating for optical distortions that occur during an image acquisition process when the distance from the camera to the scene varies. Along the same line, Morel et al. suggested to sample the blur space as well as the space of affine transformations with their Affine-SIFT or ASIFT [MG09].

Digital (photometric) images are one way of projecting our world into numbers that can be processed. New popular geometric sensors, also known as 3D scanners, allow us to capture the geometric structures of objects. In order to analyze, compare, and understand this data, the idea of aggregating local SIFT-like descriptors as bag of features was imported to surface recognition. Lowe’s diffusion process in the image domain was replaced, for example, by intrinsic diffusion on the inspected surface. One such feature, known as the heat kernel signature (HKS) measures the rate of heat dissipation from a surface point [SOG09]. The short time realization of this feature is trivially related to the Gaussian curvature. Other differential operators were proposed as local descriptors [ZBVH09], with notably the related similarity invariant curvature for surfaces which was proposed as the ratio between the magnitudes of the surface principal curvatures [RR06]. Next, the issue of which points to consider was addressed, for example through spectral analysis [RPSS10].

Treating signatures as structures in their own metric spaces was explored in [CCSG09*]. The tools that were developed to compare one shape to another, were also found to be useful in exploring intrinsic isometries by computationally mapping a surface to itself as first proposed in [RBBK07, RBBK10] and later in [OSG08]. Area preserving deformations that are as close as possible to an isometry were explored in [LDRS05, Wol09], where the former integrated bending energy to the game while the later is strictly intrinsic. The field of shape and surface matching is flourishing with reviews like [vZHCO11, IAP*11].

An important aspect of any shape correspondence measurement method is the set of transformations and deformations it can handle. So far, non-rigid shapes were treated as metric spaces that were compared using various measures imported from theoretical metric geometry [MS05, BBK08]. Local features were extracted and matched in order to initialize more computationally demanding comparison schemes, and compromises were made by embedding shapes into elementary spaces in which structures are compared and matched by relatively simple procedures. Such target spaces include Euclidean spaces [EK03, Rus07], spherical domains [BBK08], conformal disks and spheres [YT09] and topological graphs [HK03, HK06, WGZ*08], to name just a few. Local features, aggregated as bag of words, were used as a signature for efficient shape recognition rather than exact point to point matching [BBGO11].

As far as robustness to transformations has to do, the question of defining an appropriate metric for the task at hand is important in the context of shape
analysis. Researchers have been using Euclidean distances [CM91, BM92], geodesic distances [EK03, OFCD02, HSKK01, MS05, BBK06, BBK08], diffusion distances [BBK*10, RBB*10], and affine invariant versions thereof [RBB*11] to compare and match between shapes. The magnitude of the Fourier transform applied to the first derivative w.r.t. the logarithmic scale (time) of the log of heat kernel signatures, was shown to produce scale invariant local descriptors [BK10] \(^1\). Still, though the signature is local, the invariant by which it is constructed relates to global rather than local scaling. Other global invariants can be found in normalization methods like the commute time distance [QH07]. Once correspondence between two objects is achieved, the same measures could then be used to deform, morph, or warp one shape into another [KMP07].

Unlike existing solutions for the scale invariant matching problem, we introduce an invariant metric for surfaces in Section 2. We then plug the new metric into the diffusion distance framework in Section 3 that allow us to enjoy its integral qualities. Section 4 reviews implementation issues tailored for the diffusion formulation. Finally, we demonstrate the potential of the overall framework on various test cases involving local and global scale variations as well as non-rigid deformations of shapes in Section 5. The proposed integral measures are efficient to compute, and robust to noise, local and global scaling, and isometries.

2. Problem formulation

Consider \( S(u, v) \) a parametrized surface \( S : \Omega \subset \mathbb{R}^2 \rightarrow \mathbb{R}^3 \). We can measure the length of a parametrized curve \( C \) in \( S \) using either the Euclidean arc-length \( s \), or a general parametrization \( p \). The length is given by

\[
 l(C) = \int_{C \in S} ds = \int_C |C_p| dp = \int_C |S_u u_p + S_v v_p| dp \\
= \int_C \sqrt{|S_u|^2 du^2 + 2\langle S_u, S_v \rangle du dv + |S_v|^2 dv^2},
\]

from which we have the usual metric definition of infinitesimal distances on a surface

\[
ds^2 = g_{ij} d\omega^i d\omega^j,
\]

where we used Einstein summation convention, \( \omega^1 = u, \omega^2 = v \) and \( g_{ij} = \langle S_{\omega^i}, S_{\omega^j} \rangle \).

