Contrastive Logic

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Abstract

In this paper I introduce the notion of *bilogics*, namely logics interpreted over a *pair* of structures, in contrast to classical logic and many of its variations, the formulae of which are interpreted over *one* structure. In particular, I introduce and study *Contrastive Logic*, suitable for expressing *contrast* and *conformity* between the two structures involved.

A major reason for this study is striving towards an extension of *truth-conditional* semantics to cover several natural-language particles, which have been hitherto considered not to be amenable to such an extensional treatment, and were delegated to the level of non-extensional pragmatics. Examples of such particles are **but** and **already**.

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1 Introduction

The traditional semantics of most logical systems is such that a (closed) formula is interpreted as a statement about satisfaction in *one* structure, called a *model* of that formula. Satisfaction is then naturally extended for collections of formulae. The structures of interpretation are chosen in accordance with the intended application of the logic at hand. For example, assignments of truth values to propositional variables for propositional logic, first-order structures for the first-order predicate calculus, Kripke structures for modal logics, sequences of states for (linear-time) temporal logic, etc.

In this paper I introduce and investigate logical systems in which the formulae receive their natural semantic interpretation over *classes* of related structures. Thus, such (closed) formulae do not make an absolute statement about one specific structure (of the appropriate format). Rather, satisfaction is defined by a certain *relationship* among the structures in the class over which the formula is interpreted.

Note that this is essentially different from the possible-worlds semantics of modalities, which also involves a class of worlds. There, satisfaction is still a binary relation between some formula ϕ and one world w, a relation defined in terms of satisfaction of other formulae (usually subformulae of ϕ) in other worlds. It is not the case that a formula makes a *joint-statement* about the whole class. The latter is exactly the situation in the logics studied here. More on the relationship to Modal logic appears in the Conclusions section.

This paper restricts attention to logics requiring an *ordered pair* $\langle \mathcal{S}, \mathcal{A} \rangle$ of structures (of an appropriate format) for their interpretation. I refer to such logics as *bilogics*. Connectives in bilogics are referred to as *bilogical* connectives. In particular, I am interested in a special case of bilogics, called *Contrastive (bi)logic*, expressing *contrast* between the "state of affairs" in two structures over the same underlying domain (in the first-order case), or *conformity* between the two structures. By means of this logic one is able to formalize the distinction between *contrast* and *contradiction*, a distinction that was needed long ago but could not be achieved using classical logic.

A typical definition of semantic satisfaction ('$\models$') in a bilogic has the form[1]

$$\langle \mathcal{S}, \mathcal{A} \rangle \models \phi \ iff \ \mathcal{S} \models \phi_1 \ and \ \mathcal{A} \models \phi_2$$

where ϕ_1, ϕ_2 are derived from ϕ in a way dependent on its main bilogical connective. The one structure, $\mathcal{S}$, is the *standard world* while the second structure, $\mathcal{A}$, is the *actual world*. The world $\mathcal{A}$ has its usual meaning, representing a certain state of affairs, over which assertions are made in the usual logical way. The world $\mathcal{S}$ is a standard world in an application dependent way. It might represent

[1]Note that while all the examples presented are defined in terms of a conjunction of separate satisfaction in the two worlds, this need not necessarily be the case, and any other relationship is acceptable.

the *expected* state of affairs, or the *most probable* state of affairs, or the state of affairs as it *should have been* (under a certain normative prescription), or one of many other interpretations. Assertions about the standard world are also made in the usual logical way.

A major objective of the introduction of bilogics, and in particular the special case of *Contrastive Logic*, is a certain *truth functional* (though in an extended sense) formalization of certain natural-language particles, which have been considered hitherto as not amenable to such formalization. Instead, their treatment alluded to *pragmatic* rather than semantic treatment. In particular, I am interested in formalizing propositional contrastive connectives such as **but** and **surprisingly**, propositional conformal connectives such as **truly** and temporal connectives such as **already** or **still**. I elaborate on this issue in later sections.

At this point, however, I would like to stress that the paper is not an attempt to provide a comprehensive linguistic analysis as such, nor does it attempt to cope thoroughly with the bulk of available linguistic data. Rather, a major concern is forming the right abstraction out of the data and studying this abstraction. The focus is more on the approach and its relevance to semantics of natural language, than on the actual results obtained.

