


4. Given an interrogative formula $? \varphi$, where $\varphi$ is a closed indicative formulae, it is reasonable to assign it an extensional entity according to the truth value of $\varphi$. Thus, assume we assign the formula $? \varphi$ the entity \textbf{yes} iff $\varphi$ is true and \textbf{no} if $\varphi$ is false.

5. Material adequacy requires definition of answerhood, entailment and equivalence:

(a) \textbf{Answerhood} $? \psi$ is a (possible, not necessarily true) answer to $? \varphi$ iff in all structures in which $\psi$ is true, $? \varphi$ has the same interpretation.

(b) \textbf{Entailment} $? \varphi$ entails $? \psi$ iff any answer to $\varphi$ is an answer to $? \psi$.

(c) \textbf{Equivalence} $? \varphi$ is equivalent to $? \psi$ iff they have the same answers.

By these definitions, the interpretations satisfies material adequacy.

6. By the definitions, $? \varphi$ and $? \neg \varphi$ have the same answers, i.e. the set \{\textbf{yes, no}\} and are therefore equivalent.

7. However, $? \varphi$ and $? \neg \varphi$ must be assigned different interpretations.

8. Therefore, formal adequacy is not satisfied, since although $? \varphi$ and $? \neg \varphi$ are equivalent, they are assigned different interpretations.

**References**


\footnote{This is precisely the point where our bilattice approach parts ways with G&S's argument. Instead of capturing the truth value of $\varphi$, we capture the resolvedness/unresolvedness of $? \varphi$ itself. We are thus able to overcome this argument against any extensional interpretation.}
straightforward and simple account of the phenomena listed above. In addition, it allows an analysis of combined indicative and interrogative sentences, which have received only limited attention in previous accounts. All these phenomena are uniformly analyzed by using our novel bilattice construction, in a completely straightforward and natural WYSIWYG manner. It is the addition of a second dimension at the most basic level of the logic that confers upon the system its power. Incorporating this extra dimension of knowledge in addition to the classical dimension of truth at the level of the basic truth values is extremely important, since the notion of knowledge (or information) is an important part of the notion of answerability. In order to be able to answer a question, one needs certain knowledge. The incorporation of the knowledge dimension at the most basic level is an extremely helpful move. The bilattice interpretation, which simultaneously incorporates both the truth and the knowledge dimensions thus yields an improved framework for the semantics of questions. The price paid for diverging from the standard path of classical logic pays off in greatly simplifying the accounts of complex phenomena. We expect that this basic approach for the analysis of questions will also turn out to be useful in the analysis of other phenomena related to the semantics of questions. In particular, it would be interesting to explore a treatment of embedded questions, where the valuation may be relativized differently for different agents, and dynamic settings, in which the valuation may change dynamically in the course of discourse progress. Another interesting direction in which the present work may be extended is the analysis of plural questions.

Acknowledgments

We are grateful to Ian Pratt for helpful comments and discussions. The work of the second author was partially supported by the fund for the promotion of research in the Technion.

A Appendix: Rejection of an extensional analysis by G&S

In [17] there is a rejection of an extensional analysis of questions. In this appendix we present a summarized version of the G&S argument. In the main part of the paper, we have shown that the bilattice interpretation of the semantics of questions does, in fact, provide an extensional account of the meanings of questions. We present the argument here for completeness. The reasoning of [17] is based on a rejection of an extensional interpretation of (a variant of) II.Q\(^{-}\) interrogative binary choice formulae. G&S argue that an extensional interpretation of interrogative binary choice formulae may satisfy material adequacy but not formal adequacy. Therefore, it is unsuitable for the interpretation of II.Q\(^{-}\) formulae. The argument proceeds as follows:

1. Assume an extensional interpretation of II.Q\(^{-}\) binary choice formulae.

2. Assume also that answers are propositions.

3. Such an interpretation would assign extensional entities as the interpretation of binary choice interrogative formulae.
quantified question, the speaker does not convey whether she is interested in a generalization or a complete specification, allowing it to be answered either way. It is precisely the difference in verbose that may help explain the difference in the acceptability of functional/pair-list readings for downwards monotonic quantifiers. We believe the difference lies in the exhaustiveness expectations associated with the two kinds of answers. Since pair-list answers are so verbose, it is natural to assume that they carry much stronger exhaustivity requirements. For instance, if a replier answers the question (113) by giving a long list, the asker may assume the list to be complete. Thus, by default, the asker may assume that whoever is not explicitly mentioned to be not liked by e.g. Giovanni, is liked by him. Therefore, the pair-list reading resolves not only the question of who every Italian married man doesn’t like, but also who he does like, and the argument in the previous section rejecting the pair-list answer is sound.

It is precisely because they provide generalizations, that it is natural to assume weaker exhaustiveness requirements of functional answers. For instance, representing (114) in II.Q as (115), it is reasonable to assume that this answer does not implicate that it provides a complete specification of the complement question of who does like for Italian married men x. This answer seems to be neutral as to whether e.g. Giovanni likes his neighbor Maria. Thus, the functional answer does not exhaustively resolve the complement question who x likes, for all Italian married men x, and by a converse argument to the one provided in the previous section, does resolve the original question.

\[(115) \neg \exists x [\text{ITALIAN}(x) \land \text{LIKE}(x, \text{MOTHER-IN-LAW}(x))].\]

Of course, this discussion of the completeness and exhaustiveness implicatures associated with answers lacks a more formal grounding. This matter is left for further research.

11 Conclusion

In this paper we have given a new account of the semantics of NL questions. It is generally accepted that the semantics of questions should be based on a notion of answerability conditions. In this paper, we have split this task into two parts. First, we propose a method for compositionally constructing meaning representations of NL questions in an extension of Montague's II. This account is based on a straightforward extension of the Montogovian construction rules, viewing interrogative determiners as a particular case of determiners, yielding interrogative formulae. We then consider the interpretation of the resulting meaning representations in two frameworks, one according to the work of G&S and a novel interpretation based on bilattices. The work of G&S is important in setting a standard for semantic theories of questions, by imposing adequacy requirements that such theories must satisfy. We show an interpretation of the meaning representations of interrogatives in the G&S framework. This interpretation satisfies the adequacy requirements, but is very complex, giving the basic interrogatives a higher type than indicatives. More complex phenomena involving interrogatives such as coordination, multiple constituent questions and functional questions are analyzable in the G&S framework, but at the price of considerable complexity. We present an alternative interpretation of the meaning representations of NL questions in a multi-valued logic setting based on bilattices. This novel interpretation is shown to satisfy the G&S adequacy requirements for the semantics of questions. Thus, it plays by the G&S rules, so to speak. Furthermore, this approach provides a particularly elegant solution for many of the issues involved in the semantics of questions, yielding a
Formula (111) represents the non functional reading. Since the sub-formula embedded within the binding interrogative operator is a type type indicative formula, an answer to this question would be any indicative proposition which settles for every possible value $d$ for $x$, whether $d$ satisfies the embedded indicative formula. Thus, such an answer would be a specification of the individual admired most by all Englishmen, e.g. the queen. The functional reading is represented in (112), which takes advantage of the extra expressive power of II.Q. The sub-formula embedded within the scope of universal quantification over $y$ is an interrogative formula combined from indicative and a constituent interrogative formulae. By the truth table of implication, this formula is settled for each value of $y$ that is not an Englishman, or that the question whom he admires is settled. Thus, an answer to (112) is a proposition that for specifies for any $y$, who is an Englishman, the list of individuals whom he admires most. The relational answer is a concise elliptical such specification. The full answer would be a quantified proposition, e.g. Each Englishman admires his mother most. The relational force comes not from the open term his mother, but rather from the quantification in the full answer. Again, we assume that the answer is exhaustive, in the sense that it specifies the complete set of people whom each Englishman admires, in the form of a generalization.

Since we do not view relational answers as denoting functions, the problems shown for previous approaches are avoided. Also there is no need for any polyadicity. On this account, functional questions are seen as a particular case of quantifying into questions requiring no special treatment. A natural question that may be asked at this point relates to the connection between functional and pair-list readings.

10.3 Functional vs. pair-list answers

The relation between pair-list and functional readings of questions is an important factor in the analyses of [6, 4], who view pair-list readings as special cases of functional readings. According to this analysis, questions with quantifiers are to be interpreted as functional, and pair-list answers are seen as verbose specifications of functions. In this paper, we have taken a view, which may in a sense be seen as opposite. On our account, ‘quantifying-in’ is seen as the basic analysis for questions with quantifiers, allowing both pair-list and functional answers. A natural question to ask is what is the difference between the two. Chierchia argues against this sort of approach for three reasons, adapted from [14]. First, whereas pair-list readings may be seen as specifying a function, they do so very verbosely. Thus in asking a functional question, the asker is interested in the generalization provided by the functional answer. Conversely, if the asker is interested in a complete specification, a pair-list answer may fail to be exhaustive. Finally, as we saw above, downwards monotonic quantifiers do not allow a pair-list answer. However, they do allow a functional answer as in:

(113) Who does no Italian married man like?
(114) His mother-in-law.

We submit that the difference between pair-list readings and functional ones lies not in the question but in the answer. We believe that the semantic content of the question is neutral with respect to whether it is answered by a pair-list answer or by a functional one. Thus, in asking a
e, it is translated as a free variable \( W \), ranging over functions of type \((\text{e,e})\) applied to a free variable, \( x \), ranging over individuals. A functional question now “asks” for a specification of the possible values of \( W \). For example, (102) asks for functions \( W \) such that for every \( x \), if \( x \) is an Englishman, \( x \) admires \( W(x) \) most. Accordingly, (104) denotes a possible answer, as it is seen as denoting an object of the correct type. It is a function, which for every individual, yields his mother.

In fact, [6] requires different sentences to be interpreted using functions \( W^n \) from \( n \) individuals to an individual, of type \((\text{e}\times\text{e}\times\cdots\times\text{e})\text{e}\). This need arises in questions such as (105), where (106) is seen as a function from two individuals to an individual. Individuals may be seen as identity functions, \( W^0 \), on individuals.

(105) Which of his letters to her does every woman return to her lover?

(106) The ones where he asks her for money.

There are several problems with this approach to functional questions. First, rather surprisingly, so-called functional answers are not, in fact, functions, for two reasons. First, as noted by [21], such answers are of the type of individuals not functions. For example, let the translation of the term his mother be \( \text{mother-of}(x) \). Assuming \( \text{mother-of} \) is a function of type \((\text{e,e})\), which for any individual yields his mother, and \( x \) is a variable of type \( \text{e} \), then \( \text{mother-of}(x) \) is an open formula of type \( \text{e} \). It is true that in MG, when operating on these terms, one usually abstracts over the free variable (e.g., as in “quantifying-in”), however, this operation changes the type of the stand-alone term. In general, his mother is not interpreted as a function in a neutral context such as (107), where it would be counter-intuitive to claim that his mother denotes an element of a different type than \text{Mary}. We are thus led to the conclusion that his mother does not denote a function, but rather an open individual. Its functional, or rather relational interpretation stems from the quantification in the original question.

(107) Which woman kissed John?

(108) Mary.

(109) His mother.

Another respect in which “functional answers” are not functions, is the mathematical sense, since such answers allow one individual to be mapped to more than one individual. For instance, (102) may be answered by (110). This answer does not denote a function, since each individual is mapped to more than one individual. Of course, one may overcome this problem by enriching the ontology with plural entities, but this complicates the analysis.

(110) His parents.

Another shortcoming of this approach is that it leads to an infinitely polyadic interpretation of INPs as described by [6, 21]. For instance, which woman receives an infinite number of interpretations, all differing by the arity of the function \( W \) from which it is composed. In contrast, our bilattice interpretation interprets functional questions as questions of type \( \text{e} \) elements, avoiding the need for any polyadicy. The “relational” interpretation stems from the quantifiers in the question.

10.2 Our account

On our account, functional questions are treated just like any other questions with quantifiers, using either direct and indirect construction. For (102) this yields the following two translations:
However, $\varphi$ provides for each Italian married man a complete specification of whom he likes. Thus, for any value $x$, for which $v_w(\text{ITALIAN}(x)) = t$, $v_w(\exists x[\text{ITALIAN}(x) \land \varphi(x)]) = t$. This implies that for any possible world $w$ in which $v_w(\varphi) = t$, we have $v_w(\forall x[\text{ITALIAN}(x) \land \varphi(x)]) = t$. Hence, $v_w(\exists x[\text{ITALIAN}(x) \land \varphi(x)]) = t$. We thus see that the pair-list answer is not related to the question by answerhood.

This provides a semantic explanation for the unavailability of pair-list answers to questions with the negative determiner. This explanation is a straightforward application of the semantic notion of answerhood defined above. A similar argument may be used to explain the unavailability of pair-list answers to questions with other downwards monotonic quantifiers. For example [4-6]:

(99) What do at most three students like?

(100) *Paul the semantics requirement, Mary the phonology requirement, and Bill the exotic language requirement

The pair-list answer provides a specification of the things that three students like. Again, we may interpret the pair-list answer as providing a complete specification of the things they like. The reading of the question that gives less than three students wider scope may be represented as follows, where $\exists \xi \exists y[\varphi]$ is taken to mean that there are less than 3 values of $x$ such that $\varphi$.

(101) $\exists \xi \exists y[\text{STUDENT}(x) \land \exists y[\text{LIKE}(x,y)]].$

For any structure $M$ and possible world $w$, (101) is settled if there are at most three students for which the question what they like is resolved. Now, the pair list answer already provides such a specification for three students. If there is even one more student for which the question what he likes is resolved in $w$, (101) will be assigned $t$. Thus by definition, the pair-list answer does not answer the question.

We now turn to consider functional questions.

10 Functional questions

In this section, we consider functional questions such as (102). Such questions typically admit two kinds of answers, one denoting an individual as in (103), and a relational answer as in (104). We examine some previous accounts to such questions and present a novel account based on the bilattice interpretation.

(102) Who does every Englishman admire most?

(103) The queen.

(104) His mother.

10.1 Previous accounts

In handling functional questions, several authors (e.g. [4, 6, 13]) view the two kinds of answers as denoting elements of different types. Whereas (103) denotes an individual, (104) denotes a function from individuals to individuals. They therefore change the meanings associated with “gaps” and INPs in such questions. Instead of interpreting a gap as a free variable of type
As for (89), by definition, this is an interrogative formula. It will be answered if the value of
the embedded interrogative formula inside the scope of the universal quantification over y is
settled for each value d of y. By the truth table of the implication operator of the embedded
combined formula, for a particular d, the embedded formula is assigned T iff either d is not a
candidate or the question which professors recommends d is settled. Thus, the pair-list answer
does indeed answer this question. We may represent the pair-list answer as a conjunction over
all candidates, as in (92).

\begin{align*}
(92) & \forall x[\text{RECOMMENDS}_x(x, \text{SUE}) \rightarrow x = \text{SMITH}] & \\
& \forall x[\text{RECOMMENDS}_x(x, \text{MARY}) \rightarrow x = \text{BROWN}] & \\
& \forall y[\text{CANDIDATE}(y) \rightarrow ?x[\text{PROFESSOR}(x)\text{RECOMMENDS}_x(x, y)]] &
\end{align*}

9.3 Downwards monotonic quantifiers

As noted by several authors [4, 14, 31], some (if not all) downwards monotonic quantifiers\(^{18}\)
do not allow pair-list readings. The strongest example of this is the determiner no as in (93) from
[4]. The sentence exhibits a similar scope ambiguity as in the examples considered above.
The reading assigning who wider scope asks for a person that is not liked by any Italian, and is
unproblematic. It may be answered by an individual answer, e.g. Maria. However, the second
reading, which assigns who narrower scope, does not admit a pair list answer, such as (94), taken
to be an abbreviation of (95).

\begin{align*}
(93) & \text{Who does no Italian married man like?} & \\
(94) & \ast \text{Giovanni: Maria; Paolo: Francesca; etc.} & \\
(95) & \ast \text{Giovanni doesn't like Maria and only her; Paolo doesn't like Francesca and only her,} & \\
& \text{etc.} &
\end{align*}

As noted by [31], there are no known compelling syntactic or semantic explanations for this
phenomenon. A pragmatic explanation is offered by G&S. They claim that assigning no Italian
married man wider scope should be interpreted as asking for no Italian married man whom
he likes. Thus the question asks the replier to remain silent, and is therefore pragmatically
unacceptable. Of course, it would be desirable to provide a semantic explanation for this phe-

omenon, which we proceed to do presently. In fact, we show that the pair-list answer is not
related to the question according to the answerhood relation.

