Cone Visibility Decomposition of Freeform Surfaces

Gershon Elber Eyal Zussman
Department of Computer Science Department of Mechanical Engineering
Technion, Israel Institute of Technology Technion, Israel Institute of Technology
Haifa 32000, Israel Haifa 32000, Israel

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Abstract

We present a scheme to preplan the set of viewing directions that are necessary to Laser scan a freeform surface. A Laser scanner is limited by the orientation of the normal of the scanned surface and a large deviation from the normal direction can hinder the ability to detect the reflected ray of the Laser. In this work, we present a freeform surface decomposition into regions, so that each region can be properly scanned from a prescribed viewing direction. The union of all these freeform surface regions forms a coverage of the entire original surface.

keywords: Gauss Map, Symbolic Computation, Laser Scan.

1 Introduction

The appearance of an object under a particular sensing device is of fundamental interest for applications such as inspection and reverse engineering. Each and every sensor has various optical and geometrical limitations on the ranges and orientations that the sensor can detect the object under consideration. Therefore proper placement of the sensor can become a difficult planning problem. Optical sensor planning depends upon various factors including the position, orientation, and reflectivity of an object under inspection, or alternatively on the transparency of the air, and ambient lighting [13].

In order to (Laser) scan a region of an object, that region must be visible. This trivial observation can pose extreme difficulties in actual planning of a scanned region, given the geometry of the object. The level of difficulty in the planning is largely related to the number of degree of freedom in the placement process. Having only three translational degrees of freedom makes the placement problem much simpler than the case were rotational degrees of freedom are also added. Unfortunately, translation by itself is insufficient in numerous cases, as is the case for arbitrary freeform shaped surfaces.
Denote by $\mathcal{PR}$ the space of piecewise polynomial or rational functions. Let $S(u, v)$ be a regular $C^1$ freeform surface in $\mathcal{PR}$. That is $||\vec{\nabla}(u, v)|| = ||\frac{\partial S}{\partial u} \times \frac{\partial S}{\partial v}|| > 0$, where $\vec{\nabla}(u, v)$ is the unnormalized surface normal that is assumed to point into the object. Denote by $O$ the origin of the sensing device. Then, we say that a point $p_0 = S(u_0, v_0) \in S$ is locally visible from $O$ if $\langle p_0 - O, \vec{\nabla}(u_0, v_0) \rangle > 0$ [9].

We say that a point $p_0 \in S$ is globally visible from $O$ if the open segment $[p_0, O]$ does not intersect $S$. Intuitively and because $S$ is $C^1$ continuous, the local visibility constraint guarantee that for a sufficient small $\epsilon$, point $p_0 = S(u_0, v_0)$ will be (globally) visible from $O$ if $S$ is restricted to the surface region of $S(u_0 \pm \epsilon, v_0 \pm \epsilon)$. The global visibility constraint is more difficult to achieve and indeed in this work we concentrate on the local visibility problem.

**Definition 1** Let $\mathcal{G}_S$ be the Gauss map of surface $S$ [6]. $\mathcal{G}_S$ is a map from $S$ to the unit sphere, $S^2$, which takes each point $p_0 \in S$ to $S$'s unit normal, $\vec{n}_{p_0} \in S^2$. That is $\vec{n}_{p_0} = \mathcal{G}_S(p_0)$.

Trying to employ range sensors to reverse engineer or inspect freeform objects, one is faced with several optical limitations. The utilization of optical triangulation [2, 14] imposes a limit on the angle between the emitted light from the sensor and normal of the surface.

Denote by $\vartheta$ the angle between two vectors $\vec{a}$ and $\vec{b}$. Then, extending the local visibility notion, we can express the sensor’s physical constraint as,

**Definition 2** Given a scanning direction $\vec{V}$, we say that a point $p_0 \in S$ is $\alpha$-sensible from $\vec{V}$ if and only if $\alpha \leq \vartheta_{p_0}$. Further, a region $R$ is said to be $\alpha$-sensible from $\vec{V}$ if and only if every point $\vec{p} \in R$ is $\alpha$-sensible.

In the ensuing discussion, we refer to cones with their apex at the origin only, or at the center of the unit sphere $S^2$. A cone with an opening angle $\alpha$ and axis $\vec{A}_i$ is denoted by $C_{\alpha}^{A_i}$. The spherical cap that is the result of the intersection of $S^2$ and $C_{\alpha}^{A_i}$ will be denoted $S_{\alpha}^{A_i} = S^2 \cap C_{\alpha}^{A_i}$. The angle $\alpha$ is also considered the opening angle of $S_{\alpha}^{A_i}$.