The bilogical operators are defined formally based on the two (uninterpreted) structures. Only the formalization of natural language constructs by means of these operators depends on the particular interpretation of the standard world.

To give the flavor of the development to follow, let us consider the informal semantics[2] of **but**. A proposition of the form $\phi\,\mathbf{but}\,\psi$ is commonly conceived (See, for example, [15], [21], [4], [3], and [2]) to have two semantic aspects:

- An implication that both ϕ and ψ hold - an *assertive* aspect.
- An implication of a *contrast* between ϕ and ψ - a *contrastive* aspect.

This contrast is often attributed to being "contrary to expectation", or to some other grounds for the entailment of $\neg\psi$ by ϕ.

Obviously, attempting to interpret both aspects in one structure, by letting, for example, $\phi\,\mathbf{but}\,\psi \overset{df.}{=} (\phi\wedge\psi)\wedge(\phi \rightarrow \neg\psi)$, leads to an immediate contradiction. In order to formalize both aspects, I consider an *ordered pair* of structures, and interpret a **but** statement of the above form in both structures jointly. It is in combining the two structures that we gain the ability of explicating the two aspects of the meaning of **but**.

Our main idea is to define a binary contrastive connective $\phi \lhd\!\rhd \psi$, interpreted on the pair $\langle \mathcal{S},\ \mathcal{A}\rangle$ as follows:

$$\langle \mathcal{S},\ \mathcal{A}\rangle \models \phi \lhd\!\rhd \psi \ \mathit{iff}\ \mathcal{S} \models \phi\ \rightarrow\ \neg\psi\ \mathit{and}\ \mathcal{A} \models \phi\wedge\psi \qquad \textbf{(B)}$$

[2] At least part of it, as **but** is an overloaded, ambiguous word. A systematic classification of the meanings of **but**, based on their role in *discourse analysis*, is presented in [3]. Section 2.2 presents examples both of the uses of **but** not captured by my definition, as well as the intended meaning that *is* captured.

When interpreting $\mathcal{S}$ (in definition **(B)**) as the *expected world*, one can define $\phi\,\mathbf{but}\,\psi \stackrel{df.}{=} \phi \mathrel{\lhd\!\!\!\rhd} \psi$. For simplicity, I am taking expectation here to be an *extensional* notion, represented by a world. It is known that the way expectation is expressed in natural language needs *intensional* means for its full characterization. An extensional definition is a sufficient approximation for the current needs. The second conjunct in the definition captures the conjunctive assertive aspect by asserting that the conjunction $\phi\wedge\psi$ holds in the actual world. The first conjunct says that in the standard world ϕ implies $\neg\psi$, so in the standard world the conjunction $\phi\wedge\psi$ could never hold. That is how the contrastive aspect is captured: There is a contrast between the the actual world and the standard world.

An interesting issue raised by the approach is the following. In classical logics, one has the notion of the class of *all* models of a closed formula ϕ (or a class of such formulae), defined by

$$\mathcal{M}(\phi) \stackrel{df.}{=} \{\mathcal{A} \mid \mathcal{A} \models \phi\}$$

The question is, what is the analog definition for bilogics. There seem to be two natural choices.

Unanchored models: According to this approach, we let both worlds vary, and get

$$\mathcal{M}(\phi) \stackrel{df.}{=} \{\langle \mathcal{S},\ \mathcal{A}\rangle \mid \langle \mathcal{S},\ \mathcal{A}\rangle \models \phi\}$$

Anchored models: According to this approach, we keep the standard world $\mathcal{S}$ *fixed* and vary only the actual world. We get[3]

$$\mathcal{M}_{\mathcal{S}}(\phi) \stackrel{df.}{=} \{\mathcal{A} \mid \langle \mathcal{S},\ \mathcal{A}\rangle \models \phi\}$$

Both definitions are compatible with the proposed approach, and both have interesting properties. The anchored approach renders bilogics closer to modal logics, as it proposes a way of interpreting statements over one world, keeping the other one fixed. In the Conclusions section more is said about the relationship between $\mathcal{A}$ and $\mathcal{S}$.

In the rest of this paper I concentrate on *Contrastive Logics*. In Section 2 I consider the propositional and first-order framework. In section 3 I consider a linear-time temporal logic framework. Each Contrastive Logic is related to the corresponding above mentioned natural language particles. Section 4 ends with conclusions and relationship to related work.