We assign the generalized quantifier no Italian married man the meaning representation in (96),
abbreviating Italian married man to ITALIAN. The second reading of this question assigning who
narrower scope, is represented by (97).

\begin{align*}
(96) & \exists x[\text{ITALIAN}(x) \land P(x)] & \\
(97) & \exists x[\text{ITALIAN}(x) \land \forall y[\text{LIKE}_y(x, y)]] & \\
\end{align*}

Let \(\varphi\) denote the pair-list answer. We show that:

\begin{align*}
(98) & \varphi \not\models B - \exists x[\text{ITALIAN}(x) \land \forall y[\text{LIKE}_y(x, y)]] &
\end{align*}

Let \(M\) be a structure, \(w\) a possible world. By definition, a valuation \(v_w\) may assign (97) either of
the values \(\perp, T\). It assigns it the value \(T\) if there is no value of \(x\) for which both \(v_w(\text{ITALIAN}(x)) =

\(^{18}\)Following [3], a generalized quantifier \(Q\) (of type \((e, t), t\)) is downwards monotonic if whenever \(A \in Q\) and
\(B \subseteq A\), then \(B \in Q\).
that we are truly taking advantage of the expressive power of ILQ.

In order to account for an interrogative sentences such as (77), repeated here as (87), we use exactly the same two construction methods used above for indicatives, direct and indirect construction. With the exception of using an INP instead of an existentially quantified NP, the derivations proceed in the same way as for the indicative case, yielding (88) and (89). Note, that we are truly taking advantage of the expressive power of ILQ\(^+\) here, as (89) is in ILQ\(^+\), but not in ILQ\(^-\). This is because in (89), quantification is applied to an interrogative formula. This is possible only in ILQ\(^+\), where interrogatives have type t, but not in ILQ\(^-\), where they have (st).

(87) Which professor recommends each candidate?

(88) \([s_5:\exists x[\text{professor}(x) \land [\text{candidate}(y) \rightarrow \text{recommends}(x, y)]]\]

(89) \([s_5:\forall x[\text{candidate}(y) \rightarrow \exists x[\text{professor}(x) \land \text{recommends}(x, y)]]\]\

Having derived ILQ\(^+\) formulae representing the two different readings of (87), we turn to the interpretation of the formulae. The bilattice interpretation yields a straightforward interpretation for both readings. In a structure \(M\), formula (88) will be assigned the value \(T\) in any possible world \(w\), in which for each possible value \(d\) of \(x\), the embedded indicative formula is true, i.e. \(d\) is a professor that recommends each candidate. It will therefore be answered by any indicative formula \(\varphi\) such that \(\varphi\) entails for each such value \(d\) whether it satisfies the embedded indicative or not. In particular, a specification of those professors that satisfy the embedded indicative is an answer, i.e. an exhaustive specification of those professors that recommend all the candidates. For example, (90) is such an answer, as shown by (91).\(^{17}\) This entailment may be proved according to the same principles as Proposition 4 (item 1).

(90) Only Prof. smith recommends all the candidates.

(91) \(\forall x[\forall y[\text{candidate}(y) \rightarrow \text{recommends}(x, y)] \leftrightarrow x = \text{smith}] \models^B \quad \exists x[\text{professor}(x) \land \exists y[\text{candidate}(y) \rightarrow \text{recommends}(x, y)]]\]

\(^{17}\)The quantification over \(x\) in the ILQ\(^+\) representation of (90) comes from the restriction only.
This is considered an advantage in the original paper [23], but a disadvantage in [6, 20]. A solution for other quantifiers is presented by partition theories such as [14, 20], of which we review the former.

9.1.2 G&S’s approach

G&S’s approach to the meanings of questions with quantifiers views such questions not as a single question, but rather as a family of questions. For example, (77) is seen as denoting a family of questions, a different question for each candidate. Each such question relates to a particular candidate, and asks which professor recommends her. Just as in the case of coordination, discussed in Section 7, a generalized operation of quantifying may be applied to elements of types “ending in type \( t \)” (cf. [27]). This operation ‘pushes’ the quantification until it may be applied to type \( t \) elements. Applying this operation for the case of universal quantification yields correct results, partitioning the set of possible worlds into equivalence classes according to the set of professors that recommended each candidate in each possible world. However, as shown in [14], the same operation fails to yield a partition for other quantifiers. The situation is similar to the one encountered for disjunction in trying to apply generalized disjunction as described in Section 7. The solution proposed by [14] to this problem is similar as well. They employ a type-lifting operation shifting the meanings of questions into sets of sets of questions, which may be viewed as generalized quantifiers over questions. This operation is even more complex than in the case of coordination, hence we omit the details. While this does solve the problem, it does so at the price of adding another level of complexity to the theory.

9.2 Our account

As we did before, we first translate such questions into IIQ\(^+\), and then give the bilattice interpretation of the resulting formulae. The translation into IIQ\(^+\) is modeled on the translation of similar scope ambiguities in indicative sentences in MG (reviewed below), using direct and indirect construction. We will make use of the full expressional power of IIQ\(^+\), surpassing that of IIQ\(^-\).

9.2.1 Scope ambiguities in indicative sentences

We derive the scope ambiguity in (77) in a similar manner to that in (82).

(82) Some professor recommends each candidate.

Direct construction for this sentence yields (83), where the meaning of some professor is directly applied to the meaning of the VP.

(83) \([\lambda x [\text{professor}(x) \land Q(x)]](\lambda x \forall y [\text{candidate}(y) \rightarrow \text{recommends}, (x, y)])\) = \(\exists x [\text{professor}(x) \land \forall y [\text{candidate}(y) \rightarrow \text{recommends}, (x, y)]]\).

Indirect construction applies the meaning of the NP each candidate to the meaning of the gappy sentence Some professor recommends her, represented in (85). This formula has a free variable
Intuitively, one may understand the second reading as giving the NP each candidate wide scope over the interrogative which candidate. However, it is not a simple matter to account for these readings compositionally in the existing frameworks. To quote a recent paper by Szabolcsi [31]:

“The crux of the matter is that quantification is defined for domains of type t (expressions that can be true or false), and interrogatives are not such.”

A major point in this paper is that interrogatives can be made of type t, by using a non-classical domain for type t, with the result of greatly simplifying the analysis. We will briefly review some of the difficulties encountered by previous approaches. For more detailed discussion see [4]. We then show how the bilattice interpretation of interrogatives provides a straightforward interpretation of such questions. In particular, it provides a novel explanation for the availability and unavailability of ‘pair-list’ answers.

9.1 Previous accounts

We now briefly consider two previous accounts to such questions, that of [23] and that of [14]. Both theories manage to analyze such questions, but only by technically complex mechanisms.

9.1.1 Karttunen’s and Peters’ approach

In [23], the interpretation of questions with quantifiers is linked to multiple constituent questions. The two possible readings of (77) are represented by [23] as (78)–(79).

(78) \( \lambda p [\exists x [\text{PROFESSOR}(x) \land \text{true}(p) \land p = \land (\forall y [\text{CANDIDATE}(y) \rightarrow \text{RECOMMENDS}, (x, y))]]) \).

(79) \( \lambda p [\exists y \exists x [\text{CANDIDATE}(y) \land \text{PROFESSOR}(x) \land \text{true}(p) \land p = \land (\text{RECOMMENDS}, (x, y))]]) \).

Formula (78) denotes the property of propositions \( p \) such that there is some \( x \) which is a professor and \( p \) is a true proposition that asserts that for every \( y \), if \( y \) is a candidate then \( x \) recommends \( y \). This reading that is achieved by giving each narrow scope and applying the regular construction rule of [22]. The second reading of (77) is seen in [23] as equivalent to the multiple constituent question in (80).

(80) Which professor recommends which candidate?

The meaning of this question, represented in (79), denotes the property of propositions \( p \) such that there are a professor \( x \) and a candidate \( y \) such that \( p \) is the true proposition that asserts that \( x \) recommends \( y \). This representation for the second reading of (77) fits in well with Karttunen’s general semantics of questions. However, deriving it in a compositional manner is far from easy. Straightforward application of the rule for single constituent questions fails to give the required result. In fact, the representation is derived in [23] by applying the following rule, using double negation:

(81) \( \lambda p [\neg NP'(\lambda x_n [\neg Q'(p)])] \).

Applying this rule to the meaning of the NP each candidate and the gappy question which professor recommends \( x_n \) gives a formula equivalent to (79). However, as noted by [6], this rule lacks independent motivation. Also, it works correctly only for universal quantifiers (e.g. each).
which Mary did not kiss John, but in which it is unknown whether Mary kissed Bill or not. As a result, we have:

\[(71) \quad \neg \text{Kiss}_s(Mary, John) \wedge \text{Kiss}_s(Mary, John) \lor \text{Kiss}_s(Mary, Bill)\]

At this point we wish to illustrate the difference between coordination of a pair of purely interrogative sentences as considered in Section 7 and combined coordination as discussed in this section. For example, compare (65) with (72).

\[(72) \quad \text{[S-Did Mary kiss John or did she kiss Bill?]} = \text{[Kiss}_s(Mary, John)] \lor \text{Kiss}_s(Mary, Bill)\]

Whereas a negative answer such as (60) is not an answer to the combined disjunction, it is an answer to the disjunction of purely interrogative sentences. This is because questions may be resolved by negative answers as well as positive ones.

By the same methods, we may analyze answers to combined conjunctions such as (66). An answer to this question would be a proposition \(\varphi\), such that for any \(w \in [\varphi]_{M,\sigma}\), both Mary kissed John, and the question of who kissed Bill is settled.

An interesting observation about this kind of combination of an indicative with an interrogative, is that whereas a combination of an indicative followed by an interrogative is acceptable, reversing the order of the two clauses yields ungrammaticality, as in the following examples. We have no semantic explanation for this asymmetry.

\[(73) \quad \text{If who kissed Bill? then Mary kissed John.}\]

\[(74) \quad \text{Is the machine broken? or it just needs fuel.}\]

\[(75) \quad \text{Did Mary kiss Bill? or she kissed John.}\]

\[(76) \quad \text{Who kissed Bill? and Mary kissed John.}\]

9 Questions with quantifiers

In this section, we discuss constituent questions with multiple ‘wh’-terms or constituent questions with quantifiers. The meanings of such questions are notoriously difficult to derive compositionally in previous theories. We consider examples such as (77):

\[(77) \quad \text{Which professor recommends each candidate?}\]

Such questions are traditionally seen as having two different readings, distinguished by the kind of answer that may be used to answer them (see e.g. [4]):

1. Individual answer: e.g. Prof. Smith. This reading asks which professor is such that she recommends every candidate.\(^\text{16}\). Once again we accept this answer as an abbreviation of a fuller one of the form: Only Prof. Smith recommends every candidate.

2. Pair-list answer: e.g. Sue was recommended by Prof. Smith, Mary was recommended by Prof. Brown, etc. This reading asks for each candidate, which professor recommends her.

\(^{16}\)Since we ignore presuppositions of uniqueness associated with which + N, we may actually accept a list of names.
In addition, there are also examples of coordination of an indicative with an interrogative as in the following sentences. In each of them, the first clause of the sentence is indicative, and should be read so (not as a question). We show below that there is a subtle difference between such coordinations and coordinations of a pair of purely interrogative sentences.

(56) The machine is broken or does it just need fuel? [18]
(57) Mary kissed John or did she kiss Bill?
(58) Mary kissed John and who kissed Bill?

All of these examples may be represented in a straightforward manner in ILQ\(^+\), by direct translation. For example:

(59) \(\text{If Mary kissed John then who kissed Bill?} \equiv \neg \text{kiss}_v(MARY, JOHN) \rightarrow \neg \exists x [\text{kiss}_v(x, BILL)].\)

According to the truth table of implication, given in Figure 3, (59) is resolved if either its indicative antecedent is false or its interrogative consequent is resolved. Thus, a proposition \(\varphi\) answers such a formula if in any possible world \(w \in [\varphi]_{M,v}\), either Mary didn’t kiss John or the question who kissed Bill is resolved. By our analysis, it is easy to verify that the ILQ\(^+\) translations of the following indicative sentences stand in the answerhood relation to the question (59).

(60) Mary did not kiss John.
(61) Only Sue kissed Bill.
(62) If Mary kissed John then only Sue kissed Bill.

For example, it is easy to see that (60) is an answer, by the entailment in (64). Clearly, in any possible world in which (63) is true, the implication (59) is resolved.

(63) \(\neg \text{kiss}_v(MARY, JOHN)\)

(64) \(\neg \text{kiss}_v(MARY, JOHN) \equiv \nabla \text{kiss}_v(MARY, JOHN) \rightarrow \neg \exists x [\text{kiss}_v(x, BILL)].\)

The bible examples are rhetorical questions. Formally, it is easy to verify that any valuation will assign them the value \(\top\), because whenever the indicative antecedent is true, the interrogative consequent is resolved.

Similarly, we can compute the answerhood relation for coordination as in the following examples.

(65) \(\text{Mary kissed John or did she kiss Bill?} = \text{kiss}_v(MARY, JOHN) v [\text{kiss}_v(MARY, BILL)].\)

(66) \(\text{Mary kissed John and who kissed Bill?} = \text{kiss}_v(MARY, JOHN) \land \exists x [\text{kiss}_v(x, BILL)].\)

An answer to (65) would be a formula \(\varphi\), if in any possible world \(w \in [\varphi]_{M,v}\), either Mary kissed John or the question whether she kissed Bill is resolved, allowing the following answers:

(67) Mary kissed John.
(68) Mary kissed Bill.
(69) Mary kissed neither of them.

Again, it easy to verify that the ILQ\(^+\) translations of these answers stand in the answerhood relation to the translation of the original question (65). For example, it is easy to see that (67) is an answer by the following entailment:

(70) \(\text{kiss}_v(MARY, JOHN) \equiv \nabla \text{kiss}_v(MARY, JOHN) v [\text{kiss}_v(MARY, BILL)].\)

This is an instance of Proposition 8. It is interesting to note that (60) is not an answer to (65). It merely contradicts the first disjunct. To see this, it is sufficient to show a possible world in which (63) holds, but (65) is unresolved. Clearly such worlds may exist. Consider a world \(w\), in
Proof: Let $M$ be a structure and $w$ a possible world.