This paper presents a semi-optimal method to decompose a freeform surface, $S(u, v)$, into regions that are $\alpha$-sensible. The union of all these regions covers the entire freeform surface. That is, we present a decomposition approach for $\mathcal{G}_S$ using $n$ spherical caps, $S_{\alpha}^{A_i}$, $i = 1, ..., n$ such that $\mathcal{G}_S \subset \bigcup_{i=1}^{n} S_{\alpha}^{A_i}$. Then, $\vec{A}_i$, $i = 1, ..., n$ provides the necessary $n$ orientations for the sensor to successfully scan $S$.

Hence, a fundamental question to be answered is the coverage problem of $S^2$ using $n$ spherical caps of opening angle $\alpha$, a problem we address in Section 2. The solution to the coverage problem of $S^2$ is employed in Section 3, where we present a method to isolate only these spherical caps that intersect with $\mathcal{G}_S$. In Section 4, we consider the extraction problem of the $\alpha$-sensible region of surface $S$ from a given direction $\vec{A}_i$. Section 5 presents a complete example on a freeform NURBs surface while we conclude in Section 6.

All the figures and examples presented in this work were generated with the aid of the IRIT [10] solid modeling system that is developed at the Technion and Matlab [12].

2 Covering a Unit Sphere with Spherical Caps

Denote the set of spherical caps of opening angle $\alpha$ that covers the unit sphere $S^2$ by:

$$\Psi_{\alpha} = \{S_{\alpha}^{A_i}\}_{i=1}^{m},$$

(1)
such that $S^2 \equiv \bigcup_{k=1}^n S^2_{\alpha}$. We then say that $\Psi_{\alpha}$ is an $\alpha$-coverage for $S^2$.

Given $\alpha$, our goal is to compute the coverage of $S^2$ by the minimal number of spherical caps that are the result of an intersection of a cone with opening angle $\alpha$ and $S^2$. Consider a discrete collection of unit vectors $\mathcal{A} = \{\mathbf{A}_1, \mathbf{A}_2, ..., \mathbf{A}_n\}$. The least upper bound on the distance from arbitrary point $p \in S^2$ to the closest point $p_i = \mathbf{A}_i \cap S^2$ that is on $S^2$ is called the covering radius of $\mathcal{A}$ and denoted by $R_\mathcal{A}$. Thus [5],

$$R_\mathcal{A} = \sup_{p \in S^2} \inf_{\mathbf{A}_i \in \mathcal{A}} \text{dist}(p_i, p),$$

where $\text{dist}(p, q)$ is the shortest Geodesic arc on $S^2$ between the two point of $p$ and $q$.

The $\Psi_{\alpha}$ coverage (Equation 1) is obviously overlapping. One may consider the optimality of the coverage or the amount of the overlapping in this constructed coverage, that is also known as the thickness of the coverage [5].

Around each point $p_i$ is its Voronoi cell, $V(p_i)$, consisting of those points of $S^2$ that are closer to $p_i$ than any other point $p_j$, $j \neq i$ from $\mathcal{A}$. Thus,

$$V(p_i) = \left\{ p \in S^2 \mid \text{dist}(p_i, p) < \text{dist}(p_j, p), \ \forall j \neq i \right\}$$

An algorithm for the construction of the Voronoi mesh on $S^2$ is based on certain observations concerning the corners of the spherical polygon that forms the Voronoi cell, $V(p_i)$. The edges of $V(p_i)$ are great arcs that are equidistant between pairs of generating points, $p_i$, and for some other point, $p_j$, $j \neq i$. The intersection of two such edges (the corners of $V(p_i)$) are equidistant from three of the generating points. See [1] for a complete description.

The set of $k$ points, $p_i$, $i = 1, ..., k$, can be equally distributed around a sphere by using an inverse-square-law of the force of $\frac{1}{\text{dist}(p_i, p_j)^2}$. This distribution minimizes a $\frac{1}{\text{dist}(p_i, p_j)}$ potential energy function of all these points, so that in a global sense points are as 'far away' from each other as possible.