[3]The distinction was pointed out by Shai Ben-David, who expressed preference for the anchored approach.

- Every propositional formula is a CL_1 formula.

For $\phi,\ \psi$ propositional formulas:

- $\lhd\phi$ is a CL_1 formula.
- $\rhd\phi$ is a CL_1 formula.
- $!\phi$ is a CL_1 formula.
- $\phi!$ is a CL_1 formula.
- $\phi \lhd\!\rhd \psi$ is a CL_1 formula.

Table 1: Flat Propositional Contrastive Logic: Syntax

2 Propositional and first-order Contrastive Logic

2.1 Propositional Contrastive Logic

In order to interpret this logic, both structures $\mathcal{S}$ and A are assumed to consist of truth-value assignments to a countable collection of propositional variables $p,\ q,\ r$, etc. I also assume the two boolean constants **true** and **false** and the standard propositional connectives of conjunction ('$\wedge$'), disjunction ('$\vee$'), negation ('$\neg$'), implication ('$\rightarrow$'), etc. Propositional formulae are constructed as usual, ranged over by $\phi,\ \psi$ etc. Their usual truth-functional interpretation is assumed.

As a first stage, I consider CL_1, the *flat* Contrastive Logic, without nesting of contrastive operators (neither within propositional connectives nor within themselves). The syntax of CL_1 is presented in Table 1.

I now turn to the semantic definition of CL_1, presented in Table 2. I use '$\models$' to ambiguously denote satisfaction both of a classical propositional formula over one world, and for the satisfaction of a contrastive formula over two worlds, leaving the distinction to context.

Remarks on the semantics:

1. From the semantic definition, '$\lhd$' is indeed seen to be a unary *contrastive* operator, in that its argument is asserted to hold in the actual world, while not holding in the standard world. Later we shall see that, depending on the interpretation of the standard world, different situations can be captured by this contrast.

- $$\langle \mathcal{S},\ \mathcal{A}\rangle \models \phi\ iff\ \mathcal{A} \models \phi,\ for\ \phi\ propositional \qquad \textbf{(P)}$$
- $$\langle \mathcal{S},\ \mathcal{A}\rangle \models \lhd\phi\ iff\ \mathcal{S} \models \neg\phi\ and\ \mathcal{A} \models \phi \qquad \textbf{(S)}$$
- $$\langle \mathcal{S},\ \mathcal{A}\rangle \models\ \rhd\,\phi\ iff\ \mathcal{S} \models \phi\ and\ \mathcal{A} \models \phi \qquad \textbf{(C)}$$
- $$\langle \mathcal{S},\ \mathcal{A}\rangle \models\ !\phi\ iff\ \mathcal{S} \models \phi \qquad \textbf{(W)}$$
- $$\langle \mathcal{S},\ \mathcal{A}\rangle \models \phi!\ iff\ \mathcal{A} \models \phi \qquad \textbf{(A)}$$
- $$\langle \mathcal{S},\ \mathcal{A}\rangle \models \phi \lhd\!\!\!\rhd \psi\ iff\ \mathcal{S} \models \phi\ \rightarrow\ \neg\psi\ and\ \mathcal{A} \models \phi\wedge\psi \qquad \textbf{(B)}$$

Table 2: Flat Propositional Contrastive Logic: Semantics

2. Similarly, '$\rhd$' is seen to be the unary *conformity* operator, asserting that its argument is satisfied *both* in the actual world and in the standard world.

3. The operator '!' has different meaning when apllied as a prefix operator than when applied as a suffix operator. In both cases, it asserts the satisfaction of its argument in *one* of the structures: in the standard world when applied as a prefix operator, and in the actual world when applied as a suffix operator[4].

4. The operator '$\lhd\!\!\!\rhd$' is the binary contrastive operator, asserting that both its arguments are satisfied in the actual world while not both are satisfied in the standard world. Here one can clearly see the difference between *contrast* and *contradiction*, mentioned in the introduction.

5. Note that $\phi!$ is actually equivalent to the classical proposition ϕ, and *embeds* the classical propositional calculus into propositional Contrastive Logic. In the sequel, I do not distinguish between $\phi!$ and ϕ.