Let us first consider the simple case where \( S = \mathbb{R}^2 \). A scale invariant arc-length for a planar curve \( C \) is given by \( d\tau = |\kappa| ds \), where \( |\kappa| = |C_{ss}| \) is the scalar curvature magnitude. The invariance can be easily explained by the fact that the curvature

\(^1\)The scale invariant heat kernel signature SI-HKS\((s, \xi)\) of the surface point \( s \) at time \( t \) is defined as a function of the heat kernel signature by

\[
\text{SI-HKS}(s, \xi) = \left| \mathcal{F} \left( \frac{d}{d\eta} \log(\text{HKS}(s, \eta)) \right) \right|,
\]

where \( \mathcal{F} \) denotes the Fourier transform, the HKS\((s, t)\) is defined in Eq. (9), and \( \eta = \log(t) \).
is defined by the rate of change of the angle $\theta$ of the tangent vector with respect to the Euclidean arc-length, namely 

$$\frac{d\theta}{ds} = \kappa,$$

from which we have the scale invariant measure $d\theta = \kappa ds$, or in its monotone arc-length form $d\tau = |\kappa| ds$.

The next challenge would be dealing with a less trivial surface. By its definition, the curvature of a planar curve is inversely proportional to the radius of the osculating circle $\kappa = \rho^{-1}$ at any given point along the curve. Thus, for a curve on a non-flat surface we need to find such a scalar that would cancel the scaling effect. Recall that for surfaces there are two principal curvatures $\kappa_1$ and $\kappa_2$ at each point. These scalars, or combinations thereof could serve for constructing normalization factors that modulate the Euclidean arc-length on the surface for scale invariance.

This is the case for 

$$d\tau = |\kappa_1| ds = \frac{1}{|\rho_1|} ds,$$

or 

$$d\tau = |\kappa_2| ds = \frac{1}{|\rho_2|} ds.$$

We could also define similarity or scale invariant arc-length using the mean curvature $2H = \kappa_1 + \kappa_2$ and the Gaussian curvature $K = \kappa_1 \kappa_2$. Thus, 

$$d\tau = \frac{K}{H} ds = \frac{2}{|\rho_1 + \rho_2|} ds,$$

would be similarity (scale) invariant, as well as 

$$d\tau = \sqrt{|K|} ds = \frac{1}{\sqrt{|\rho_1 \rho_2|}} ds.$$

Yet, among all above possible options, only the last one is intrinsic in the sense that it is also invariant to isometric transformations of the surface. We therefore consider the following local scale-invariant isometric metric

$$\tilde{g}_{ij} = |K| \langle S_{\omega^i}, S_{\omega^j} \rangle,$$

so that 

$$d\tau^2 = |K| \left( \langle S_u, S_u \rangle du^2 + 2 \langle S_u, S_v \rangle dudv + \langle S_v, S_v \rangle dv^2 \right),$$

as our candidate for a scale invariant arc-length.

Given the surface normal 

$$\vec{n} = \frac{S_u \times S_v}{|S_u \times S_v|},$$

the second fundamental form is defined by 

$$b_{ij} = \langle S_{\omega^i \omega^j}, \vec{n} \rangle = \frac{\det(S_{\omega^i \omega^j}, S_u, S_v)}{\sqrt{g}},$$

from which we have the Gaussian curvature $K \equiv b/g$ where $b = \det(b_{ij})$ and $g = \det(g_{ij})$. 
Using the above notations we can write our similarity (scale) invariant metric \( \tilde{g}_{ij} \) as a function of the regular metric \( g_{ij} \) and the second fundamental form \( b_{ij} \). Recall that the metric structure is positive definite, that is \( g > 0 \). It allows us to write

\[
\tilde{g}_{ij} = |K| g_{ij} = \frac{|b|}{g} g_{ij}.
\]

It is interesting to note that the recent affine invariant metric explored in [RBB*11] could be written using similar notations as

\[
g_{ij}^{\text{equi-affine}} = |K|^{-1/4} b_{ij} = \left( \frac{g}{|b|} \right)^{1/4} b_{ij},
\]

projected onto a positive definite metric-matrix. The equi-affine intrinsic metric can be coupled with the scale-invariant one to produce a full affine invariant metric for surfaces given by

\[
g_{ij}^{\text{affine}} = |K^{\text{equi-affine}}| g_{ij}^{\text{equi-affine}} = |K^{\text{equi-affine}}| \left( \frac{g}{|b|} \right)^{1/4} b_{ij},
\]

where the Gaussian curvature is extracted from the affine invariant metric, that is, \( K^{\text{equi-affine}} = b^{\text{equi-affine}}/g^{\text{equi-affine}} \), and \( |g^{\text{equi-affine}}| = \sqrt{g|b|} \). Note that as we have been using intrinsic measures to construct this metric it would also be invariant to isometries within that space.