1. Let $w \in \llbracket \varphi \lor \psi \rrbracket_{M,w}$. Since both $\varphi$ and $\psi$ are interrogative, by the truth-table for $\lor$, one of $\llbracket \varphi \rrbracket_{M,w}$ or $\llbracket \psi \rrbracket_{M,w}$ must be $T$. Thus, $w \in \llbracket \varphi \rrbracket_{M,w}$ or $w \in \llbracket \psi \rrbracket_{M,w}$ and therefore, $w \in \llbracket \varphi \lor \psi \rrbracket_{M,w}$. The other direction is similar.

2. Similar by following the usual definition of the conjunction $\varphi \land \psi = \neg (\neg \varphi \lor \neg \psi)$. ■

The following requirements of empirical adequacy are also satisfied. A conjunction entails both its conjuncts, a disjunction is entailed by both of its disjuncts and a conjunction entails the corresponding disjunction. Thus, this analysis satisfies all the requirements while extensions retain type $t$.

Proposition 8 Let $\varphi, \psi$ be indicative $ILQ^+$ formulae.

1. $\varphi \land \psi \models_B \varphi$ and $\varphi \land \psi \models_B \psi$
2. $\psi \models_B \varphi \lor \psi$ and $\varphi \models_B \varphi \lor \psi$
3. $\varphi \land \psi \models_B \varphi \lor \psi$.

Proof: Immediate from Proposition 7. ■

8 Combining indicative and interrogative formulae

The bilattice interpretation of $ILQ$ allows us to analyze sentences, composed of combined indicative and interrogative sentences. As mentioned by [18], such sentences have received very little attention in the literature. Perhaps the most natural examples of such combinations occurs with implication between an indicative and an interrogative as in the following sentences:

(54) If Mary kissed John then who kissed Bill?

(55) If $x$ is equal to 5 then what is $x + 1$?

Biblical examples are found in Amos III 3-5 (The King James Version):

"Can two walk together, except they be agreed? Will a lion roar in the forest, when he hath no prey? will a young lion cry out of his den, if he have taken nothing? Can a bird fall in a snare upon the earth, where no gin is for him? shall [one] take up a snare from the earth and have taken nothing at all?"

Similar examples can be found in Proverbs VI 27-28:

"Can a man take fire in his bosom, and his clothes not be burned? Can one go upon hot coals, and his feet not be burned?"

32
the answer to the question is positive and a block in which it is negative. The intension of each conjunct interrogative formula in $M$, in a possible world $w$ is the block in the partition to which $w$ belongs. Thus, each of $\textbf{[} \varphi \textbf{]}_{M,w,e}^{GS}$ and $\textbf{[} \psi \textbf{]}_{M,w,e}^{GS}$ is a set of possible worlds. We can take the intersection of the intensions of the conjuncts as the intension of the conjunction:

$$\textbf{[} \varphi \land \psi \textbf{]}_{M,w,e}^{GS} = \textbf{[} \varphi \textbf{]}_{M,w,e}^{GS} \cap \textbf{[} \psi \textbf{]}_{M,w,e}^{GS}.$$  

This intersection is the set of possible worlds in which the answer to each of the individual questions is the same as in $w$. Thus, the conjunction of the two questions induces a partition on the set of possible worlds as well. In each block of the partition, the pair of answers to the two questions is the same. This is no longer a binary partition, but rather a partition into (a maximum of) four blocks. While this works for conjunction, an analogous analysis for disjunction fails. Defining the intension of the disjunction as the union of the intensions of the disjuncts divides the set of possible worlds into blocks (the unions of the blocks of the original binary partitions), but these blocks are, in general, overlapping, and do not yield a partition. A possible world may simultaneously belong to more than one block. This is undesirable since it gives wrong predictions with respect to answerhood and interrogative entailment, for example, a proposition may formally answer such a question, without distinguishing between different blocks, i.e., be included in more than one block.

$$\textbf{[} \varphi \lor \psi \textbf{]}_{M,w,e}^{GS} = \textbf{[} \varphi \textbf{]}_{M,w,e}^{GS} \cup \textbf{[} \psi \textbf{]}_{M,w,e}^{GS}.$$  

A similar problem occurs with constituent questions, where conjunction may be defined as intersection, but defining disjunction as union of the intensions fails. This problem is solved in [15] by introducing a type-shifting operation, shifting the type of the intension of an interrogative from type $(s,t)$ to type $((s,t),t)$. This type-shifting operation is similar to the lifting of the meaning of an NP from type $e$ to type $((e,t),t)$. However, the resulting type in this case is the formidable type $((s,t),t)$. Note that this type is used in a fixed possible world. In a structure $M$, the type will be $(s,((s,t),t))$. Instead of joining the intensions of the disjunct questions, which are blocks of possible worlds, one first lifts each disjunct into the set of properties of the block and then takes the union of these sets of properties. The result is the set of properties shared by both blocks of possible worlds. Each such property is a set of sets of possible worlds. The result is the set of sets of possible worlds in which the block is a member. While this solves the problem, it does so at a price of immensely complicating the analysis.

### 7.2 Our account

The bilattice approach to the meanings of interrogative ILQ formulae yields a straightforward account of coordination, by viewing these formulae as having type $t$, without the need for any type-shifting. According to the definition of valuations in Section 6, the valuation of a conjunction/disjunction is defined using the corresponding bilattice operation on the knowledge dimension, not the truth dimension. As shown by the following proposition, this account satisfies the formal adequacy requirement.

**Proposition 7**

1. $\textbf{[} \varphi \lor \psi \textbf{]}_{M,e} = \textbf{[} \varphi \textbf{]}_{M,e} \cup \textbf{[} \psi \textbf{]}_{M,e}$
2. $\textbf{[} \varphi \land \psi \textbf{]}_{M,e} = \textbf{[} \varphi \textbf{]}_{M,e} \cap \textbf{[} \psi \textbf{]}_{M,e}$.
Let us also assume that $A \models_{M,w_1} B G$, and $A \models_{M,w_2} -B G$, meaning that in $w_1$, $A$ implies that the yogurt is good and in $w_2$, $A$ implies that the yogurt is not good. Now assume $A$ is true in both $w_1$, $w_2$. Clearly, $G \models_{M,w_1} [?] Y G]$, and $-G \models_{M,w_2} [?] Y G]$. Therefore, $A \models_{M,[?] Y G}]$. Thus, $A$ resolves the question according to the bilattice interpretation, even though it gives a positive answer in $w_1$ and a negative one in $w_2$. In both possible worlds the question is settled.

According to the G&S interpretation, they are not related by the answerhood relation, since the question partitions the set of possible worlds into (singleton) blocks: $[?] Y G]]^{G&S}_{M,w_1} = \{w_1\}$ and $[?] Y G]]^{G&S}_{M,w_2} = \{w_2\}$. Since $A$ is true in both possible worlds, its intension contains both of them. It is therefore not wholly included in either of the blocks.

This counter-example implies a genuine difference in the results which the two approaches yield. We prefer to view this question and answer pair as related by answerhood. Note that such indirect answers are possible on both interpretations. Thus, in both interpretations we may answer the question without actually saying whether the yogurt is good or not. It is only by virtue of the G&S extension, i.e. the truth value of $Y G$ that we can find out the intuitively straight answer. However, as shown by G&S, this notion of answer, which is so acceptable intuitively does not satisfy the adequacy requirements. A similar asymmetric result may be shown for interrogative entailment as well.

## 7 Coordination

In this section, we turn our attention to coordination of questions, as in (47)–(48) from [15].

(47) Does John walk? And/Or does Mary walk?

(48) Who walks? And/Or who talks?

In these examples, there is a conjunction or disjunction operator having wide scope over two interrogative sentences.\(^\text{15}\)

### 7.1 The G&S account

Since interrogatives are assigned type $(s,t)$ according to the G&S interpretation, they cannot be conjoined using regular conjunction/disjunction, which operates on type $t$ elements. However, it is possible to use a generalized conjunction/disjunction operation along the lines of [27]. This is the motivation behind the formal adequacy requirement of interpreting conjunction as set intersection and disjunction as set union. As shown by [15], this works for conjunction, but not for disjunction. For example, consider coordination of binary choice interrogative formulae as in (51).

(51) $[?] [\varphi] \lor [?] [\psi]$.

According to the G&S interpretation of IIQ, a binary choice interrogative induces a binary partition on the set of possible worlds of a structure $M$, separating them into a block in which

\(^{15}\) The reverse scope relations yield different questions as in (47)–(48). Since the treatment of these questions is rather straightforward, we will not discuss them here.

(49) Does John walk and/or Mary walk?

(50) Who walks and/or talks?
Since the semantic notions are defined relative to structures, any attempt at comparison must first relate the different structures used by the two interpretations. This is not trivial, because the domains for type \( t \) are different in the two kinds of structures. The G&S interpretation uses two values, whereas the bilattice interpretation uses five. Of course, we could view the G&S valuations as valuations into \( \text{FIVE} \), which happen to assign only the two classical truth values. However, such valuations are omniscient, rendering all questions immediately resolved. We will therefore impose the following requirement for relating the two kinds of structures. A pair of \( \text{IIQ}^- \) and \( \text{IIQ}^+ \) structures will be called \emph{compatible} if they agree on all indicative entailments:

**Definition 22** Let \( M^- \), \( M^+ \) be an \( \text{IIQ}^- \) structure and an \( \text{IIQ}^+ \) structure, respectively. \( M^- \) and \( M^+ \) are compatible iff for any indicative type \( t \) \( \text{IIQ}^- \) formulae \( \psi, \varphi : \psi \models^G S \varphi \iff \psi \models^B_{M^+} \varphi \).

We can now relate answerhood relative to compatible structures. It turns out that answerhood according to the G&S interpretation entails answerhood according to the bilattice interpretation, but not the other way around.

**Proposition 6** Let \( \psi \) be an indicative \( \text{IIQ}^- \) formula and \( \varphi \) an interrogative \( \text{IIQ}^- \) formula. And let \( M^- \), \( M^+ \) be a pair of compatible \( \text{IIQ}^- \) and \( \text{IIQ}^+ \) structures. Then, if \( \psi \models^G S \varphi \) then \( \psi \models^B_{M^+} \varphi \).

**Proof:** Let \( \psi \models^G S \varphi \). There are two cases:

1. If \( \varphi \) is a binary choice interrogative formula, then \( \varphi \equiv \exists \varphi' \), where \( \varphi' \) is indicative. By definition, there is a possible world \( w \) in \( M^- \) such that \[ \models^G S_{M^-, w, \sigma} \subseteq \models^G S_{M^-, w, \sigma} \]. Without loss of generality, assume \[ \models^G S_{M^-, w, \sigma} = t \]. Then, \( \psi \models^G S \varphi \). Since \( M^- \), \( M^+ \) are compatible, \( \psi \models^B_{M^+} \varphi \). Since \( \varphi \equiv \exists \varphi' \), we get \( \psi \models^B_{M^+} \varphi \).

2. If \( \varphi \) is a constituent interrogative formula, then \( \varphi \equiv \exists x \varphi' \), where \( \varphi' \) is indicative. By definition, there is a possible world \( w \) in \( M^- \) such that \[ \models^G S_{M^-, w, \sigma} \subseteq \models^G S_{M^-, w, \sigma} \]. Assume \[ S = \models^G S_{M^-, w, \sigma} \]. Thus, for any tuple \( d \), if \( d \in S \), then \( \psi \models^G S \varphi' [d/x] \) and if \( d \notin S \), \( \psi \models^G S \neg \varphi' [d/x] \). Since \( M^- \), \( M^+ \) are compatible, we get the same entailments for \( M^+ \), meaning that in any possible world \( w \) in which \( v_w(\varphi) = t \), \( \exists x \varphi' \) is resolved. 

The other direction does not hold i.e. there are compatible \( \text{IIQ}^- \) and \( \text{IIQ}^+ \) structures \( M^- \), \( M^+ \), such that in \( M^+ \) \( \psi, \varphi \) are related according to the answerhood relation (of the bilattice interpretation), but in \( M^- \) they are not (according to the G&S interpretation). An example that illustrates this is the following question and answer pair.

(45) Q: Is this yogurt good?
(46) A: Its expiration date is the 20th of July.

Let us denote by \( YG \) the proposition that the yogurt is good. The meaning of the question is represented by \( \exists y G(y) \), and the meaning of the answer by \( A \) (with the obvious anaphora resolution of \( ts \)). Let \( M^- \), \( M^+ \) be compatible \( \text{IIQ}^- \) and \( \text{IIQ}^+ \) structures, both of which contain just two possible worlds, \( w_1 \), in which the date is before the 20th of July, and \( w_2 \) in which it is well after
2. Intuitively, if one knows all the possible values of \( x \) that make \( \varphi \) true, then one also knows if there is such a value.

3. This is intuitively correct, since knowing all the values of \( x \) that make \( \varphi \) true entails knowing whether \( a \) is such a value.

4. This is to be expected since the order of the variables should not be important.

5. Our interpretation gives the intuitively correct result here. Knowing the values of \( x \) which satisfy the conjunction does not entail knowing the values of \( x \) which satisfy just one of the conjuncts. Many of the previous approaches fail here. See [17] for details.

6. We have thus fulfilled the promise from the remarks following Definition 1 of showing these two formulae to be equivalent.

7. We thus allow applying the interrogative binary operator ‘?\( x \)’ to formulae which do not contain a free occurrence of \( x \) and get a sensible result. In particular, \( ?x?x[\varphi] \equiv^B?x[\varphi] \)

8. Another debt from the remarks following Definition 1.

9. Yet another debt.

10. This is true for any combination of indicative and interrogative \( \psi, \varphi \). In particular, taking both \( \varphi, \psi \) to be interrogative, it means that if \( \varphi \) is never resolved, then it entails any other question.\(^{14}\)

6.5 Comparison with G&S

After presenting both the G&S and the bilattice interpretations of IIQ, it is natural to ask what is the relationship between the two. One major difference lies in the relation between questions and their answers. As we have seen, the G&S interpretation may be viewed in a sense as including the information content of answers within the meaning of the question. This information is retained within the partition. Our approach, on the other hand leads to a clear separation between questions and their answers. Questions are assigned a truth value as are indicative sentence. Questions and their possible answers are related via the answerhood relation. It is therefore worthwhile to compare the semantic notions of answerhood and entailment of the G&S and bilattice interpretations.

Such a comparison leads to a technical difficulty, since the two interpretations apply to different variants of IIQ. However, it is easy to verify that every IIQ\(^-\) formula is also in IIQ\(^+\), but is assigned a different type. We can compare the two approaches by focusing on IIQ\(^-\) formulae by concurrently interpreting them according to the G&S interpretation, using their original IIQ\(^-\) type assignment, and according to the bilattice interpretation, using the IIQ\(^+\) type assignment.

\(^{14}\)A literary example exhibiting this phenomenon is the following excerpt from Atlas Shrugged by Ayn Rand:

“We’re not taking any chances,” said the engineer. “Whoever’s responsible for it, he’ll switch the blame to us if we move. So we’re not moving till somebody tells us to.” “And if nobody does?” “Somebody will turn up sooner or later.” “How long do you propose to wait?” The engineer shrugged. “Who is John Galt?” “He means,” said the fireman, “don’t ask questions nobody can answer.”
9. ??[?x[?]] \equiv_b ?x[?], ??[?x[?]] \equiv_b ?x[?],

10. If \( \models_{\mathcal{M},a} \psi \), then \( \phi \models_{\mathcal{M}} \psi \) for any \( \phi \).

Proof:

1. Negation on indicatives switches truth/falsehood, but not knowledge. Thus for any valuation \( v \), \( v(?[?]) = T \) iff \( v(?) \in \{f, t\} \) iff \( v(\neg \phi) \in \{f, t\} \) iff \( v(?[\neg \phi]) = T \). Similarly for \( ?[\neg \phi] \) and \( ?[\phi] \).