As a first attempt to understand the contrastive (and conformal) connectives, it is shown that they are not really *independent*, and can be interdefined. For

[4] Clearly, this distinction reflects the position of the two structures in the ordered pair over which CL_1 is interpreted.

that purpose, the ambiguity of logical notation is extended, and the equivalence of two contrastive propositions is defined.

Definition: Two contrastive propositions ϕ and ψ are *equivalent*, denoted by $\phi \Leftrightarrow \psi$, iff for every $\langle \mathcal{S},\ \mathcal{A} \rangle$, $\langle \mathcal{S},\ \mathcal{A} \rangle \models \phi\ iff\ \langle \mathcal{S},\ \mathcal{A} \rangle \models \psi$.

One immediately obtains the following equivalences by a simple truth values calculation.

Proposition:

1.
$$\lhd \phi \ \Leftrightarrow\ \phi \lhd\!\!\rhd \phi \ \Leftrightarrow\ \mathbf{true} \lhd\!\!\rhd \phi \tag{1}$$

   This defines unary contrast in terms of binary contrast.

2.
$$\phi \lhd\!\!\rhd \psi \ \Leftrightarrow \lhd (\phi \wedge \psi) \tag{2}$$

   This is a definition of binary contrast in terms of unary contrast.

Below several more of the properties of contrastive connectives are formulated.

Proposition: $\phi \lhd\!\!\rhd \psi \ \Leftrightarrow\ \psi \lhd\!\!\rhd \phi$

Proof: Immediate[5] from the definition, by observing the equivalences $\phi \wedge \psi \ \Leftrightarrow\ \psi \wedge \phi$ and $\phi \rightarrow \neg \psi \ \Leftrightarrow\ \psi \rightarrow \neg \phi$. A discussion of the commutativity of the binary contrast operator appears in the Conclusions section.

A *bilogical tautology* is a bilogical proposition ϕ that is valid for *every pair* of assignments to the propositional variables in ϕ. This fits the unanchored approach mentioned above, were both assignments vary independently. One could also consider tautologies with respect to a fixed standard world. I do not pursue this issue further here.

Proposition: For no propositional ϕ and ψ is $\phi \lhd\!\!\rhd \psi$ a bilogical tautology.

Proof: Assume $\phi \lhd\!\!\rhd \psi$ is a bilogical tautology. By definition, $\phi \wedge \psi$ has to be a tautology (in $\mathcal{A}$), hence both ϕ and ψ have to be tautologies. But then, $\phi \rightarrow \neg \psi$ can not be a tautology (in $\mathcal{S}$).

Note that the situation changes once contrastive propositions are embedded within negation. Also note that the binary contrastive connective does have bilogical contradictions (i.e., bilogical propositions that hold for no pair of assignments). For example, if ψ is any usual propositional contradiction, then $\phi \lhd\!\!\rhd \psi$ is a contradiction for any ϕ. In particular, so is $\phi \lhd\!\!\rhd \mathbf{false}$.

[5] One could even have seen it more immediately, if in (**B**) we had the equivalent conjunct $\mathcal{S} \models \neg(\phi \wedge \psi)$, stressing symmetry.

2.2 Defining contrastive and conformal natural-language particles

Let us return now to the issue of defining the meaning of natural-language particles in a (generalized) truth functional way, using contrastive logic. As already mentioned in the introduction, by taking $\mathcal{S}$ to be the *expected world*, one can define **but** by $\phi\,\mathbf{but}\,\psi \stackrel{df.}{=} \phi \mathrel{\triangleleft\!\triangleright} \psi$. This definition captures the intuitive characterization [2], that

> " ... **but** means **and** + something else."

where it was difficult previously to capture this "something else".

As already mentioned, there is no attempt for a comprehensive, linguistically adequate, chracterization of all uses of **but**. Rather, I focus here on that use that fits the intuitive characterization above. In terms of the description in [15], we formalize the meaning resembling that of *although*, not the meaning resembling the (colloquially used) *while*, or *whereas*. Thus, the first of the following two statements is captured, and not the second:

1. He is hungry but not eating
2. My dog is black but my cat is white

A related discussion by W. Abraham appears in [1]. There, there is another use not treated here, of asserting a certain "approximation" with negating "achievement", as in the following example.

He is not a genius, but he is very able.

In particular, the use of **but** as an indication of alternative, as in

I do not like apples but pears

(the equivalent of the German *Sondern* and the Hebrew *Ella*), is also excluded. Note also, that I am only treating **but** as a sentential connective (*coordinator*). Thus, uses such as

All the runners but one have finished the race

are not handled either. These examples should clarify the scope of use I am trying to formalize.