Another possible intrinsic measure for surfaces embedded in \( \mathbb{R}^3 \) is

\[
\kappa_{\min}^2 = \min(\kappa_1^2, \kappa_2^2) = \left( \sqrt{H^2 - K} - |H| \right)^2.
\]

It could be used to define the scale invariant metric

\[
\tilde{g}_{ij} = \kappa_{\min}^2 \langle S_{\omega}, S_{\omega} \rangle = \kappa_{\min}^2 g_{ij},
\]

that we will not explore in this paper.

3. SCALE INVARIANT DIFFUSION GEOMETRY

Diffusion geometry was introduced by Bérard et al. [BBG94]. It uses the Laplace-Beltrami operator \( \Delta_g \) of the surface as a diffusion or heat operator. Here, without giving up its isometry nature w.r.t. the metric, we consider the operator \( \Delta_{\tilde{g}} \) to construct a scale invariant diffusion geometry. The diffusion distance between two surface points \( s, s' \in S \) is given by the surface integral over the difference between two heat profiles of dissipation from two sources, one located at \( s \) and the other at \( s' \). The heat profile on the surface from a source located at \( s \), after heat has dissipated for time \( t \), is given by the heat kernel

\[
h_{s,t}(\hat{s}) = \sum_i e^{-\lambda_i t} \phi_i(s) \phi_i(\hat{s}),
\]
where \( \phi_i \) and \( \lambda_i \) are the corresponding eigenfunctions and eigenvalues of \( \Delta_{\tilde{g}} \), that satisfy \( \Delta_{\tilde{g}} \phi_i = \lambda_i \phi_i \). The diffusion distance is then defined as

\[
d^2_{g,t}(s,s') = \| h_{s,t}(\hat{s}) - h_{s',t}(\hat{s}) \|_{\tilde{g}}^2
= \int_S (h_{s,t}(\hat{s}) - h_{s',t}(\hat{s}))^2 da(\hat{s})
= \sum_i e^{-2\lambda_i t} (\phi_i(s) - \phi_i(s'))^2.
\]

(3)

By integrating over the time \( t \) of the diffusion distance constructed upon the regular metric \( g \), we can obtain a global scale invariant version of the regular diffusion distance [QH07]. This quantity is also known as the commute time distance, and is given by

\[
d^2_{CT}(s,s') = \int_0^\infty d^2_{g,t}(s,s') dt
= \sum_i \frac{1}{2\lambda_i} (\phi_i(s) - \phi_i(s'))^2.
\]

(4)

This is indeed an elegant setting for global scale invariance derived from the regular metric \( g \). Yet, our goal a bit more ambitious, that is, a measure that would be invariant to both local and global scale deformations. From a differential viewpoint we redefine the surface geometry. For that goal, the new geometric structure \( \tilde{g} \) is combined with the diffusion distance framework that stabilizes potential numerical variations in the new metric through its integral averaging nature.

4. Implementation considerations

In our experiments we assumed a triangulated surface. We follow the decomposition of the Laplace-Beltrami diffusion operator proposed in [RBB*11]. In order to compute diffusion distances we need the eigenfunctions and eigenvalues of the scale invariant operator \( \Delta_{\tilde{g}} \). For that goal we could use the finite elements method (FEM) for triangulated surfaces, see [Dzi88, Reu10].

The eigendecomposition \( \Delta_{\tilde{g}} \phi = \lambda \phi \) weak form is given by

\[
\int_S \psi_k \Delta_{\tilde{g}} \phi da = \lambda \int_S \psi_k \phi da,
\]

(5)

for \( \{ \psi_k \} \) a sufficiently smooth basis of \( L^2(S) \), in our case first order finite element functions, and \( da = \sqrt{g} dudv \) is a surface area element w.r.t. the metric \( \tilde{g} \). The finite element function \( \psi_k \) is equal to one at the surface vertex \( k \) and decays linearly to zero in its 1-ring. The number of basis elements is thus equal to the number of vertices in our triangulated surface. Assuming vanishing boundary conditions, we readily have that

\[
\int_S \psi_k \Delta_{\tilde{g}} \phi da = \int_S (\nabla \psi_k, \nabla \phi)_{\tilde{g}} da
\]
\[
\int_S \tilde{g}^{ij} (\partial_i \phi)(\partial_j \psi_k) \, da = \lambda \int_S \psi_k \phi \, da.
\]