Hence, given $k$, we first find the best distribution of $k$ points on $S^2$ by exploiting this minimization or relaxation procedure of the potential energy function. Then, this equal distribution is employed in the generation of the Voronoi mesh. The later is used to derive the actual radius of the coverage, $R_\mathcal{A}$. Clearly, $R_\mathcal{A}$ is directly related to the opening angle $\alpha$, and this set of points defines $\Psi_{\alpha}$. One needs to construct a table, $\mathcal{T}$, of the value of $\alpha$ (or $R_\mathcal{A}$) for a given $k$ and compute the exact location of these $k$ points on $S^2$. For $k = 4, 6, 8, 12$ and $20$, the points should be positioned at the vertices of the respective regular platonic solid. For other values of $k$, numerical minimization or relaxation methods must be employed as already suggested. See [5] for more. Henceforth, we assume the existence of table $\mathcal{T}$ that needs to be constructed only once.

Given a sensor with a specific $\alpha$-sensibility, one can find the nearest from below covering radius using table $\mathcal{T}$, and extract the best coverage of the unit sphere by spherical caps for that $\alpha$. Figure 1 shows on example of equally distributed points on $S^2$. This distribution was computed using the energy relaxation technique that was proposed in this section. The next problem to consider is to find the best coverage of the Gauss map, $\mathcal{G}_S$, by spherical caps. This problem is discussed in the following section.

## 3 Computation of the Desired Orientations of the Sensor

The following projection from $S^2$ to the plane $z = 1$ is crucial to our discussion:
Figure 1: A set of 66 points that covers $S^2$ with spherical caps of $15^\circ$ degrees.

**Definition 3** The central or gnomonic projection $[11]$ of a point $p = (p_x, p_y, p_z) \in S^2$ onto the plane $z = 1$ equals,

$$\mathcal{M}(p) = \begin{pmatrix} p_x \\ p_y \\ p_z \\ 1 \end{pmatrix}.$$  

Because the two antipodal points, $p \in S^2$ and $-p \in S^2$, are projected to the same location in the plane $z = 1$, we hereafter assume that only the upper hemisphere of $S^2$ is considered, with $p_z > 0$.

The unnormalized normal field of $S$, $\overline{n}(u, v)$, can be computed symbolically. The projection of $\overline{n}(u, v)$ on $S^2$, denoted $\mathcal{P}(\overline{n}(u, v))$, is no longer in $PR$ and in [9] an arbitrary precise piecewise linear approximation is employed. The boundary of $\mathcal{P}(\overline{n}(u, v))$ provides the boundary of $\mathcal{G}_S$ and is formed from either the boundary of the tensor product vector field of $\mathcal{P}(\overline{n}(u, v))$, or its central silhouette [9]. Given a prescribed $\alpha$, one needs to detect and isolate those cones in the spherical $\alpha$-coverage, $\Psi_\alpha$, that also intersect with $\mathcal{G}_S$. Unfortunately, the boundary of $\mathcal{G}_S$ can assume highly complex shape, and hence we take a conservative approach that would yield almost optimal conservative answer but
with additional robustness, by employing the central convex hull of \( G_S \) computed in the plane \( z = 1 \), \( \mathcal{CH}(\mathcal{M}(G_S)) \) [9].

Assume that \( G_S \) fits into a hemisphere. Otherwise, and because \( S(u, v) \) is \( C^1 \) continuous, one can always subdivide \( S \) until \( G_S \), of all subdivided surfaces, \( S_i \), fits into a hemisphere. Without loss of generality, assume \( G_S \) fits into the \( z > 0 \) hemisphere. In fact, for numerical reasons we require \( G_S \) to be \( \alpha \) degrees away for the equator, or to fit into the northern spherical cap of radius \((90 - \alpha)\). In [3, 4], a method is proposed to detect the proper rotation that is necessary to bring an arbitrary \( G_S \) into the \( z > 0 \) hemisphere, provided \( G_S \) is indeed hemispherical.

Recall that \( C_\alpha^\lambda \) is a cone with opening angle \( \alpha \) and axis \( \mathbf{A} \). Then,

**Definition 4** The radial \( \alpha \) offset of region \( R \subset S^2 \), denoted \( Off_\alpha(R) \), equals,

\[
Off_\alpha(R) = S^2 \cap \left( \bigcup_{p \in R} C_\alpha^p \right).
\]

A cone \( C_\alpha^p \) intersects \( \mathcal{CH}(G_S) \) if and only if \( \mathcal{M}(\mathbf{A}) \in \mathcal{M}(Off_\alpha(\mathcal{CH}(G_S))) \). In other words, one can compute the radial \( \alpha \) offset of \( G_S \) and detected and isolate all the axes in the spherical coverage that was computed in Section 2 that are also contained in \( Off_\alpha(\mathcal{CH}(G_S)) \).