The exclusion of an extensional, truth-conditional treatment of particles like **but** is a recurrent theme in linguistics and philosophy of language. Below are some typical quotations of statements to that effect.

[2]: " ...This falls outside the truth conditional semantics"

[6], p. 277: "Observe that logic does not recognize a special adversative[his term for my 'contrastive' - N.F.] connective. Indeed, if one tries to define **but** in terms of truth-values, one arrives at a scheme identical with that for logical **and**. ... Since, on the other hand, there is a clear linguistic difference between **and**

and **but** ... this provides a further indication for the fact that logical definitions are inadequate to account for the semantic values of coordinators of natural language."

[9]: "Finally, there appears to be in natural languages items like **but** in English, which may be best analyzed in terms of a truth-functional core-meaning (where **but** = **and**) plus a non-truth-functional overlay ..."

I believe that such claims are not substantiated. While it is rather clear that no logical treatment can ever capture the *full* richness of meanings of *any* natural-language particle, and there is a limit to the power of extensional, truth-functional characterization, I do not agree to the current placement of this limit, and I strive to push it forward, extending the scope of applicability of such approaches.

I do so under the belief that linguistic research itself will benefit too from such an endeavor, similarly to the benefits acrud by classical logic in the (albeit partial) characterization of **and**, **or** and **not**, the modal-logic characterization of **possibility** and **necessity**, the temporal-logic characterization of **eventually**, **until**, etc. In addition, extensional approaches should enhance the *computational* aspects of natural language understanding.

Using Contrastive Logic one can deal with contrast in a purely extensional, albeit slightly extended truth functional manner, not alluding to hearer or listener oriented pragmatics. For an approach explicitly advocating the latter, see, for example, [7] and [5].

I am not concerned here with the details of the nature of the expected world $\mathcal{S}$, and in questions like what makes it expected. These are more properly dealt with either linguistically or philosophically. Some comments about a way to relate expectation to actuality appear in the Conclusions section. I do mention in passing, that this notion encapsulates at least two different grounds for expectation. The first, *global* expectation, arises due to some general rules. For example in

He is over ninety but can run a Marathon.

the expectation is due to some general rules a bout aging. However, in

Q: Do you collect Irish stamps?

A: I collect old ones but I do not collect new ones.

a *local* expectation (not due to any general rule) is generated by the question, by which collecting Irish stamps suggests collecting all of them or collecting none.

This approach can also accommodate the view of **but** in [23], that

"... its logical function is to overwrite defaults."

We just have to take $\mathcal{S}$ as the default world.

Note, however, that this is an oversimplification. As is well-known from studies in AI, one can not assume a *unique* default world. Consider the following example. Suppose we have the following three default rules:

1. Adults are normally employed
2. Students are normally unemployed
3. Students are normally adults

Applying the above simplified approach, we get

1. x is an adult **but** x is not employed

   for an x who is an exception of the first rule. Similarly,

2. x is a student **but** x is employed

   for an x who is an exception of the second rule.

In trying to formulate all the default rules as general rules in the expected world $\mathcal{S}$, e.g., as:

$$\mathcal{S} \models \forall x : \mathit{adult}(x) \rightarrow \mathit{employed}(x)$$

$$\mathcal{S} \models \forall x : \mathit{student}(x) \rightarrow \neg \mathit{employed}(x)$$

$$\mathcal{S} \models \forall x : \mathit{student}(x) \rightarrow \mathit{adult}(x)$$

We immediately see that no such $\mathcal{S}$ may exist, as these three conditions are contradictory.

Thus, to cope better with default situations, a family of expected worlds, possibly partially ordered by a preference relation, is needed.

In the sequel, I retain the unique expected world, which suffices for my needs.

Note that under the above presented definition of **but**, the *absence* of contrast between the two parts of a contrastive statement renders the statement false! For example, consider an account of a chess game, where

I played white but you played black.

Since there is no contrast between the two players playing opposite colors (rather the contrary holds), this statement turns to be false. Compare with [3] (p. 167), where a similar sentence is considered “odd”, not false. This has a clear bearing on the meaning of negating contrastive statements, discussed below.