Approximating the eigenfunction \( \phi \) as a linear combination of the finite elements \( \phi = \sum_l \alpha_l \psi_l \) leads to

\[
\int_S \tilde{g}^{ij} (\partial_i \sum_l \alpha_l \psi_l)(\partial_j \psi_k) \, da = \sum_l \alpha_l \int_S \tilde{g}^{ij} (\partial_i \psi_l)(\partial_j \psi_k) \, da,
\]

which should be equal to

\[
\lambda \int_S \psi_k \sum_l \alpha_l \psi_l \, da = \lambda \sum_l \alpha_l \int_S \psi_l \psi_k \, da.
\]

We thus need to solve for \( \alpha_l \) that satisfy

\[
\sum_l \alpha_l \int_S \tilde{g}^{ij} (\partial_i \psi_l)(\partial_j \psi_k) \, da = \lambda \sum_l \alpha_l \int_S \psi_l \psi_k \, da,
\]

or in matrix form \( \mathbf{A} \alpha = \lambda \mathbf{B} \alpha \), defined by the above elements.

Another option is numerically approximating the Laplace-Beltrami operator on the triangulated mesh and then computing its eigendecomposition. There are many ways to approximate \( \Delta \tilde{g} \), see [WMKG07] for an axiomatic analysis of desired properties and possible realizations, and [PP93] for the celebrated cotangent weights approach. Lévy [Lévy06] formalized the decomposition of the Laplace Beltrami operator in cotangent-weight form as a generalized eigendecomposition problem, while cubic finite elements for computing the eigendecomposition of the Laplace Beltrami were suggested in [Reu10]. A comprehensive and useful introduction to geometric quantities approximation on triangulated surfaces can be found in [MDSB03].

5. Experimental results

The first experiment demonstrates the invariance of the eigenfunctions of the Laplace Beltrami operator with the new metric \( \tilde{g} \). Figures 1 and 2, present the first eigenfunctions, \( \phi_1, \phi_2, \ldots \), textured mapped on the surface using the usual metric \( g_{ij} = \langle S_{\omega^i}, S_{\omega^j} \rangle \) (left frames) and the scale invariant one \( \tilde{g}_{ij} = |K|g_{ij} \) (right frames). The upper row of each frame shows the original surface, while the second row presents a deformed surface using isotropic inhomogeneous distortion field in space (local scales). Color represents the value of the eigenfunction at each surface point.

We next experiment with scale invariant heat kernel signatures [SOG09,BBGO11]. The heat kernel signature (HKS) at a surface point is a linear combination of all
Figure 1. Three eigenfunctions of $\Delta_g$ (left) and the invariant version $\Delta_{\tilde{g}}$ (right) for the armadillo with local scale distortions. Unlike the regular metric, the scale invariant metric preserves the correspondence between the matching eigenfunctions.

Figure 2. Four eigenfunctions of $\Delta_g$ (left) and the invariant version $\Delta_{\tilde{g}}$ (right) for the horse with local scale distortions. Unlike the regular metric, the scale invariant metric preserves the correspondence between the matching eigenfunctions.
eigenfunctions given by

\[ \text{HKS}(s, t) = \sum_i e^{-\lambda_i t} \phi_i^2(s) \]  

at that point. Figure 3 displays the invariance of the HKS signature for several times, textured mapped onto a centaur and its locally scaled version and normal horse and one with enlarged head that looks like a mule. Figure 4 displays the inconsistency of corresponding signatures when using the regular metric (left) and the consistency achieved with the invariant one (right). The signatures are extracted at three points as indicated in the figure: two finger tips; one on the right and one on the left hand of the centaur, and the horseshoe of front left leg. In the graphs, the signature value at each time \( t \) is scaled with respect to the integral of the signatures at that time over the (invariant) surface area, i.e. \( \text{HKS}(s, t) / \int_S \text{HKS}(s, t) da(s) \) as done in [SOG09] for presentation prepossess. As can be observed the proposed metric produces invariant non-trivial signatures.