The radial \( \alpha \) offset can be computed directly on \( S^2 \). For the convex hull computation, we exploit the approach taken in [9] and use central projection (Definition 3). The central projection projects points on \( S^2 \) onto the plane \( z = 1 \). Hence, the mapping is one to one for the open hemisphere of \( \{ p \mid p \in S^2 \text{ and } z > 0 \} \). Moreover, great circles on \( S^2 \) are mapped to straight lines in the plane \( z = 1 \). Therefore, one is able to compute the radial convex hull of \( G_S \) by projecting \( G_S \) onto the plane \( z = 1 \) and exploiting regular planar convex hull algorithms in that plane.

We now need to compute the radial \( \alpha \) offset of this convex hull. In the plane \( z = 1 \), the radial \( \alpha \) offset becomes a variable radius offset problem. Denote by \( P_{xz} \) the \( xz \) \((y = 0)\) plane. As we approach the equator of \( z = 0 \) on \( S^2 \), a radial traversal of \( \epsilon \) degrees along the great arc of \( P_{xz} \cap S^2 \) is going to be centrally projected to an arbitrary large distance in the \( z = 1 \) plane. More precisely and using first order Taylor expansions for \( \cos(\epsilon) \approx 1 \) and \( \sin(\epsilon) \approx \epsilon \), the traversal for \( p_z \) becomes (similar for \( p_x \)) \( p_z \rightarrow p_z \cos(\epsilon) - p_z \sin(\epsilon) \approx p_x - p_z \epsilon \). Then, one gets from Definition 3,

\[
\mathcal{M}(p_x - p_x \epsilon, p_y, p_x \epsilon + p_z) - \mathcal{M}(p_x, p_y, p_z)
\]

\[
= \left( \frac{p_x - p_x \epsilon}{p_x \epsilon + p_z}, \frac{p_y}{p_x \epsilon + p_z}, 1 \right) - \left( \frac{p_x}{p_x}, \frac{p_y}{p_z}, 1 \right)
\]

\[
= \left( \frac{p_x - p_x \epsilon}{p_x \epsilon + p_z} - \frac{p_x}{p_x}, \frac{p_y}{p_x \epsilon + p_z} - \frac{p_y}{p_z}, 0 \right)
\]

\[
= \left( \frac{(p_x^2 + p_z^2) \epsilon}{p_x^2 + p_z p_x \epsilon}, \frac{p_x p_y}{p_x^2 + p_z p_x \epsilon}, 0 \right)
\]

\[
p_z \rightarrow 0 \Rightarrow (\infty, \infty, 0), \quad (4)
\]

Hence, the radial \( \alpha \) offset can be difficult to compute on the plane \( z = 1 \). Nevertheless, Equation (4) can be used to hint on the offset amount one needs to apply in the plane \( z = 1 \) for an
equivalent radial $\alpha$ offset on $S^2$, a result we exploit to approximate the variable radius offset of the convex hull of $G_S$ in the plane $z = 1$.

Alternatively, given $CH(G_S) \subset S^2$, one can compute for point $p$ on the boundary curve of $CH(G_S)$ the normal, $N_p$, to that boundary in the tangent plane of $S^2$, pointing outside of $CH(G_S)$. Traversing an angular distance of $\alpha$ along the great arc through $p$ in the direction of $N_p$, one gets at the offset location of $p$. Repeating this process for all the points in the piecewise linear approximation of $CH(G_S)$ yields $Off_s(CH(G_S))$. Because $CH(G_S)$ is obviously radially convex, and since the offset of a convex polygon is convex, the resulting radial $\alpha$ offset on $S^2$ is also radially convex.

Finally, by centrally projecting the set of axes of the cones that form the coverage of the northern hemisphere of $S^2$, $\{ A_i \mid C_{i \alpha} \in \Psi_\alpha \text{ and } A_i > 0 \}$, where $A_i$ is the $z$ coefficient of $A_i$, onto the plane $z = 1$, the axes that are found to be projected into $M(Off_s(CH(G_S)))$ are the ones to select. This operation can be conducted by simply testing for a point $(M(A_i))$ inclusion in a polygon $(M(Off_s(CH(G_S))))$, for example, using the Jordan theorem [6].

Algorithm 1 summarizes the entire process, returning the set of directions, $\Omega$, that an $\alpha$-sensible scanner must be placed at for a complete scan of the surface $S$.