The whole approach can be refined by switching to a three-valued version of the logic, allowing also “undefined” truth values, and incorporating [19] a theory of presuppositions. (See [21] for a criticism of the presuppositional account of contrast, but with a less satisfactory alternative means of treatment than provided here). As already mentioned, another possible extension is towards an intensional characterization of expectation. Such extensions are not treated here.

Note also that our explication of **but** is symmetric. Asymmetry enters the definition as soon as ϕ and ψ involve temporal operators. A more comprehensive discussion of this issue appears in the Conclusions section. The definition also satisfies the *compositionality* criterion posed in [9], a necessary condition for presence of a truth-functional connective in natural language. The interpretation for a bilogic of their third criterion, namely that of *confessionality*, is not clear. If it should be applicable *separately* to each structure, than clearly our definition of **but** violates it. It does hold if taken jointly for both structures.

To show the power of the approach, I consider some more applications of it at the purely propositional contrastive level. A later section considers temporally-related contrast.

Consider the unary natural-language particle **surprisingly** ϕ. It also has two aspects of its meaning.

- An implication that ϕ holds - an *assertive* aspect.
- An implication of an *astonishment* of ϕ - an *unexpectedness* aspect.

If we once again interpret $\mathcal{S}$ as an *expected* world, one can define **surprisingly** $\phi \overset{df.}{=} \lhd\phi$. Thus, surprise is captured as denial of expectation.

As an interesting consequence of the bilogical equivalences stated above one gets the following equivalences for natural language expressions:

1. $$\textbf{surprisingly } \phi \;\Leftrightarrow\; \phi \textbf{ but } \phi$$

   and
   $$\textbf{surprisingly } \phi \;\Leftrightarrow\; \textbf{true but } \phi$$

2. Another consequence is in the opposite direction.

   ϕ **but** $\psi \;\Leftrightarrow\;$ **surprisingly** $(\phi \wedge \psi)$.

Thus, **surprisingly** can be *defined* in terms of **but** and vice-versa. This captures an intuitive understanding of contrast as a surprisingly holding conjunction, and of surprise as a self-contrast, a *diagonalization* of binary contrast (i.e., equating its two arguments), and also a contrast to an absolute truth.

Consider now the unary natural language particle **truly** ϕ (or sometimes also **indeed** ϕ), which also has two aspects of meaning:

- An implication that ϕ holds - again the *assertive* aspect.

- An implication of an *expectedness* of ϕ - a *conformity* aspect.

Thus, it is possible now to define **truly** $\phi \stackrel{df.}{=} \rhd\phi$ when again interpreting $\mathcal{S}$ as the *expected* world.

If we interpret $\mathcal{S}$ as a *wishful* world[6] one may define **hopefully** $\phi \stackrel{df.}{=} !\phi$, while for $\mathcal{S}$ the *unwishful* world, we may put[7] **it is feared that** $\phi \stackrel{df.}{=} !\phi$.

2.2.1 Nested contrastive propositions

As such composite statements are hardly used in natural language, we do not have very clear intuitive guidelines as to how to extend CL_1 to the full *Contrastive Logic* CL.

However, it is a rather common situation where a logic is inspired by some natural language phenomenon, but after the formation of the right logical abstraction, the logic is "detached" from its origin and treated mathematically, "smoothed" wherever needed. Thereby, it may provide interpretation to constructs that do not reflect any natural language analogon, and could not arise via a translation of the natural language to the logic.

As an example, consider *DRT* (Discourse Representation Theory[8] [13]) described here using the linear notation of [8] (Vol. II, Section 7.4).

A DRS of the form $\langle\ \{x\ \},\ \{p(x)\}\ \rangle$ can never arise from a translation of a natural language sentence, but yet admits interpretation (via embedding in models). The *simplest* DRSs that can arrise are of the forms

$$\langle\ \{x\ \},\ \{boy(x),\ cry(x)\}\ \rangle$$

(corresponding to A boy cries), or

$$\langle\ \{x\ \},\ \{x = John,\ cry(x)\}\ \rangle$$

(corresponding to John cries).