Figure 3. The heat kernel signature at different times textured mapped onto the surface for the regular metric (left frames) and the invariant one (right frames). The four shapes at each row, left to right, capture the HKS values at \( t = 10, 50, 100, \) and \( 500 \), respectively.
Next, we extract Voronoi diagrams for 15 points selected by the farthest point (2-optimal) sampling strategy starting from the tip of the nose as the first point. In this example, length is measured using diffusion distance with either a regular metric or the invariant one. Yet again, the invariant metric produces the expected result where the correspondence between the two surfaces is independent of the local scaling deformations. This is obviously not the case for the regular metric.

Finally, we experimented with heat kernel signatures computed with the proposed metric within the ShapeGoogle recognition framework applied to the SHREC’10 shape retrieval benchmark. That database is the only one in which there suppose to be local scale variations. In fact, the distortions in that benchmark appear like dilation operations rather than scaling. Still, Table 1 demonstrates that the proposed framework can handle even these deformations while being robust to articulations referred to as isometries in the table, as well as topological noise that is handled by the diffusion part of the signature. The results are comparable to SG3 (relating to the SI-HKS of [BK10]) in the SHREC’10 framework [BBC∗10].

6. Conclusions

We introduced a new metric that gracefully handles changes in size at virtually any scale, that is, local to global as well as articulations that have relatively small effects on the Gaussian curvature. It was combined with diffusion distances and used to construct heat time kernels, which are in fact semi-differential scale invariant signatures for surfaces. Our future plans are to use the proposed measures and
computational tools to study the geometric relations between existing species in nature. We plan to use the local scale invariance framework to explore humankind shape-similarity to apes, then to mammals in general including marine mammals like whales and dolphins, and continue to birds, reptiles, and fish. By enriching the set of transformations we can handle with efficient computational tools while comparing shapes, we could provide a quantitative evidence to the morphometric nature of Charles Darwin’s qualitative theory of evolution [Dar59].

Table 1. Performance of the \( \tilde{g} \)-HKS on SHREC’10 shape retrieval benchmark with the ShapeGoogle framework (mAP in %).

<table>
<thead>
<tr>
<th>Transformation</th>
<th>Strength</th>
<th>1</th>
<th>&lt;2</th>
<th>&lt;3</th>
<th>&lt;4</th>
<th>&lt;5</th>
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</table>

Figure 5. Voronoi diagram using diffusion distances for farthest point sampling each surface with 15-points applying the regular metric (left two surfaces) or the invariant version (right).
The Laplace Beltrami operator is defined as \( \Delta_g \equiv \frac{1}{\sqrt{g}} \partial_i \sqrt{g} g^{ij} \partial_j \), where

\[
(g^{ij}) = (g_{ij})^{-1} = \frac{1}{g} \begin{pmatrix} g_{22} & -g_{12} \\ -g_{12} & g_{11} \end{pmatrix},
\]

is the inverse metric matrix. The mean curvature vector can then be written as

\[
2H \hat{n} = (\kappa_1 + \kappa_2) \hat{n} = \Delta_g S = \frac{1}{\sqrt{g}} \partial_i \sqrt{g} g^{ij} \partial_j S,
\]

where \( \partial_i \equiv \partial / \partial \omega_i \); for example \( \partial_1 \equiv \partial / \partial u \).

For a surface given as a graph \( z = f(x, y) \), we have

\[
K = \frac{f_{xx} f_{yy} - f_{xy}^2}{(1 + f_x^2 + f_y^2)^2} = \frac{\det(\text{Hess}(f))}{(1 + |\nabla f|^2)^2},
\]

\[
H = \frac{(1 + f_{xx}) f_y^2 - 2 f_{xy} f_x f_y + (1 + f_{yy}) f_x^2}{(1 + f_x^2 + f_y^2)^{3/2}} = \text{div} \left( \frac{\nabla f}{\sqrt{1 + |\nabla f|^2}} \right),
\]

where \( \text{Hess}(f) \) is the Hessian of \( f(x, y) \).

The mean curvature is given by

\[
H = \frac{g_{22} b_{11} - 2 g_{12} b_{12} + g_{11} b_{22}}{2g} = \frac{1}{2} b_{ij} g^{ij},
\]

where \((g^{ij}) = (g_{ij})^{-1}\) is the inverse metric matrix. The two principal curvatures can now be written as a function of \( H \) and \( K \),

\[
\kappa_1 = H + \sqrt{H^2 - K} \\
\kappa_2 = H - \sqrt{H^2 - K}.
\]

That allows us to express the second choice of a scale invariant metric by Equation (2).

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