Algorithm 1

Input:
$S(u, v)$, input surface;
$\alpha$, physical limit of sensing angle;

Output:
$\Omega$, a set of directions to orient the sensor at, for a complete scan of $S$.

Algorithm:
$\Psi_\alpha \leftarrow$ Coverage of $S^2$ with spherical caps of opening angle $\alpha$
$\{ A_i \}_{i=1}^m \leftarrow$ Set of axes of the spherical caps in $\Psi_\alpha$
$G_S \leftarrow$ Piecewise linear approximation of gauss map of $S$
$CH(G_S) \leftarrow$ convex hull of $G_S$
$M(Off_s(CH(G_S))) \leftarrow$ radial $\alpha$ offset of $CH(G_S)$, projected onto plane $z = 1$
$\Omega \leftarrow \{ A_i \mid M(A_i) \in M(Off_s(CH(G_S))) \}$

4 The Extraction of the $\alpha$-Sensible Regions

Once the $\Omega$ set has been established, the regions $R_i$ of $S(u, v)$ that contain normals that deviate by at most $\alpha$ degrees from each and every direction $A_i \in \Omega$ must be determined. Because the set of cones $\{ C_{i \alpha} \mid A_i \in \Omega \}$ covers $G_S$, we are guaranteed that the union of the regions, $\bigcup_i R_i$, covers $S$. Note, however, that $R_i$, $\forall i$, need not be simply connected or even connected at all.

The cosine of the angle between $A_i$ and $\pi(u, v)$, the unnormalized normal field of $S(u, v)$, equals $\frac{\langle\pi(u, v), A_i\rangle}{\|\pi(u, v)\|}$. For surface point $(u_0, v_0)$ to be $\alpha$-sensible from $A_i$, the angle between $\pi(u_0, v_0)$ and $A_i$ must be less than $\alpha$. Because the cosine function is monotone for angles between zero and $\pi/2$, one
can rewrite this conditions as,
\[
\cos(\alpha) = \frac{\langle \vec{m}(u,v), \vec{A}_i \rangle}{\|\vec{m}(u,v)\|}.
\] (5)

Squaring both sides of Equation (5), one gets,
\[
\cos^2(\alpha) \left( n_x^2(u,v) + n_y^2(u,v) + n_z^2(u,v) \right) = \left( n_x(u,v)\vec{A}_{ix} + n_y(u,v)\vec{A}_{iy} + n_z(u,v)\vec{A}_{iz} \right)^2,
\] (6)

where \((n_x, n_y, n_z)\) and \((\vec{A}_{ix}, \vec{A}_{iy}, \vec{A}_{iz})\) denotes the coefficients of \(\vec{m}\) and \(\vec{A}_i\), respectively.

Equation (6) is computable and representable in \(\mathcal{PR}\) [8]. Moreover, one can further simplify it by assuming \(\vec{A}_i\) is collinear with the \(z\) axis. A simple rotation transformation of vector \(\vec{A}_i\) to the \(z\) axis that is applied to \(S\) can clearly achieve this goal. Let,
\[
\mathcal{F}(u,v) = \cos^2(\alpha) \left( n_x^2(u,v) + n_y^2(u,v) \right) + \left( \cos^2(\alpha) - 1 \right) n_z^2(u,v).
\]

Then, Equation (6) becomes,
\[
\mathcal{F}(u,v) = 0.
\] (7)

Given a freeform surface, \(S(u,v) \in \mathcal{PR}\). Equation (7) is a zero set of a function in \(\mathcal{PR}\). The fact that \(\mathcal{F}(u,v) \in \mathcal{PR}\) enables one to exploit the stable properties of the NURBs representation, the convex hull containment, and evaluation and subdivision. These stable properties make it possible to consistently compute the zero set. These stable properties are crucial due to the increasing orders that result from symbolically [8] computing \(\mathcal{F}(u,v)\). If \(S(u,v)\) is a surface of degrees \(k\) by \(l\), \(\vec{m}(u,v)\) is a vector field of degrees \(2k - 1\) by \(2l - 1\) and \(\mathcal{F}(u,v)\) is a scalar field of degrees \(4k - 2\) by \(4l - 2\).

Figure 2 shows a simple convex surface, along with its unnormalized normal field, \(\vec{m}(u,v)\), and the regions that are found \(\alpha\)-sensible from a viewing direction of the \(z\) axis. That is, surface regions with normals that deviate from the \(z\) axis by no more than \(\alpha\) degrees. Figure 3 shows several examples of different viewing directions of the same surface as in Figure 2, along with the regions that are found \(\alpha\)-sensible from that viewing direction.