2. To see that \( \models_b ??x[?x(x)] \), assume by way of contradiction that this is false in some structure \( M \) and possible world \( w \). Then, \( v_w(?[?x[?x(x)]]) = \bot \) and \( v_w(?x[?x(x)]) = T \). In this case, \( v_w(\exists x[?x(x)]) \in \{ \bot, u \} \). Therefore, for all \( d \in D \), \( v(\varphi(d)) \in \{ \bot, u \} \). Then, by definition, \( v(?x[?x(x)]) = \bot \). This is a contradiction.

3. \( ?x[?x(x)] \models_b ??x[?x(a)] \). Assume the opposite, then for some \( w \), \( v_w(?) \in \{ \bot, u \} \) and \( v_w(?x[?x(x)]) = T \). But then by definition, for all \( d \in D \), \( v_w(?) \in \{ f, t, T \} \). In particular, for \( a \). This is a contradiction.

4. \( ?x(?y[?x(x)]) \models_b ??y[?x(x, y)] \). Assume otherwise, then without loss of generality, we can assume that for some structure \( M \) and possible world \( w \), \( v_w(?) \in \{ \bot, u \} \) and \( v_w(?x[?x(x, y)]) = \bot \). But since \( v_w(?) \in \{ \bot, u \} \), contradiction.

5. \( ?x[?x(x)] \models_b ??x[?x(x)] \).

As a counter example, let \( M \) be a structure and \( w \) a possible world, where \( v_w(?) = f \) for all \( d \in D \) and \( v_w(?) = u \). In \( w, v_w(?x[?x(x)]) = T \) and yet \( v_w(?x[?x(x)]) = \bot \).

6. \( \models_b ?[?x[?]] \) since \( ? \) does not affect the value of interrogative formulae.

7. To see that if \( x \) does not appear free in \( \phi \), then \( ?x[?x] \equiv_b ?[?x] \), let \( v \) be a valuation. By Definition 13, \( v \) assigns both formulae a truth value in \{ \bot, \top \}. Now, by definition, \( v(?x[?]) = \top \) if for some \( d \in D \), \( v(\varphi[d/x]) = \top \). If \( x \) does not appear free in \( \phi \), then \( \varphi[d/x] = \varphi \) for all \( d \in D \). Thus, \( v(?x[?]) = \top \) if \( v(\varphi) = \bot \). But since \( v(?[?]) = \bot \), we get that \( v(?x[?]) = v(?[?]) \).

8. This is similar to the previous case.

9. \( ?x[?x[?]] \equiv_b ?x[?x] \) since \( ? \) does not change the value of interrogative formulae. To see that \( ?x[?x[?]] \equiv_b ?x[?x] \), notice that \( v(?x[?x[?]]) = \bot \) if for some \( d \in D \), \( v(?x[?x[?]]) = \bot \) if \( v(\varphi[d/x]) = u \) if \( v(?x[?]) = \bot \).

10. This is immediate from the definition.

Comments:

1. This is as expected, since knowing the answer to a positively phrased question is the same as knowing the answer to a negatively phrased one.
3. If \( \varphi \) is an interrogative formula, such that \( \models_M \varphi \), then for any \( \psi \), \( \models_M \varphi \).

4. \( \forall x[\varphi(x) \rightarrow x = a], ?x[x = a] \models_M ?x[\varphi(x)] \).

Proof:

1. \( \models_B ?x[\varphi(x)] \) since no valuation satisfies \( \psi \).

2. To see that \( \models_B ?x[\varphi(x)] \), assume that \( v \) is a valuation and \( w \) is a possible world, s.t. in \( w \), \( v_w (\exists x[\varphi(x)]) = t \). In this case, \( v_w (\exists x[\varphi(x)]) = f \), which implies that for all \( d \in D \), \( v_w (\varphi(x)[d/x]) = f \), which in turn implies that \( v_w (?x[\varphi(x)]) = T \).

3. Immediate.

4. Let \( v \) be a valuation and \( w \) a possible world, such that \( v_w (\forall x[\varphi(x) \rightarrow x = a]) = t \) and \( v_w (?x[x = a]) = T \). Then for any value of \( x \), \( v_w (x = a) \) is known to be either \( t \) or \( f \). Hence, for any value of \( x \), \( v_w (\varphi(x)) \) is known to be either \( t \) or \( f \). Thus, \( v_w (?x[\varphi(x)]) = T \).

Comments:

1. One special case of this is if \( \psi \) is assigned the value \( u \) in each possible world, corresponding to the NL answer \textit{I do not know}. Similarly, if \( \psi \) is contradictory.

2. This corresponds to the NL answer \textit{no one} or \textit{nothing}.

3. This is the formal counterpart of \textit{rhetorical questions}, i.e. questions that are always resolved. Since rhetorical questions are not really asking anything, they may be answered by any sentence.

4. This is the justification behind seeing (43) as an answer to (40).

Proposition 5 (Interrogative entailment and equivalence)

1. If \( \varphi \) is indicative, then \( ?[\varphi] \equiv_B ?[\neg \varphi] \), \( \models_B ? x[\varphi] \equiv_B ? x[\neg \varphi] \)

2. \( ?x[\varphi(x)] \models_B ?x[\exists x[\varphi(x)]] \)

3. \( ?x[\varphi(x)] \models_B ?x[\varphi(a)] \)

4. \( ?x ?y[\varphi(x, y)] \equiv_B ?y ?x[\varphi(x, y)] \)

5. \( ?x[\varphi(x) \land \psi(x)] \equiv_B ?x[\varphi(x)] \)

6. \( ?[\varphi] \equiv_B ?[\varphi] \)

7. Let \( \varphi \) be indicative. If \( x \) does not appear free in \( \varphi \), then \( ?x[\varphi] \equiv_B ?[\varphi] \)

8. Let \( \varphi \) be interrogative. If \( x \) does not appear free in \( \varphi \), then \( ?x[\varphi] \equiv_B \varphi \)
(41) Mary and Sue.
(42) \texttt{K\textsc{iss}_4(M\textsc{ary},JO\textsc{hn})\&K\textsc{iss}_4(S\textsc{ue},JO\textsc{hn})}.

In addition, answers are required to be exhaustive. An answer to a question such as (40) must not only specify the full list of domain elements who kissed John, but also the full list of those who did not do so. By pragmatic considerations, it is quite natural to interpret (41) as being an abbreviation for (43). Thus, our final ILQ\textsuperscript{+} representation of (41) is (44).

(43) Only Mary and Sue kissed John.
(44) \(\forall x[\texttt{K\textsc{iss}_4(x,JO\textsc{hn})}\iff x = M\textsc{ary}\lor x = S\textsc{ue}]\).

### 6.4.2 Formal adequacy

Formal adequacy requires that entailment be reduced to set inclusion, and that equivalence be reduced to equivalence. This is shown by the following simple corollary of Definition 20. Our definitions also satisfy the formal adequacy requirement with respect to coordination as shown in Section 7.

**Proposition 3** Let \(\varphi, \psi\) be type t ILQ\textsuperscript{+} formulae, and let \(M\) be a structure.

\begin{itemize}
  \item \(\models_M^B \varphi\) iff for any assignment \(\sigma\), \([\varphi]_{M,\sigma} \subseteq [\psi]_{M,\sigma}\)
  \item \(\equiv_M^B \varphi\) iff for any assignment \(\sigma\), \([\varphi]_{M,\sigma} = [\psi]_{M,\sigma}\)
\end{itemize}

### 6.4.3 Generalized entailment adequacy

According to Definition 20 the relations of indicative entailment, interrogative entailment and answerhood are all seen as instances of a single generalized entailment relation, thus satisfying the generalized entailment adequacy criterion.

### 6.4.4 Empirical adequacy

Checking material and formal adequacy was straightforward. Checking empirical adequacy requires checking many different examples (most are from [17]). In the following propositions, let \(\varphi, \psi\) be type t ILQ\textsuperscript{+} formulae.

**Proposition 4 (Answerhood)**

1. \(\psi \models_M^B ?x[\varphi(x)]\) where \(\psi\) is an indicative formula such that \([\psi]_{M,\sigma} = \emptyset\).
2. \(\neg \exists x[\varphi(x)] \models_M^B ?x[\varphi(x)]\), where \(\varphi\) is indicative.
Since in our system, indicative formulae may be assigned \( u \), signifying the value “unknown”, \( \psi \lor \neg \psi \) is not a tautology. However, in any structure and possible world where \( ?[\psi] \) is satisfied, the disjunction is satisfied as well. For a real-world scenario, assume a contest in which John participates, and consider a structure \( M \) with two possible worlds \( w_1, w_2 \), representing two time points, one before the contest, and the other well after the contest has taken place and the results published. Let \( \psi \) be the proposition that John won the contest. Since in \( w_1 \), the contest hasn’t yet taken place, it is natural to assume that \( v_{w_1}(\psi) = \bot \), therefore the disjunction is not satisfied. However in \( w_2 \), \( ?[\psi] \) is settled, making the disjunction true as well.

### 6.4 Adequacy

We now show that the bilattice interpretation of \( \text{ILQ} \) satisfies the adequacy criteria of Section 4, including: material, formal, generalized entailment and empirical adequacy.

#### 6.4.1 Material adequacy

The bilattice interpretation satisfies the material adequacy requirement by virtue of Definition 20, by which the required semantic notions are defined. The answerhood relation corresponds to the intuitive notion of being a possible answer to a question. By Definition 20, \( \psi! \) is an answer to \( \varphi? \) in a structure \( M \), if for every possible world \( w \in W \) in which \( \psi! \) is true, \( \varphi? \) is settled.

For binary-choice interrogative formulae, of the form \( ?\varphi \), where \( \varphi \) is indicative, both \( \varphi \) and \( \neg \varphi \) are possible answers. For a constituent interrogative \( ?x[\varphi] \), an answer is any proposition that exhaustively specifies which set of possible values of \( x \) satisfy \( \varphi \), and which values do not. Entailment between interrogatives corresponds to the intuitive requirement. \( \psi? \) entails \( \varphi? \) in \( M \), iff for each \( w \in W \), iff \( \psi? \) is settled in \( w \), \( \varphi? \) is also settled in \( w \). Equivalence is defined as mutual entailment.

It is important to note that these semantic notions are defined as relations between \( \text{ILQ}^+ \) formulae, rather than NL sentences. It is possible to define induced relations on NL sentences by the corresponding relations between the \( \text{ILQ}^+ \) representations. For instance, an indicative sentence answers a question if their \( \text{ILQ}^+ \) representations stand in the answerhood relation. Since the answerhood relation is defined in terms of type \( t \) formulae, categorial answers such as (41), are viewed as elliptical. Thus, as a first step, we may represent the meaning of (41) as (42).

(40) **Who kissed John?**
1. $\varphi$ is satisfied in $M$, $w$ relative to $\sigma$, denoted $\models_{M,w,\sigma}^B \varphi$, iff $w \in \{ \varphi \}_{M,\sigma}$.

2. $\Sigma$ is satisfied in $M$, $w$ relative to $\sigma$, denoted $\models_{M,w,\sigma}^B \Sigma$, iff for all $\psi \in \Sigma$, $\models_{M,w,\sigma}^B \psi$.

This definition applies both to indicative and to interrogative formulae alike (as well as to other type $t$ formulae).\(^\text{13}\) It is easy to see that a formula $\varphi$ is satisfied in a possible world $w$ iff it is assigned a designated value there. This is because $\varphi$ is satisfied in $w$ iff $w$ is in the intension of $\varphi$, the latter being the set of possible worlds in which $\varphi$ is assigned a designated value. This means that indicative and interrogative formulae are satisfied in different ways. An indicative formula is satisfied in a possible world if it is assigned the value $t$ there. An indicative formula is satisfied by being assigned the value $\top$.

**Definition 19 (Validity)**

1. $\varphi$ is valid in $M$, written $\models_M^B \varphi$, iff for all $w \in W$ and assignments $\sigma$, $\models_{M,w,\sigma}^B \varphi$.

2. $\varphi$ is valid, written $\models^B \varphi$, iff for any structure $M$, $\models_M^B \varphi$.

**Definition 20 (Consequence)**

1. $\varphi$ is a consequence of $\Sigma$ (equivalently, $\Sigma$ logically entails $\varphi$) in a structure $M$, denoted by $\Sigma \models_M^B \varphi$, if for all $w \in W$, and assignments $\sigma$, if $\models_{M,w,\sigma}^B \Sigma$, then $\models_{M,w,\sigma}^B \varphi$.

2. $\Sigma \models^B \varphi$ if for any structure $M$, $\Sigma \models_M^B \varphi$.

**Definition 21 (Equivalence)** $\psi, \varphi$ are equivalent in $M$, denoted $\psi \equiv_M^B \varphi$ iff $\{ \psi \} \models_M^B \varphi$ and $\{ \varphi \} \models_M^B \psi$.

Since indicative and interrogative formulae are satisfied by being assigned different designated values, we get several different kinds of entailment relations. We now wish to view these different relations as the semantic relations required by the material adequacy criterion of Section 4. Let $\varphi \models^B \psi$. We get the relations in Figure 4.

Of the four resulting relations, indicative entailment is familiar from classical logic. Interrogative entailment and answerhood are discussed in Section 4. If $\varphi$ and $\psi$ are related by the answerhood relation, then we say $\varphi$ answers $\psi$. Examples of these relations are given in Section 6.4.4. The final relation of mixed entailment is not required by the formal adequacy criterion, but is a natural by-product of the generalized consequence relation. An example of such reasoning is the following entailment, where $\psi$ is a type $t$ indicative formula.

\[ (39) \ ?[\psi] \models^B \psi \lor \neg \psi. \]
known (by the semi-oracle) to be either false or true. In both cases, the intuitive understanding is that since the semi-oracle knows whether \( \varphi \) is true or false, the question corresponding to \( ?[\varphi] \) is resolved in its view. The question remains unresolved, i.e. \( \nu(?[\varphi]) = \perp \), in case the semi-oracle doesn't know whether \( \varphi \) is true or false. A constituent interrogative formula, \( ?x[\varphi] \), is resolved, i.e. assigned \( \top \), iff the semi-oracle knows the truth value of \( \varphi[d/x] \) for any possible value \( d \) for \( x \), regardless of whether this value is true or false.

In assigning the value \( \top \) to an interrogative formula, we only retain the information that the formula is settled, not how it is settled. This is a major point of difference from G&S, who in some sense retain the actual answer, i.e. either the actual truth value or the set of domain elements that satisfy the corresponding indicative formula. However, as shown in the appendix, the extensional part of this approach fails to satisfy the adequacy criteria, leading G&S to adopt a more complex intensional interpretation. Our approach does not run into these problems, and manages to remain extensional while satisfying the adequacy criteria as shown below. We define a formal answerhood relation, which connects questions with their possible answers.

Definition 13 keeps a clear separation between the way in which the connectives and quantifiers are interpreted for indicative and interrogative formulae. For indicative formulae, the operators act as in strong Kleene three-valued logic. For interrogative formulae, they operate on the knowledge dimension and not on the truth dimension. Thus, the disjunction of two interrogative formulae yields a resolved interrogative iff at least one of the disjuncts is resolved. Similarly, existential quantification over an interrogative \( \exists x[\varphi] \) yields a resolved formula just in case \( \varphi \) is resolved for all possible values of \( x \).

