Euler–Maclaurin Expansions for Integrals
with Endpoint Singularities:
A New Perspective

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Abstract

In this note, we provide a new perspective on Euler–Maclaurin expansions of (offset) trapezoidal rule approximations of the finite-range integrals \( \int_a^b f(x) \, dx \), where \( f \in C^\infty(a, b) \) but can have general algebraic-logarithmic singularities at one or both endpoints. These integrals may exist either as ordinary integrals or as Hadamard finite part integrals. We assume that \( f(x) \) has asymptotic expansions of the general forms

\[
\begin{align*}
  f(x) & \sim \sum_{s=0}^{\infty} P_s (\log(x - a))(x - a)^{\gamma_s} \quad \text{as } x \to a^+, \\
  f(x) & \sim \sum_{s=0}^{\infty} Q_s (\log(b - x))(b - x)^{\delta_s} \quad \text{as } x \to b^-,
\end{align*}
\]

where \( P_s(y) \) and \( Q_s(y) \) are some polynomials in \( y \). Here the \( \gamma_s \) and \( \delta_s \) are complex in general and different from \(-1, -2, \ldots\). The results we obtain in this work generalize, and include as special cases, those pertaining to the known special cases in which \( f(x) = (x - a)^p \log^p(x - a) g_a(x) = (b - x)^q \log^q(b - x) g_b(x) \), where \( p \) and \( q \) are nonnegative integers and \( g_a \in C^\infty[a, b] \) and \( g_b \in C^\infty(a, b] \). In addition, they have the pleasant feature that they are expressed in very simple terms based only on the asymptotic expansions of \( f(x) \) as \( x \to a^+ \) and \( x \to b^- \).

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