5 A Complete Example

This section provides a complete example for all the different stages of Algorithm 1. The selected freeform surface is a section of a propeller shown in Figure 4. Two different views of the freeform surface are provided.

Figure 5 shows the normal field of this surface. In Figure 5 (a), the normal field along with the boundary and central silhouettes in light gray are shown while in Figure 5 (b), the projection of these two sets of curves onto \(S^2\) is shown in dark gray. Note \(\vec{m}(u,v)\) fits into the upper hemisphere. This projection onto \(S^2\) provides the boundaries of \(G_S\).

Figure 6 (a) shows the central projection of the boundaries and central silhouettes of the normal field onto the plane \(z = 1\), in light gray. In Figure 6 (b), the convex hull of the boundaries and central silhouettes is also presented, in dark gray.

Figure 7 shows the radial offset of the convex hull in both the plane \(z = 1\) and on the unit sphere of \(S^2\).
Figure 2: The convex surface, $S(u,v)$, in (a), with its unnormalized normal field, $\pi(u,v)$, in (b), is analyzed to detect the regions in $S$ that have surface normals that deviates less than $\alpha$ from the $z$ axis. These regions are contained in the cone oriented along the $z$ axis with an opening angle of $\alpha$ as seen in (b). In (c), these regions in the original surface $S(u,v)$ are enhanced.

Figure 3: Different viewing directions for the same surface as in Figure 2 along with the regions in the surface that have a normal deviation of less than an angle $\alpha$ degrees.
Figure 4: This freeform surface which is a portion of a propeller is used as a test case in this work. Two different views are provided.

Finally, 8 shows two views of the axes in the coverage $\Psi_{\alpha}$ that are also found to be in the offset of the convex hull of boundaries and central silhouettes of the normal field of $S$.

The exploitation of the convex hull of $G_S$ instead of $G_S$ significantly simplified the computation, because the exact boundaries of $G_S$ were never computed. Nevertheless, the last stage of the algorithm, when one figures out the regions of $S$ that are $\alpha$-sensible from a given direction $A_i$ (Section 4), can be used to eliminate the directions that yields empty such regions. Every direction, $A_i$, such that $F(u, v) = 0$ in Equation (7) yields the empty set can be purged away. The resulting set of directions, while still not optimal, will typically be significantly smaller.

6 Conclusion

We have presented a method to decompose a freeform surface into regions, so that each region is $\alpha$-sensible from a selected direction. The selection of the directions is made out of an almost optimal relaxed distribution of points on $S^2$ and guarantee a complete coverage of the surface.

In this work, we did not address the question of the scanner’s position, only its orientation. Once a region $R_i$ of surface $S$ is known to be $\gamma$-sensible from a prescribed direction, for some angle $\gamma$, and the angle of view of $R_i$ from the scanner is $\beta$, then a scanner that is $\alpha$-sensible, where $\alpha = \beta + \gamma$ can completely scan $R_i$. Nevertheless, an optimal solution to this scanner’s placement problem that takes into account the angle of view as well as the depth of view is still an open question.

Another difficulty that needs to be considered is the question of local vs. global visibility. While solving the local visibility is useful, it is by no means sufficient. One must employ regular hidden surface visibility tools for freeform surfaces [7] from each scanner’s viewing direction and verify the visibility of the region that is to be scanned from that direction.

7 Acknowledgment

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Figure 5: The unnormalized normal field of the propeller section from Figure 4, along with its boundary and central silhouette in thick light gray lines are presented in (a). In (b), the projection onto $S^2$ of the boundary and central silhouette of the unnormalized normal field is shown in thick dark gray.

References


Figure 6: The boundary and central silhouettes of $G_S$ in thick dark gray lines is projected onto the plane $z = 1$ in thick light gray lines, in (a). In (b), The convex hull in thick dark gray lines is shown for the projected $G_S$ in light gray, again computed in the plane $z = 1$.


    http://www.cs.technion.ac.il/~gershon/irit/home/irit_index.html
Figure 7: The offset in thick light gray lines of $G_S$, both in the plane $z = 1$ and on $S^2$.


Figure 8: The final set of directions from $\Psi_\alpha$ that covers $\mathcal{G}_S$ in thick gray lines. Two views are provided with (a) presents the regular view of Figures 5 to 7 and (b) shows the entire Gauss map. See also Figure 1.