A valuation that assigns closed atomic formulae just the truth values \( f, t \) but not \( u \) will be called an omniscient valuation. For such valuations all questions are resolved.

**Definition 17 (Omniscient valuation)** An omniscient valuation, denoted by \( \Omega \), assigns any closed atomic (indicative) formula of type \( t \) a value in \( \{f, t\} \).

**Proposition 2** Let \( \Omega \) be an omniscient valuation.

1. If \( \psi \) is an indicative ILQ\(^+\) formula, then \( \Omega(\psi) \in \{f, t\} \).
2. If \( \varphi \) is an interrogative ILQ\(^+\) formula, then \( \Omega(\varphi) = \top \).

### 6.3 Semantic notions

We can now define the notions of satisfiability, validity and consequence. These notions are important for the formal adequacy criterion. Corresponding notions are defined for bilattices in [1, 2]. In the following definition, let \( M \) be a structure, \( w \) a possible world and \( \sigma \) an assignment function. Let \( \varphi, \psi \) be type \( t \) ILQ\(^+\) formula, and let \( \Sigma \) be a set of such formulae.

**Definition 18 (Satisfaction)**
3. If \( x \) is of type \( a \), then \( v_w(\exists x[\psi]) = \lor_{d \in D_a} v_w(\psi[d/x]) \)

4. \( v_w(\exists x[\psi]) = \exists(\psi) \)

5. If \( x \) is of type \( a \), then \( v_w(\exists x[\psi]) = \begin{cases} \bot & \text{if for some } d \in D_a : v_w(\psi[d/x]) \in \{\bot, a\} \\ T & \text{otherwise.} \end{cases} \)

6. If \( \alpha \) is of type \( (a, t) \) and \( \beta \) is of type \( a \), then \( v_w(\alpha(\beta)) = [\alpha]_{M,w,\sigma}([\beta]_{M,w,\sigma}) \).

**Definition 14** If \( v_w \) is a valuation relative to a possible world \( w \), \( \varphi \) is an open formula of type \( t \), and \( \sigma \) is an assignment function, then \( v_w(\varphi)_\sigma \) is a valuation relative to the assignment \( \sigma \). If \( x_1, x_2, \ldots, x_n \) are the free variables in \( \varphi \), then \( v_w(\varphi)_\sigma = v_w(\varphi[\sigma(x_i)/x_i, i = 1, \ldots, n]) \).

In addition to the extension of an ILQ\(^+\) expression \( \alpha \), we also define its intension relative to a structure \( M \) and an assignment function \( \sigma \), denoted \( [\alpha]_{M,\sigma} \):\(^{12} \)

**Definition 15** (Intension of ILQ\(^+\) expressions) If \( \alpha \) is an expression of type \( a \), then \( [\alpha]_{M,\sigma} \) is that function \( h \in D_a^{\mathcal{W}} \) such that for all \( w \in \mathcal{W} : h(w) = [\alpha]_{M,w,\sigma} \).

In MG, since the intension of type \( t \) formulae is a characteristic function from a set of possible worlds to the values \( \{f, t\} \), it is common to speak of the intension of such formulae as a set of possible worlds, i.e., the set of worlds in which the extension of the formula is the truth value \( t \). It is convenient to retain this set-theoretic view of intensions. Since our valuations give more than two values, we cannot directly translate characteristic functions into sets. We can however do so by defining the intension of a type \( t \) formula as the set of possible worlds in which its extension is a designated truth value. We thus define, by abuse of notation:

**Definition 16** (Intension of type \( t \) ILQ\(^+\) formulae) If \( \varphi \) is a type \( t \) formula, then its intension in a structure \( M \), relative to an assignment function \( \sigma \), denoted \( [\varphi]_{M,\sigma} \) is \( \{w \in \mathcal{W} | [\alpha]_{M,w,\sigma} \in D\} \).

The following proposition is an immediate corollary of Definition 13:

**Proposition 1** Let \( \psi! \) be an indicative ILQ\(^+\) formula, and let \( \varphi? \) be an interrogative one. Then, for any valuation \( v \), \( v(\psi!) \in \{f, u, t\} \) and \( v(\varphi?) \in \{\bot, T\} \).

The two possible values of \( \bot, T \), when assigned to an interrogative formula, reflect the intuitive notion of a question being unresolved or resolved. For instance, let \( \varphi \) be a closed indicative binary choice type \( t \) formula. The formula \( ?[\varphi] \) is resolved, i.e., \( v(?[\varphi]) = T \), just in case \( \varphi \) is

\(^{12}\)Even though we introduce intensions our account remains extensional in the sense that as in MG, extensions are computed relative to a possible world, and intensions are functions from possible worlds to extensions. In contrast, G&S's account uses intensions even in a fixed possible world.
1. $D_e = D$
2. $D_e = \mathcal{W}$
3. $D_t = \textsc{FIVE}$
4. $D_{(a,b)} = D_b D_a$.

We now define the extension of an ILQ$^+$ expression $\alpha$, relative to a structure $M$, a possible world $w$ and an assignment function $\sigma$, denoted $[\cdot \alpha]_{M,w,\sigma}$.

**Definition 11 (extension of ILQ$^+$ expressions)**

1. If $\alpha$ is a constant (of type $a$), then $[\cdot \alpha]_{M,w,\sigma} = I(\alpha)(w)$.
2. If $\alpha$ is a variable (of type $a$), then $[\cdot \alpha]_{M,w,\sigma} = \sigma(\alpha)$.
3. If $\alpha$ is of type $(a,b)$ and $\beta$ is of type $a$, then $[\cdot \alpha(\beta)]_{M,w,\sigma} = [\cdot \alpha]_{M,w,\sigma} (\cdot \beta)_{M,w,\sigma}$
4. If $\alpha$ is of type $a$ and $u$ is a variable of type $b$, then $[\cdot \lambda u[\alpha]_{M,w,\sigma}$ is that function $h \in D_{(b,a)}$ such that for all $d \in D_b : h(d) = [\cdot \alpha]_{M,w,\sigma[d/u]}$
5. If $\alpha$ is of type $t$, then $[\cdot \alpha]_{M,w,\sigma} = v_w(\alpha)_t$, where $v_w(\alpha)_t$ is defined in Definition 14.

Valuations of type $t$ formulae are defined for atomic formulae in Definition 12. Valuations uniquely determine maps from the set of closed formulae of type $t$ to $\textsc{FIVE}$ according to Definition 13. We use the notation $\varphi[d/x]$ to denote the formula $\varphi$ after replacement of every free occurrence of $x$ by $d$.

**Definition 12** A valuation $v_w$ relative to a possible world $w$ in a structure $M$ is a mapping from the set of ground atomic (indicative) formulae of type $t$ to $\{f, u, t\}$.

The intuition behind this definition is that a valuation to an atomic formula reflects the amount of truth/knowledge assigned by the semi-oracle to the proposition denoted by it. The semi-oracle may know the proposition to be true, know it to be false, or not know whether it is true or false, in which case it is assigned the value $u$.

**Definition 13** A valuation $v_w$ relative to a possible world $w$ in a structure $M$ determines a unique map, also denoted $v_w$, on the set of all closed formulae of type $t$ to $\textsc{FIVE}$ according to the following conditions. Let $\psi, \varphi$ be closed type $t$ ILQ$^+$ formulae.

1. $v_w(\varphi \lor \psi) = v_w(\varphi) \lor v_w(\psi)$
2. $v_w(\neg \psi) = \neg v_w(\psi)$
Definition 9 A structure $M$ for $ILQ^+$ is a tuple $(D, W, V, I)$, where:

1. $D$ is a nonempty domain set.
2. $W$ is a set of possible worlds.
3. $V$ is a function, which for every $w \in W$ yields a valuation $v_w$ from the set of type $t$ $ILQ^+$ formulae to $FIVE$, according to Definition 13 below.
4. $I$ is an interpretation function, which assigns each $ILQ^+$ constant of type $a$ and world $w \in W$ a domain element of type $a$.

We assume all possible worlds are mutually accessible. The same domain $D$ is shared by all these worlds. The domains of interpretation for all types of $ILQ^+$ elements are given by the following definition:

Definition 10
the interpretation of IIQ. The motivation behind these truth tables is that the connectives are interpreted differently for indicative truth values than for interrogative truth values. Negation operates like ‘¬’ on the indicative truth values and like ‘¬’ on the interrogative truth values. Applying ‘?’ to any truth value yields the value T ("resolved") when applied to either the indicative values t, f. The motivation here is that if an indicative formula is known by the semi-oracle to be either true or false, then the corresponding binary choice formula is considered resolved. Otherwise, it is unresolved. Applying the interrogative operator to interrogative formulae does not change their truth value. Finally, disjunction, being a binary operator, may operate either on purely indicative values, where it operates like ‘∪’, or on purely interrogative values, where it operates like ‘∪’ or on a combination of indicative and interrogative values. For the latter case, disjunction yields an interrogative value: T if either of the values is a designated one and ⊥ otherwise. We shall see the application of these operators in the sequel.

The IIQ operators ‘∧’, ‘→’ are defined in Definition 1 in the usual manner using ‘∨’, ‘¬’. Like disjunction, they may operate either on a pair of indicative truth values, or on a pair of interrogative values, or on a combination of both. One may verify that conjunction operates like ‘Π’ on a pair of indicative values and like ‘□’ on interrogative values. On a combination of an indicative and an interrogative value, conjunction yields an interrogative value: T if both values are designated, ⊥ otherwise. The truth tables of these operations are also given in Figure 3.

At this point we wish to remark on our choice of the bilattice FIVE for the interpretation of IIQ. This choice is not obvious, since there is a simpler bilattice, called FOUR, which is the simplest bilattice. FOUR is similar to FIVE, but without the value u. Initially, we began by considering FOUR for the interpretation of IIQ. However, we have come to the conclusion that the analysis is much more elegant using five truth values. This is because two interrogative values are needed for accounting for the difference between resolved and unresolved interrogatives. Three indicative values are needed, as two (t, f) are standard and the third (u) is necessary to account for propositions that are unknown. The value u is “responsible” for interrogative formulae being unresolved in the following sense: given an indicative formula φ, [φ] is unresolved just in case φ is assigned the value u. Without the value u, all indicative formulae are either true or false, hence all interrogative formulae are resolved. Furthermore, it is essential that the indicative and interrogative values be kept separate. This is because the operators ‘?’; ‘¬’; ‘∨’ are defined on the truth values themselves. It is convenient to have them operate differently on indicative and interrogative values. For example, as shown below, it is possible to have disjunction operate in the standard way for indicative disjuncts, while simultaneously giving the desired results both for a disjunction of interrogatives, and for a combination of an indicative and an interrogative. Otherwise, three different disjunction operators would have to be defined, and used according to syntactic factors. This greatly complicates the analysis. In order to be able to define disjunction as a single model-theoretic operation (not only as an IIQ syntactic operation), it is essential that the truth values of indicatives be separate from those of interrogatives. For this reason we use five distinct truth values.

### 6.2 Structures for interpreting IIQ

We now define structures for interpreting IIQ⁺.
(b) — is a lattice homomorphism from \((B, \bowtie, \oplus, \perp, \top)\) to \((B, \oplus, \bowtie, \top, \perp)\) and from \((B, \sqcap, \sqcup, f, t)\) to itself.

\[
\begin{array}{c}
\top \\
f \\
\downarrow \\
\downarrow \\
\downarrow \\
\downarrow \\
u \\
\downarrow \\
t \\
\downarrow \\
\perp \\
t
\end{array}
\]

Figure 2: The bilattice \textbf{FIVE}

\textbf{FIVE}, shown in Figure 2, contains five truth values. Intuitively, these five values can be divided into two groups: three indicative truth values: \(f\) (false), \(u\) (unknown) and \(t\) (true)\(^\text{11}\), and two interrogative truth values: \(\perp\) (unresolved) and \(\top\) (resolved). The motivation behind these truth values is that the semi-oracle may assign an indicative formula one of the indicative truth values, and an interrogative formula one of the interrogative truth values. Several operations are defined for bilattices. ‘\(\bowtie\)’ and ‘\(\oplus\)’ are respectively the meet and join of the knowledge dimension. ‘\(\sqcap\)’ and ‘\(\sqcup\)’ are the meet and join on the truth dimension. The operator ‘\(\sim\)’ switches truth/falsehood but does not affect the amount of knowledge. Conflation switches knowledge but does not affect the amount of truth.

The meet and join operations induce two partial ordering relations on bilattices, \(\leq_t\), to be thought of as a partial order with respect to the amount of truth assigned to a formula, and \(\leq_k\), to be thought of as a partial order with respect to the amount of knowledge assigned to it.

The induced partial orders on \textbf{FIVE} are the following:

- \(\leq_t\) - represented in Figure 2 as the horizontal arrow. \(f \leq_t \perp, u, \top \leq_t t\)
- \(\leq_k\) - represented in Figure 2 as the vertical arrow. \(\perp \leq_k u \leq_k f, t \leq_k \top\).

Note that the knowledge dimension plays a double role here. For the indicative values, \(u\) represents less knowledge than both \(f\) and \(t\). For the interrogative truth values, \(\perp\) represents less knowledge than \(\top\). In classical logic, the value \(t\) has a special status with regard to semantic notions of validity and consequence. In bilattices, it is common to choose a set \(D\) of designated truth values (see [1]). We follow standard practice in choosing the set of designated values of \textbf{FIVE} as \(D = \{t, \top\}\).

For the interpretation of the IIQ\(^+\) connectives, we will not use the standard operators of bilattices, but instead define three new operations: ‘\(\sim\)’, ‘\(\tilde{\top}\)’ and ‘\(\tilde{\perp}\)’, which correspond to the respective IIQ syntactic connectives. We give the full truth tables for these operations in \textbf{FIVE} in Figure 3. We omit the truth tables for the original bilattice connectives as we will not use them in

\(^{11}\) Note the typographical difference between the truth value \(t\) and the type \(t\).
In contrast to classical bivalent logic, which may be seen as having a single dimension of truth/falsehood, bilattices allow viewing systems of multiple truth values as being ordered along two different dimensions, which may be thought of as the dimension of truth and the dimension of knowledge. The main motivation behind using bilattices for the interpretation of interrogatives is that the knowledge dimension, embedded intrinsically within them appears at the core of the intuitive notion of answerability conditions. Just as truth conditions of an indicative sentence restrict the possible state of affairs, to those states in which the sentence is true, so do we want answerability conditions to restrict those states of affairs in which an interrogative sentence is resolved (cf. [11]). It is natural to analyze resolvedness in terms of knowledge. The resulting interpretation satisfies the adequacy requirements of Section 4.

Using knowledge as an explicit part of the model raises a philosophical concern. Whereas truth/falsehood may in some sense be considered an objective measure, knowledge requires an agent, someone who knows. Our formal model will not contain such an explicit agent. Rather, we use a valuation function, which assigns ILQ formulae one of five values, similar to valuations used in classical two-valued logic. Whereas a valuation in classical logic may be thought of as an oracle, which gives the correct truth value to each proposition, in our system, we think of this valuation as a “semi-oracle”, having only partial knowledge. The partiality of knowledge is needed, since in a state of full knowledge, all questions are resolved. This valuation/semi-oracle will assign indicative formulae either the value true/false as in the classical case, or a third value representing “unknown”. For interrogative formulae, the valuation will give either the value “resolved”, representing the fact that the semi-oracle knows the answer to the question, or “unresolved”, representing the fact that the semi-oracle does not know it. The meaning of an interrogative formula is the set of conditions (i.e. possible worlds) in which the semi-oracle does indeed know the answer to the question.

6.1 Bilattices - basic definitions

We first give the basic definitions of bilattices, focusing on the bilattice FIVE. We then define structures for the interpretation of ILQ\(^*\), and semantic notions of answerability and interrogative entailment for it.

Definition 8 (Bilattice) A bilattice with conflation is a structure 
\((B, \sqcap, \sqcup, \otimes, \oplus, \neg, \sim, f, t, \bot, \top)\), such that:

1. \((B, \sqcap, \sqcup, f, t)\) and \((B, \otimes, \oplus, \bot, \top)\) are both complete lattices with \(f\) and \(t\) being bottom and top elements of the first and \(\bot\) and \(\top\) of the second.

2. \(\sim: B \rightarrow B\) is a mapping with:
   
   (a) For any \(b \in B\), \(\sim(\sim b) = b\), and
   
   (b) \(\sim\) is a lattice homomorphism from \((B, \sqcap, \sqcup, f, t)\) to \((B, \sqcup, \sqcap, t, f)\) and from \((B, \otimes, \oplus, \bot, \top)\) to itself.

3. \(-: B \rightarrow B\) is a mapping (called conflation) with:

   (a) For any \(b \in B\), \(-(-b) = b\), and
in any possible world. Note that according to this definition, a contradiction is an answer to any interrogative formula. Formally:

**Definition 5 (ILQ^- Answerhood)** If $\psi$ is of type $t$, and $\varphi$ is of type $(s,t)$, then: 
$\psi \models^G_M \varphi$ iff there is a possible world $w \in W$ such that $\models^G_M \psi \subseteq \models^G_M \varphi$.

Entailment of interrogative formulae is defined as follows: $\psi$ entails $\varphi$ in a structure $M$ iff for all possible worlds $w$ of $M$, the intension of $\psi$ in $w$ is included in the intension of $\varphi$ in $w$. Note the difference between this definition and the regular definition of indicative entailment, due to the difference in basic types. In indicative entailment, the intension of one formula should be included in the intension of the other. Here, this condition should hold for each possible world. Intuitively, an interrogative formula entails another iff every block in the partition induced by the first is included in a block of the partition induced by the second one. Formally:

**Definition 6 (ILQ^- Entailment)** If $\psi, \varphi$ are of type $(s,t)$, then: 
$\psi \models^G_M \varphi$ iff for all assignments $\sigma$, $\forall w \in W : \models^G_M \psi \subseteq \models^G_M \varphi$.

Equivalence between interrogative formulae is defined as equivalence of the intensions for each possible world $w$, i.e. entailment in both directions.

**Definition 7 (ILQ^- Equivalence)** $\psi \equiv^G_M \varphi$ iff for all assignments $\sigma$, 
$\forall w \in W : \models^G_M \psi = \models^G_M \varphi$.

For empirical examples of these relations, see [17]. While the G&S interpretation of ILQ satisfies the adequacy requirements, it does so at the price of a high level of complexity. The main source of this complexity is viewing the meanings of questions as partitions, causing an asymmetry in the types assigned to indicative and interrogative formulae. This will have grave consequences for the treatment of more complex NL questions, as shown in subsequent sections. These considerations led us to search for a simpler extensional interpretation of interrogative formulae, which satisfies the adequacy criteria of Section 4, while retaining the symmetry in type assignment between indicative and interrogative formulae. We present such an interpretation in the following section.

### 6 Bilattice interpretation of ILQ

In this section we present a new extensional interpretation of ILQ^+, based on bilattices [7, 9, 10]. Bilattices provide a general algebraic framework for multiple-valued logic. We will focus on a bilattice called **FIVE**. This interpretation runs against the argument of [17] against an extensional interpretation of interrogatives. Interestingly, [17] mention that their argument holds even in three-valued logic systems. However, we show that in a system of five values, it is possible to give an extensional interpretation. In principle, the approach presented here may also be applied to other bilattices which have additional truth values, allowing finer distinctions.
Definition 4 (ILQ⁻ Intensional interpretation) Let \( \varphi \in IL \) be of type \( t \).

1. \( \llbracket ?[\varphi] \rrbracket_{M,w,\sigma}^G = \{ w' \in W \mid \llbracket ?[\varphi] \rrbracket_{M,w',\sigma}^G \} \)

2. \( \llbracket ?[\varphi] \rrbracket_{M,w,\sigma}^G = \{ w' \in W \mid \llbracket ?[\varphi] \rrbracket_{M,w',\sigma}^G \} \).

The motivation behind this definition is the following. In a structure, the set of possible worlds \( W \) represents a set of alternative epistemic possibilities. A hypothetical agent, having only partial knowledge of the state of affairs in the real world may entertain several different alternatives, each represented as a possible world. An interrogative formula partitions \( W \) into equivalence classes, blocks of possible worlds. Two possible worlds are equivalent (belong to the same block) iff the extension of the interrogative formula is the same in both, i.e., if the question is answered in the same way in both.

To illustrate this interpretation consider the following NL questions and their ILQ representations. First, consider a yes/no question such as (32), represented in ILQ as (33). The extension and intension of this formula are given in (34) and (35).

(32) Did Mary kiss John?

(33) \( \llbracket s\text{-Did Mary kiss John?} \rrbracket = \llbracket ?[\text{KISS}_s(\text{MARY, JOHN})] \rrbracket \)

(34) \( \llbracket ?[\text{KISS}_s(\text{MARY, JOHN})] \rrbracket_{M,w,\sigma}^G = \llbracket ?[\text{KISS}_s(\text{MARY, JOHN})] \rrbracket_{M,w}^G \)

(35) \( \llbracket ?[\text{KISS}_s(\text{MARY, JOHN})] \rrbracket_{M,w,\sigma}^G = \{ w' \in W \mid \llbracket ?[\text{KISS}_s(\text{MARY, JOHN})] \rrbracket_{M,w',\sigma}^G \} = \llbracket ?[\text{KISS}_s(\text{MARY, JOHN})] \rrbracket_{M,w}^G \).

The extension is the truth value of the corresponding indicative formula. The intension in a world \( w \) is the set of possible worlds \( w' \) in which the formula \( \text{KISS}_s(\text{MARY, JOHN}) \) has the same truth value as in \( w \). Interrogative binary choice formulae divide \( W \) into two equivalence classes, one in which the truth value of the corresponding indicative formula is true, and one in which it is false. As an example of a constituent question, consider (36). Its extension and intension are presented in (37) and (38).

(36) \( \llbracket s\text{-Who kissed John?} \rrbracket = \llbracket ?x[\text{KISS}_s(x, \text{JOHN})] \rrbracket \)

(37) \( \llbracket ?x[\text{KISS}_s(x, \text{JOHN})] \rrbracket_{M,w,\sigma}^G = \{ \sigma'(x) \mid \sigma' \approx x \sigma \text{ and } M, w, \sigma' \models \text{KISS}_s(x, \text{JOHN}) \} \).

(38) \( \llbracket ?x[\text{KISS}_s(x, \text{JOHN})] \rrbracket_{M,w,\sigma}^G = \{ w' \in W \mid \llbracket ?x[\text{KISS}_s(x, \text{JOHN})] \rrbracket_{M,w',\sigma}^G \} \).

The extension relative to a structure \( M \), world \( w \) and assignment function \( \sigma \) is the set of individuals \( d \) in the domain, that are such that \( \text{KISS}_s(d, \text{JOHN}) \) holds in \( M, w \). The intension is the set of possible worlds \( w' \), such that the extension of the formula is the same in \( w' \) and \( w \), i.e. the set of possible worlds \( w' \) in which the same people kissed John as in \( w \). It thus partitions \( W \) into equivalence classes, according to who kissed John in those worlds. These equivalence classes are the sets of possible worlds in which nobody kissed John, those in which only Mary kissed John, those in which only Mary and Sue kissed John etc.

The following notions of answerhood, entailment and equivalence are defined by [17], and shown to satisfy the adequacy criteria. An indicative formula \( \psi! \) of type \( t \) is an answer to an interrogative formula \( \varphi? \) of type \( (s, t) \), iff its intension (a set of possible worlds) is completely included in one of the equivalence classes of the partition induced by the interrogative formula.
expressions and for the extension and in tension of ILQ formulae, but with different subscripts. For example, not be conjoined or quantified over using the regular operations of conjunction or quantification. As we shall see, this difference makes the treatment of complex questions much simpler on our account than on the G&S account.

Looking at the construction methods of Section 3, we see that the expressive power of II.Q\(^{"-}\) is in fact sufficient for the translation procedure from simple NL questions to II.Q. The crucial property of II.Q\(^{"-}\) that allows this is that it allows \(\lambda\)-abstraction over interrogative formulae (not allowed in [17]). \(\lambda\)-abstraction is necessary e.g. in the meaning of which, \(\lambda P\lambda Q[? x [P(x) \land Q(x)]]\).

Structures for II.Q are the same as structures used for II in MG. A structure is a tuple \((D, W, V, I)\) where \(D\) is a non-empty set, its domain (from which non-empty interpretation domains for each type may be constructed), \(W\) is a non-empty set of possible worlds, all mutually accessible, \(V\) is a function, which for any possible world \(w\) yields a valuation of type \(t\) formulae into \{false, true\} and \(I\) is an interpretation function. The domain is common to all possible worlds. The interpretation of indicative II.Q formulae is the usual II interpretation. In a given structure and possible world the extension of an indicative formula \(\psi\) of type \(t\), written \([\psi]_{M,w,\sigma}^G\) is a truth value.\(^{10}\) Its extension in a structure \(M\), written \([\psi]_{M,\sigma}^G\) is the subset of possible worlds in which its extension is the truth value true. For interrogative formulae, the picture is more complex. G&S claim that we cannot interpret interrogatives extensionally. Even in a fixed possible world, we have to look at their intentions. This is not an accidental property of G&S's analysis. In fact they give an argument in [17], summarized in the appendix, according to which, no adequate extensional interpretation of questions is possible. In Section 6, we show that in spite of this argument, it is possible to give an adequate extensional interpretation of interrogatives by using truth-values in a non-classical way. The extension of an interrogative formula \(\varphi\), written \([\varphi]_{M,w,\sigma}^G\) is computed in two stages, by going through its extension, \([\varphi]_{M,\sigma}^G\).

The extension of a binary choice interrogative formula \(?[\varphi]\) is just the truth value of \(\varphi\). The extension of a constituent interrogative formula of the form \(?x[\varphi]\) is just the set of possible domain element tuples, to which \(x\) may be assigned, as given in Definition 2. We use the notation \(\sigma' \approx_x \sigma\) to signify that assignments \(\sigma, \sigma'\) differ at most with respect to the value they assign to the variable \(x\). Similarly, we write \(\sigma' \approx_x \sigma\) to signify that assignments \(\sigma, \sigma'\) differ at most with respect to the values they assign to the variable tuple \(x\).

**Definition 3 (II.Q\(^{-}\) Extensional interpretation)** Let \(\varphi \in II\) be of type \(t\).

1. \([?[\varphi]]_{M,w,\sigma}^G = [\varphi]_{M,w,\sigma}^G = \begin{cases} \text{true} & \text{if } M, w, \sigma \models \varphi \\ \text{false} & \text{otherwise} \end{cases}\)

2. \([?[x]\varphi]_{M,w,\sigma}^G = \{ (\sigma'(x_1), \sigma'(x_2), \ldots, \sigma'(x_n)) \mid \sigma' \approx_x \sigma \text{ and } M, w, \sigma' \models \varphi \}.\)

The extension of an II.Q\(^{-}\) interrogative formula in a given possible world, \(w\), is the set of possible worlds in which the extension of the formula is the same as in \(w\). It is thus a type \((s,t)\) entity.

\(^{10}\)Note that we use the square semantic brackets in two different roles, both for denoting the meaning of NL expressions and for the extension and intension of II.Q formulae, but with different subscripts.
un-intuitive, since in everyday parlance, we do not speak of such entailment. However, it is exactly the relation used in the common scientific endeavor of reducing one question to another. For instance, in Computer Science, in the study of computational complexity, one frequently reduces one decision problem to another. Once such an entailment relation is defined, equivalence between questions may be defined as mutual entailment. Any system which satisfies formal adequacy may be used to formalize linguistic predictions regarding answerhood, entailment and equivalence. Given two sentences, we can formally verify whether they stand in one of these relations. As shown in [17], many of the theories in the literature, including both sets of proposition theories and categorial theories fail to satisfy these adequacy criteria. It is only the theory of G&S themselves which does. We therefore consider an interpretation of a variant of IIQ according to G&S, in the next section.

5 G&S interpretation of IIQ

In a series of papers [12, 14, 16, 17], G&S present an account of the semantics of NL questions. This account is based on a formulation of the meanings of questions in a logical language, similar in several aspects to IIQ, but less expressive. In this section, we show how a G&S style interpretation may be assigned to a variant of IIQ, called IIQ−, defined as follows:

**Definition 2 (IIQ− Syntax)** The set \( W_{IIQ^-} \) of IIQ− formulae is the smallest set of formulae satisfying the following conditions:

1. If \( \varphi \) is an IL formula, then \( \varphi \in IIQ^- \) and is of the same type as in IL.
2. If \( \varphi \in IIQ^- \) is of type \( t \), then \( ?[\varphi] \in IIQ^- \) and is of type \( (s,t) \).
3. If \( \varphi \in IIQ^- \) is of type \( t \), and \( x_1, x_2, \ldots, x_n \) are variables of any types, then
   
   \[ ?x_1, x_2, \ldots, x_n[\varphi] \in IIQ^- \] is of type \( (s,t) \).
   
   For \( \bar{x} = (x_1, x_2, \ldots, x_n) \), we use the abbreviation \( ?\bar{x}[\varphi] \).
4. If \( \varphi \in IIQ^- \) is of type \( b \), and \( x \) is a variable of type \( a \), then \( \lambda x[\varphi] \in IIQ^- \) and is of type \( (a,b) \).
5. If \( \varphi \in IIQ^- \) is of type \( (a,b) \) and \( \psi \) is of type \( a \), then \( \varphi(\psi) \in IIQ^- \) and is of type \( b \).

We abbreviate \( ?x_1, x_2, \ldots, x_n[\varphi] \) to \( ?\bar{x}[\varphi] \). Note that whereas in IIQ+, the binding interrogative operator is applied to variables one at a time, IIQ− uses a single binding interrogative operator, binding several variables en bloc. The reason for this difference is that in IIQ−, the binding interrogative operator is of type \( (t,(s,t)) \). When applied to a formula of type \( t \), it yields an interrogative formula of type \( (s,t) \). It cannot therefore be applied to interrogative formulae, which are of type \( (s,t) \). In IIQ+, however, the binding interrogative operator is of type \( (t,t) \), accepting a type \( t \) (indicative or interrogative) formula and yielding a type \( t \) interrogative formula.

Another important consequence of the difference in the types assigned to interrogative formulae in the two variants of IIQ is that since interrogative IIQ+ formulae are of type \( (s,t) \), they may
After deriving ILQ representations of NL questions, we turn to the model-theoretic interpretation of these formulae. At this point, we should ask what would constitute an adequate such interpretation. Adequacy criteria for such an interpretation are discussed in the next section.

4 Adequacy criteria for the interpretation of ILQ

In considering the issue of what constitutes an adequate semantics of NL questions, [17] present some adequacy criteria which any such semantics should satisfy. These criteria impose constraints on the model-theoretic interpretation of NL questions. Since in this paper we have split the task of assigning a semantics to NL questions into two parts, translation of NL to ILQ and model-theoretic interpretation of ILQ, these criteria constrain the second part.

- **Material adequacy**: A model-theoretic interpretation of questions should specify notions of answerhood, entailment and equivalence between the meanings of NL sentences, both indicative and interrogative:
  - *Answerhood* is a relation between the meanings of an indicative and an interrogative sentence. Adequate semantics of NL questions should define such a relation as a formal counterpart of the intuitive notion of an indicative sentence being a possible (though not necessarily true) answer to the question.
  - *Entailment* between the meanings of questions should be defined according to the following criterion: the meaning of one question should entail the meaning of another if whenever the first is resolved, so is the second.
  - *Equivalence* between the meanings of two questions should be defined such that the meanings of two questions are equivalent iff they entail each other.

- **Formal adequacy**: Equivalence of meaning should reduce to identity of the meanings of questions. Entailment should reduce to ‘inclusion’ on the semantic values. Similarly, conjunction and disjunction of interrogatives should be defined standardly in terms of set intersection and set union.

- **Generalized entailment adequacy**: Both answerhood and entailment between questions should be instances of a generalized entailment relation, defined uniformly for both indicative and interrogative formulae.

- **Empirical adequacy**: Several “test cases” are given by G&S, where the semantic notions of equivalence, entailment, and answerhood are checked to see if they correspond to native speaker intuitions.

At first sight, the requirement of material adequacy may appear objectionable. While answerhood and equivalence appear to be useful notions, entailment between questions is rather

---

8While [17] does not give this criterion, it is implicitly contained in their account, as they use the same notation (|=r) for both question entailment and answerhood.

9Empirical adequacy is not mentioned explicitly as an adequacy criterion in [17], but it is clear from the presentation that it is a desirable property.
This formula denotes a set of propositions \( p \), such that there is a value of \( x \) such that \( x \) is a woman and \( p \) is a true proposition denoting that \( x \) kissed John. These propositions are the true answers to the question. This construction rule for questions is indeed reminiscent of MG’s rule of “quantifying in”. However, unlike the original rule for quantified NPs, the meaning of the INP is not applied directly to the meaning of the gappy sentence. Rather, it is applied within the scope of the \( \lambda \)-abstraction over propositions \( p \). This complication leads to problems once multiple constituent questions are considered, as shown in Section 9.

Instead of this rule, we use a simpler one, directly applying the meaning of the INP to that of the gappy sentence as in (28) (this time representing which woman using the binding interrogative operator instead of the existential quantifier).

\[
(27) \quad [s_{\text{Which woman kissed John?}}]
= \\
\lambda p [\lambda Q [\exists x (\text{WOMAN}(x) \land Q(x))] (\lambda u [\text{TRUE}(p) \land p = \lambda (\text{KISS}_u(u, \text{JOHN}))])]
= \\
\lambda Q [\exists x (\text{WOMAN}(x) \land Q(x))] (\lambda u [\text{TRUE}(p) \land p = \lambda (\text{KISS}_u(u, \text{JOHN}))]).
\]

The result from indirect construction is the same as that from direct construction. This is to be expected since the sentence is unambiguous. This result is considerably simpler than (27) of [22]. Notice once more, that even though the meaning of the question is derived by manipulating ILQ formulae, this is only a convenience. We would get the same results if we were to apply the model-theoretic interpretation of the INP, which is a type \((e,t, t)\) function to the meaning of the gappy sentence. Here, as in MG, a gappy sentence leads a double life. It starts off as an open formula, which relative to an assignment, denotes a type \( t \) object. However, before operating on it, we apply \( \lambda \)-abstraction over the free variable \( a \), transforming it into a type \((e, t)\) function, regardless of any assignments. Whereas it is convenient to do so in a formal language, the same manipulation could in principle be performed on the model-theoretic entities themselves. Applying the meaning of the INP to the meaning of the gappy sentence, after abstraction over \( a \), yields a type \( t \) element. For further discussion, see [21].

We now come to the meaning of who, which is given in (29). This meaning can be seen as derived by applying the meaning of which to a predicate which is true of any individual\(^6\) \( \lambda y [\text{TRUE}]\).\(^7\) The relation between who and which \( N \) is thus the same as that between someone and some \( N \).

\[
(29) \quad [s_{\text{who}}] = \lambda Q [\forall x (Q(x) \land \lambda y [\text{TRUE}](x))] = \lambda Q [\forall x (Q(x))].
\]

The meaning of (11) may be derived by either direct construction as in (30) or by indirect construction as in (31), yielding the same result.

\[
(30) \quad [s_{\text{Who kissed John?}}] = [s_{\text{who}}] [\lambda x (\text{KISS}_x (x, \text{JOHN}))]
= \\
\lambda Q [\forall x (Q(x)) (\lambda y [\text{KISS}_y(y, \text{JOHN})))] (\lambda u [\text{KISS}_u(u, \text{JOHN}))] = \lambda Q [\forall x (Q(x)] (\lambda u [\text{KISS}_u(u, \text{JOHN}))] = \lambda Q [\forall x (Q(x)] (\lambda u [\text{KISS}_u(u, \text{JOHN}))] = \lambda Q [\forall x (Q(x)] (\lambda u [\text{KISS}_u(u, \text{JOHN}))] = \lambda Q [\forall x (Q(x)] (\lambda u [\text{KISS}_u(u, \text{JOHN}))] = \lambda Q [\forall x (Q(x)] (\lambda u [\text{KISS}_u(u, \text{JOHN}))].
\]

\(^6\)A finer analysis could impose a further condition that \( y \) denotes a person, interpreting who as which person. In a sense, we interpret who as an abbreviation for which entity.

\(^7\)The ILQ constant \text{TRUE} is assigned the truth value \( t \) both in the G&S interpretation and in our novel bilattice interpretation.
3.2.2 Direct construction

In the direct construction method, the meaning of an INP is directly applied to the meaning of a VP, in the same way as the meaning of an ordinary NP is applied to the meaning of a VP in MG. An example of this process is shown in (22). In this derivation, the meaning of the VP is given as in MG, by type raising John to $\lambda P[P(\text{John})]$, and functionally applying meaning of the verb kiss to it.

(22) $[s.]\text{Which woman kissed John} = [\text{wh-NpWhich woman}][[vP\text{kissed John}] =
\lambda P[?x[\text{WOMAN}(x) \wedge Q(x)]](\lambda y[\text{KISS}_x(y, \text{JOHN})]) = ?x[\text{WOMAN}(x) \wedge \text{KISS}_x(x, \text{JOHN})].$

Through functional application of the meaning of the INP to the meaning of the VP, we arrive at the meaning of the complete interrogative sentence. The resulting formula in (22) may informally be understood as asking for values of $x$ such that $x$ is a woman and $x$ kissed John. Note that here too, even though we use ILQ as an intermediate representation, we may derive the meaning of the question directly, in model-theoretic terms. We apply a function of type $[e, t]$ (the meaning of the INP) to an element of type $t$ (the meaning of the VP), and get a type $t$ element (the meaning of the question).

3.2.3 Indirect construction

A second method of deriving the meaning representations of constituent questions corresponds to the indirect construction of quantified NPs, via “quantifying in”. This construction method is similar to the single construction method given in [22], only simpler. We present both methods simultaneously, exemplifying them by deriving once again the meaning of (16). The first step in applying this method is deriving the meaning of a gappy sentence such as (23).

(23) She kissed John.

The meaning of this sentence is presented in (25) by applying the meaning of the pronoun to the meaning of the VP kissed John. In (24), $u$ is some fresh variable.

(24) $[\text{NPHe/She/It}] = \lambda P[P(u)].$

We assign the following interrogative formula to the “gappy-S”. Note the free occurrence of ‘$u$’ in the result:

(25) $[s.-\text{gappShe kissed John}] = [\text{NPShe}][[vP\text{kissed John}] =
\lambda P[P(u)](\lambda y[\text{KISS}_x(y, \text{JOHN})]) = \lambda y[\text{KISS}_x(y, \text{JOHN})](u) = \text{KISS}_x(u, \text{JOHN}).$

We construct the meaning of the question (16) by combining the meanings of the INP which woman, represented in (20), with the meaning of the gappy sentence (23), represented in (25). Recall that (25) has a free variable $u$. In constructing the meaning of the full question, we use $\lambda$-abstraction over $u$. It is in the method of combining the meanings of the INP and the gappy sentence that the analysis of [22] and ours part ways. In [22], they are combined by a complex rule, summarized in [23] as:

(26) $Q' = \lambda P[\text{INP}(\lambda u[\text{true}(p) \wedge p = ^\wedge (S')])]$.

For example, for (16), we get (27) (representing the meaning of which woman in the same way as a woman).

---

5By this novelty convention we avoid the tedious pedantic pronoun-numbering he$_n$ as done originally by Montague.
3.2.1 Interrogative NPs

In order to give the meaning representation \([\text{wh-}NP\text{who}]\) of who, we first represent the meaning of NPs of the form which N, e.g. which woman as in (16).

(16) Which woman kissed John?

We construct the meaning of INPs similarly to the construction of the meanings of noun phrases as *generalized quantifiers*. The interrogative determiner which will have a meaning representation of the same type as determiners of regular generalized quantifiers, i.e. (second order) binary relations between properties of individuals. This is a useful property of our analysis, since interrogative determiners have similar distribution as indicative determiners and share some of their semantic properties. The meaning representation \([\text{wh-}\text{Det}\text{which}]\) of which will have the same form as that of a, except that instead of using the existential quantifier, we use ILQ's binding interrogative operator, as shown in (17)–(18).

\[
(17) [\text{Det}a] = \lambda P \lambda Q [a(P, Q)] = \lambda P \lambda Q [\exists x (P(x) \land Q(x))]
\]

\[
(18) [\text{wh-}\text{Det}\text{which}] = \lambda P \lambda Q [\text{which}(P, Q)] = \lambda P \lambda Q [\exists x (P(x) \land Q(x))],
\]

Just like (17), so does (18) denote a Curried two-place function. Given \(P\) and \(Q\), both of type \((\text{e}, \text{t})\), it yields an element of type \(\text{t}\). Informally, this function may be read as one which given \(P\) and \(Q\), “asks” for the values of \(x\) that satisfy both \(P\) and \(Q\). Note that in the bilattice interpretation, this conjunction operator is not interpreted as standard boolean conjunction, but rather as a special operation defined below. The meaning of an INP such as which woman is represented in (20). It is derived by applying the meaning of the interrogative determiner which to the meaning of the noun woman, given in (19).\(^3\)

\[
(19) [\text{N}\text{woman}] = \lambda y [\text{WOMAN}(y)]
\]

\[
(20) [\text{wh-}\text{NP}\text{which woman}] = [\text{wh-}\text{Det}\text{which}][[\text{N}\text{woman}]] = \\
\lambda P \lambda Q [\text{which}(P, Q)] [\lambda y [\text{WOMAN}(y)]] = \lambda P \lambda Q [\exists x (P(x) \land Q(x))][\lambda y [\text{WOMAN}(y)]] = \\
\lambda Q [\exists x [\text{WOMAN}(x) \land Q(x)]].
\]

We may informally understand (20) as a function, which given a (property) \(Q\) of type \((\text{e}, \text{t})\) “asks” for values of \(x\) such that \(x\) is a woman and has the property \(Q\). This formula is of the same form and type as the meaning representation of a woman in MG, as given in (21), the only difference being the use of the binding interrogative operator instead of the existential quantifier.\(^4\) Thus, it has type \((\text{e}, \text{t}), \text{t}\). However, unlike [22], who interprets INPs precisely as existentially quantified NPs, using \(\exists\), here we use the interrogative operator. As will be shown in Section 6, the type \(\text{t}\) elements used in the interpretation of interrogative formulae will be non-classical, pertaining not to the amount of truth they contain, but to whether they are resolved or not.

\[
(21) [\text{N}P\text{a woman}] = [\text{Det}a][[\text{N}\text{woman}]] = \\
\lambda P \lambda Q [a(P, Q)][\lambda y [\text{WOMAN}(y)]] = \lambda P \lambda Q [\exists x (P(x) \land Q(x))][\lambda y [\text{WOMAN}(y)]] = \\
\lambda Q [\exists x [\text{WOMAN}(x) \land Q(x)]].
\]

\(^3\)We use the same notation in different font for both NL predicates and their IIQ correlates (instead of using an apostrophe). For instance, the IIQ representation of the noun woman is the IIQ type \((\text{e}, \text{t})\) constant woman.

\(^4\)Combining the interrogative determiner which with a singular \(N\) imposes a presupposition of uniqueness with respect to the possible answers. We follow several authors (e.g. [6]) in seeing this issue as pragmatic, and thus exclude it from the representation. For a discussion of uniqueness presuppositions imposed by questions, see [30].
3.1 Yes/no questions

In constructing the meaning representation of ‘Yes/No’ questions in ILQ, we begin with the meaning of prepended auxiliary verbs such as did/does/will. The meaning of these verbs is a \((t,t)\) function. Since at this stage in the presentation we have not yet presented our interpretation of type \(t\), we postpone the presentation of this function as well. This meaning is represented in ILQ using the interrogative operator as in (12).

\[\text{[wh-sux Did]} = \lambda \varphi [?\varphi].\]

This operator may be applied to the meaning of sentences. For example, given the indicative sentence in (13), represented in ILQ in (14) (as in MG), the corresponding interrogative sentence (10) is derived by applying (12) to (14) as in (15). The final formula may be intuitively be understood as “asking” whether \(\text{kiss}_*(\text{MARY, JOHN})\) holds.

\[(13) \quad \text{Mary kissed John}\]

\[(14) \quad [[s\text{Mary kissed John}] = \text{kiss}_*(\text{MARY, JOHN})]\]

\[(15) \quad [[s\text{Did Mary kiss John?}] = [\text{wh-sux Did}][[s\text{Mary kissed John}]] = \lambda \varphi [?\varphi](\text{kiss}_*(\text{MARY, JOHN})) = ?[\text{kiss}_*(\text{MARY, JOHN})].\]

The interpretation of such ILQ+ formulae depends on the fact that the domain of interpretation for type \(t\) is not the classical one. We will use a system with five truth-values, and assign such interrogative formulae non-classical truth-values. Note that ‘?’ should be interpreted here as an operator of type \((t,t)\). Its truth-table will be given in Section 6. Thus, as claimed above, the interpretation of binary-choice questions may be performed directly at the model-theoretic level. Applying the interrogative operator ‘?’ to a type \(t\) entity yields another type \(t\) entity. In this regard, this operator is interpreted similarly to negation. Note that semantically, this operator may be applied to any type \(t\) formula, corresponding either to an indicative or an interrogative sentence. Over-generation is avoided by syntactically restricting auxiliaries of this kind to apply only to indicative sentences.

3.2 Constituent questions

The meanings of NL constituent questions are derived based on the meanings of interrogative noun phrases (INPs). The latter are modeled after the meanings of NPs as generalized quantifiers. After assigning a meaning to INPs, we combine them with the meaning of the rest of the sentence in one of two ways: direct or indirect construction, modeled after the similar two construction methods for regular quantified NPs in MG. Allowing both construction methods allows the analysis of scope phenomena, as illustrated in Section 9. The direct method works by interpreting the INP in situ. The indirect method works by using a substitutional rule similar to that of MG’s substitutional rules of “quantifying in”. The indirect construction rule is similar to, but simpler than the (single) construction method used by [22]. We assign both ‘wh-elements’ such as who/what and INPs such as which woman the syntactic category wh-NP. We assign interrogative determiners such as which or what the syntactic category wh-Det.
We follow MG in assigning transitive verbs the complex type \(((\e, t), t, (e, t))\). This has the advantage of giving a uniform treatment of both proper names and quantified NPs in object position, as both are seen as denoting a property of properties of individuals. We use the predicate symbol \(kiss\) as one accepting two arguments, one of type \(e\) and one of type \(((e, t), t)\).

We use the following familiar notational convention: If \(\delta\) is an expression of type \(((e, t), t, (e, t))\), then \(\delta(x, y) = \text{def} \delta(x, \lambda P[P(y)])\).

There are several important points to notice with regard to Definition 1:

- Unlike G&S's system, Definition 1 allows interrogative operators to be embedded within formulae. For example, the following are well-formed:
  - \(\exists x?y[p(x, y)], \forall x?y[p(x, y)], \text{in addition to } ?y?x[p(x, y)], ?y?y[x[p(x, y)]]\).
  - \(??[\varphi]\). We will later see that \(??[\varphi] \equiv ??[\varphi]\).\(^2\)
  - \(??x[\varphi], ?x'[?x[\varphi]]\). We will later see that \(??x[\varphi] \equiv ?x[\varphi], ?x'[?x[\varphi]] \equiv ?x[\varphi]\).
  - \(??y[\varphi]\). We will later see that \(??y[\varphi] \equiv y[?x[\varphi]]\).
- Definition 1 allows formulae of the form \(??x[\varphi]\) even if \(x\) does not appear free in \(\varphi\). In particular, it allows formulae of the form \(??x[?x[\varphi]]\). We will later see that if \(x\) does not appear free in \(\varphi\), then \(??x[?x[\varphi]] \equiv ?[\varphi]\). Similarly, \(??x[?x[\varphi]] \equiv ?x[\varphi]\). In addition, if \(\varphi\) contains a free occurrence of \(x\), then \(??x[?x[\varphi]]\) depends on a value assignment to \(x\), as usual.

The main difference between II.Q\(^+\) and II.Q\(^-\), the latter being more closely related to the language used by G&S, is the type assignment to basic formulae constructed by the use of the interrogative operators. These are assigned type \(t\), whereas in II.Q\(^-\), they are assigned type \((s, t)\). As a result, II.Q\(^+\) allows regular operations which apply to type \(t\) entities, such as coordination or quantification to be applied to interrogative formulae as well as indicative formulae. Even though such formulae are assigned type \(t\), the domain of this type will not be the classical two-valued boolean algebra.

### 3 Translating NL into ILQ

We now present a compositional method for constructing II.Q interrogative formulae for representing meanings of English questions. This line of investigation has been followed by several researchers in the Montogovan tradition, most notably [22]. Here we give a different analysis than [22], the main difference being the way in which meanings are derived for NL questions. Our compositional derivation allows a better handling of scope ambiguities in questions with quantifiers and in view of the additional expressiveness of the meaning representation language II.Q\(^+\) is able to deal with a wider range of NL constructs. We consider two kinds of questions, yes/no questions such as (10) and constituent questions such as (11). We assign both kinds of questions the syntactic category ‘S?’.

(10) Did Mary kiss John?

(11) Who kissed John?

\(^2\)The equivalence relation will be defined after giving the interpretation of II.Q\(^+\) formulae.
4. If $\varphi \in ILQ^+$ is of some type $b$, and $x$ is a variable of some type $a$, then $\lambda x[\varphi] \in ILQ^+$ and is of type $(a, b)$.

5. If $\varphi \in ILQ^+$ is of type $(a, b)$ and $\psi$ is of type $a$, then $\varphi(\psi) \in ILQ^+$ and is of type $b$.

6. $\land, \rightarrow$ and $\forall$ are defined using $\neg, \lor$ and $\exists$.

The free variables of an ILQ formula are determined as for IL, with the only addition that the binding of an interrogative operator does indeed bind its variable. When appending quantifiers, $\lambda$ operators or interrogative operators, we freely omit the extra square brackets, writing e.g. $?x?y[\varphi]$ instead of $?!x[?y[\varphi]]$. We call type $t$ IL formulae indicative formulae. If $\varphi$ is an indicative formula, then a formula of the form $?\varphi$ is called a binary choice interrogative formula and a formula of the form $?x?y?$ is called an interrogative constituent formula. We will sometimes use the notation $\varphi_!$ to denote an arbitrary indicative formula and $\varphi_?$ to denote an arbitrary interrogative formula (either binary choice or constituent). This is not part of the formal syntax of ILQ, but a convenient notational device. We will retain the same distinctions and notation for ILQ as well.\(^1\)

At this point, we do not give the model-theoretic interpretation of ILQ. However, the systematic definition of the syntax required us to assign types to ILQ formulae, which restrict the range of possible model-theoretic interpretations. Even though ILQ formulae are assigned types, the domains of these types are not yet specified. These domains are not necessarily the same as those of IL. The reader should bear in mind that any ILQ formula will eventually come to denote an element of the domain of the appropriate type. Since ILQ shares IL's types, such elements will either be atomic elements of the proper domain or functions thereof. Until the semantics is given, we will not know how to read ILQ formulae. We therefore give a temporary informal interpretation of them to facilitate their reading. Indicative formulae may be intuitively read in the familiar way of IL. A formula of the form $?\varphi$, where $\varphi$ is an IL formula may be read (in anthropomorphenic terms) as “asking” whether $\varphi$ holds. A formula of the form $?x[\varphi]$ may be read as “asking” for values of $x$, such that $\varphi$ holds for $d$. We emphasize that this is only a rough intuitive understanding of the formulae. Their formal interpretation will be given later on. The following are some simple examples. More complicated examples of mixed interrogative and indicative sub-formulae are presented later on.

(5) $?[\text{HAPPY}(\text{JOHN})]$ is an interrogative binary choice formula of type $t$. Intuitively, it asks whether John is happy.

(6) $?x[\text{HAPPY}(x)]$ is an interrogative constituent formula. Intuitively, it asks for a specification of those elements which have the property of being happy.

(7) $?x[\exists y[\text{KISS}, (x, y)]]$ is an interrogative constituent formula, which intuitively asks for those $x$'s such that there is some $y$, which was kissed by $x$ (the '*' is explained below).

(8) $?y[\text{HAPPY}(x)]$ is a type $t$ formula, which intuitively asks “is everyone happy?”.

(9) $\forall x[?\text{HAPPY}(x)]$ is an interrogative binary choice formula, which intuitively asks for everyone, is he happy.

---

\(^1\)Note that the indicative and interrogative formulae do not exhaust the range of ILQ formulae. There are even type $t$ formulae which are neither indicative nor interrogative. For example, a variable $\varphi$ of type $t$ cannot be said to be either indicative or interrogative. Variables are considered well-formed formulae as in IL.
linguistic theories of the semantics of questions, and in particular both the “sets of propositions” and the “categorial” approaches fail to satisfy these criteria. In fact, of existing approaches, it is only the theory of G&S themselves, which succeeds in this task. We consider in Section 5 a model-theoretic interpretation of a variant of ILQ, called ILQ\(^{-}\), according to the theory of G&S. Unfortunately, the G&S theory is quite complex. In particular, it yields an intensional interpretation even for simple extensional questions. Unlike indicative sentences, the denotation of which is a type \(t\) element, the denotation of a question is an element of type \((s,t)\), where type \(s\) is the type of possible worlds. Thus, even in a fixed possible world, the meaning of a question is a set of possible worlds. According to [17], this intensionality is not accidental. In fact, they present an argument, which we summarize in the appendix, ruling out any adequate extensional semantics for questions. In spite of this argument, we present an alternative interpretation of ILQ\(^{+}\) in Section 6, based on bilattices. Not only is this account extensional, assigning questions denotations of type \(t\), it is also shown to be adequate by the criteria of Section 4. The assignment of type \(t\) to questions is made possible by our reinterpretation of the domain of type \(t\) as the bilattice FIVE instead of its usual interpretation over the (simplest) boolean algebra of the two truth-values \textit{true} and \textit{false}. We apply this novel interpretation to coordination in Section 7, combined indicatives and interrogatives in Section 8, quantifying into questions in Section 9 and functional questions in Section 10.

## 2 ILQ

We start our presentation of the semantics of NL questions by presenting IIQ, a logic for representing the meanings of NL indicative and interrogative sentences, as well as combined indicative and interrogative sentences. We present the syntax of a variant of IIQ, called IIQ\(^{+}\). We deliberately postpone giving the semantics of IIQ to Section 6. The relation of IIQ\(^{+}\) to the language used by G&S is discussed in Section 5. IIQ\(^{+}\) is an extension of MG’s II. It shares II’s basic types and the method for constructing compound types from them. IIQ enhances II by the addition of two new operators defined on type \(t\) formulae: the \textit{interrogative operator} ‘?’, used to represent the meanings of yes/no questions and the \textit{binding interrogative operator} ‘?\(\cdot\)’. Other than the addition of these two new operators, IIQ\(^{+}\) shares II’s vocabulary, including II’s connectives, quantifiers and operators, and for each type \(a\), an infinite set of constants \(CON_a\) and variables \(VAR_a\). We use the following variable names, possibly with numeric subscripts: \(x, y\) are variables of unspecified type. \(\alpha, \nu\), are of type \(e\). \(P, Q\) are of type \((e, t)\). In addition, IIQ\(^{+}\) (like II) contains the symbol for identity. IIQ\(^{+}\) syntax is defined as follows:

**Definition 1 (IIQ\(^{+}\) Syntax)** The set of IIQ\(^{+}\) formulae is the smallest set of formulae satisfying the following conditions:

1. If \(\varphi\) is an II formula, then \(\varphi\in IIQ^{+}\) and is of the same type as in II.

2. If \(\varphi, \psi \in IIQ^{+}\) are of type \(t\), then the following are also type \(t\) IIQ\(^{+}\) formulae:
   \[-\varphi, \varphi \lor \psi, \varphi, [\varphi].\]

3. If \(\varphi \in IIQ^{+}\) is of type \(t\), and \(x\) is a variable, then the following are also type \(t\) IIQ\(^{+}\) formulae: \(\exists x[\varphi], \varphi[x[\varphi]].\)
is based on a two-stage interpretation as shown in Figure 1. NL questions are first compositionally translated into a formal language, an extension of Montague’s IL called Intensional Logic with Questions (ILQ). The translation procedure is simpler and more straightforward than previous accounts. This formal language is then interpreted in model-theoretic terms. We present a model-theoretic interpretation of ILQ based on bilattices [7, 9, 10]. Bilattices are algebraic structures which contain two partial order relations. They provide a framework for multi-valued logic, incorporating reasoning with conflicting or partial information. Bilattices have been successfully applied to several domains, most notably to the semantics of logic programming. In Linguistics, they have been used by [25] to handle propositional attitudes and by [29] to account for pragmatic phenomena. Their application to the semantics of interrogatives is new. As in MG, the use of the formal language as an intermediate representation is a convenient device, adding to the perspicuity of the analysis. However, it is not a necessary part of it (see [5] for a discussion). We could just as well have given a direct model-theoretic interpretation of NL terms. We will stick to this terminological distinction, reserving the term meaning representation for the ILQ translation of NL terms, the term interpretation for the model-theoretic analysis of ILQ formulae, and semantics/meaning for the combination of the two.

![Figure 1: Semantic interpretation of NL questions](image)

In this paper, we discuss extensional, direct questions in a static setting, as we believe that these form the basic core elements a semantic theory of questions should deal with. We consider both so-called “yes/no” questions and constituent questions, that refer to verb arguments, but not additional adjuncts such as time, place etc. For a discussion of temporal questions see [26]. For example, we deal here with questions such as (1)–(2), but not (3)–(4). We believe that the account presented here may provide a sound basis for the analysis of intensional or embedded questions, as well as the dynamics of questions and answers in dialog.

1. Who(m) did Mary kiss?
2. Who kissed John?
3. What did Mary seek?

After presenting an analysis of these basic questions, we consider some more complex examples, showing how our novel account provides a particularly elegant solution to some outstanding problems in the semantics of questions, including coordination, quantifying into questions and functional questions. In addition, this interpretation may also be applied to combined indicative and interrogative sentences, which have received only limited attention in previous approaches.

The outline of the paper follows the translation and model-theoretic interpretation steps of the system. In Section 2 we present the syntax of a variant of ILQ, called ILQ⁺. In Section 3, we present a compositional method for translating NL questions to ILQ. We then consider the model-theoretic interpretation of ILQ. In Section 4 we review a set of adequacy criteria for the model-theoretic interpretation of NL questions, adopted from [17]. As shown by [17], most
1 Introduction

The semantics of natural language (NL) interrogative sentences, henceforth abbreviated to questions, has received considerable attention in the linguistic and philosophical logic literature. See [17, 18] for surveys. In the linguistic literature, it is generally accepted that the semantics of questions should provide answerability conditions for them in direct analogy with truth conditions for indicative sentences. Answerability conditions for a particular question are conditions which are required to hold in order to answer a question. There is less agreement on how answerability conditions should be defined, with different researchers interpreting this notion differently. The linguistic approaches are divided into three different varieties in [17]: partition theories, sets of proposition theories and categorial theories. Partition theories view the meaning of questions as partitioning the set of possibilities into different mutually exclusive subsets, which jointly exhaust the full set of possibilities. As a major proponent of this view, we will review the work of Groenendijk and Stokhof (henceforth G&S) [12, 14, 16, 17]. Sets of proposition theories view the meaning of a question as a set of propositions that answer the question. We will focus on the work of [22, 23] which belongs to this variety. Categorial theories (e.g. [19, 28]) also analyze the meaning of questions in terms of their answers, but these answers are not required to be propositions. Instead, they may be elements of other non-sentential categories, depending on the question. We will not discuss these theories here. In this paper, we present a novel account of the semantics of questions in a Montague Grammar (MG) style framework [8, 24]. This account
Bilattices and the Semantics of Natural Language Questions

Rani Nelken Nissim Francez
Computer Science Department
The Technion
Haifa 32000, Israel

November 19, 1998

“Can two walk together except they be agreed?” Amos III 3.

Abstract

We propose a novel semantic theory of NL questions in a Montogovian framework. This theory is composed of a compositional translation method into a formal logical meaning representation language, and a model-theoretic interpretation of this language. The main innovation is in the interpretation of type t as a bilattice, instead of the traditional boolean interpretation. This novel account is shown to satisfy a set of adequacy criteria imposed on the semantics of questions by [17], while being simpler than existing accounts. It provides a straightforward analysis of complex questions including coordinated questions, combined indicative and interrogative sentences, quantifying into questions, and functional questions.

Contents

1 Introduction 2

2 ILQ

3 Translating NL into ILQ
3.1 Yes/no questions ........................................... 7
3.2 Constituent questions ...................................... 7
3.2.1 Interrogative NPs ....................................... 8
3.2.2 Direct construction ..................................... 9
3.2.3 Indirect construction .................................. 9

4 Adequacy criteria for the interpretation of ILQ 11

5 G&S interpretation of ILQ 12

6 Bilattice interpretation of ILQ
6.1 Bilattices - basic definitions .............................. 16
6.2 Structures for interpreting ILQ ........................... 18
6.3 Semantic notions .......................................... 22
6.4 Adequacy .................................................. 24
6.4.1 Material adequacy .................................... 24