I start by defining the "easier" *nesting* of contrastive propositions within standard propositional connectives. This is done very much as expected, by interpreting the propositional connectives in their standard way at the meta-level. We thus get:

$$\langle\mathcal{S},\ \mathcal{A}\rangle \models \phi\wedge\psi\ iff\ \langle\mathcal{S},\ \mathcal{A}\rangle \models \phi\ and\ \langle\mathcal{S},\ \mathcal{A}\rangle \models \psi$$

$$\langle\mathcal{S},\ \mathcal{A}\rangle \models \phi\vee\psi\ iff\ \langle\mathcal{S},\ \mathcal{A}\rangle \models \phi\ or\ \langle\mathcal{S},\ \mathcal{A}\rangle \models \psi$$

$$\langle\mathcal{S},\ \mathcal{A}\rangle \models \neg\phi\ iff\ \langle\mathcal{S},\ \mathcal{A}\rangle \not\models \phi$$

[6] This proposal is due to Ira Forman.
[7] Or, more colloquially, **I'm afraid that** ϕ.
[8] Readers not familiar with this theory may skip this example without any substantial loss.

The negation case is of interest. Negating a contrastive proposition of the form ϕ **but** ψ, by this definition, is actually

$$\mathcal{A} \not\models \phi \wedge \psi \ \textit{or} \ \mathcal{S} \not\models \phi \rightarrow \neg\psi$$

i.e., either negating one of ϕ, ψ in the actual world (negating the assertive aspect), *or negating the contrast itself*, by negating the implication in the standard world.

Once we have contrastive negated propositions, we may obtain contrastive tautologies by negating contrastive contradiction. As an example, consider

Proposition:

$$\models \neg(\mathbf{false} \lhd\!\!\rhd \phi)$$

Proof: Trivial, since $\mathcal{A} \not\models \mathbf{false} \wedge \phi$.

Since neither the disjunction nor the negation of **but** sentences is common practice in natural language, it is hard to see whether this extension to negation fits such application. One situation where such a negation might be used is in a dialogue as follows:

Mother: I told you to stay home but you did not listen.

Child: No!

How can we interpret the childs response, negating the mother contrastive statement? Is the child really meaning that either the mother didn't tell him to stay home, or he did listen, or there is no contrast between her telling him and his not listening to her?

As a consequence of the lack of intuitive direction from natural-language usage, the situation here differs from related problematic situations for negation where it induces presupposition preservation. A typical example is negating sentences with definite articles (interpreted as a claim of existence and uniqueness). Thus

It is not the case that the (current) king of France is bald

is *naturally* interpretable in the case of absence of presupposition failure as

The (current) king of France is not bald

There does not seem to be an analogous simple interpretation for negating contrast. Hence, I retain here logical negation, as defined above.

Note that conjunctions[9] of contrastive statements seem to be more natural and more commonly used, as in

I slept eight hours but I am still tired and I ate a large meal but I am still hungry.

Next, we come to the more complicated issue of nesting the contrastive operators within themselves. The problem is, that our definition of the satisfaction of a contrastive statement assumes that the component (or components, in the

[9] Assuming a reading under which a conjunction binds weaker than contrast.

- $$\langle \mathcal{S},\ \mathcal{A}\rangle \models \lhd\phi \ iff\ \mathcal{S} \models \neg\alpha(\phi)\ and\ \mathcal{A} \models \alpha(\phi) \qquad \textbf{(S)}$$
- $$\langle \mathcal{S},\ \mathcal{A}\rangle \models\ \rhd\phi \ iff\ \mathcal{S} \models \alpha(\phi)\ and\ \mathcal{A} \models \alpha(\phi) \qquad \textbf{(C)}$$
- $$\langle \mathcal{S},\ \mathcal{A}\rangle \models\ !\phi \ iff\ \mathcal{S} \models \alpha(\phi) \qquad \textbf{(W)}$$
- $$\langle \mathcal{S},\ \mathcal{A}\rangle \models \phi!\ iff\ \mathcal{A} \models \alpha(\phi) \qquad \textbf{(A)}$$
- $$\langle \mathcal{S},\ \mathcal{A}\rangle \models \phi \lhd\!\rhd \psi \ iff\ \mathcal{S} \models \alpha(\phi)\ \rightarrow\ \neg\alpha(\psi)\ and\ \mathcal{A} \models \alpha(\phi)\wedge\alpha(\psi) \qquad \textbf{(B)}$$

Table 3: Full Propositional Contrastive Logic: Semantics

binary case) may be interpreted over one structure! To overcome this difficulty, I introduce a syntactic operator $\alpha(\phi)$, which isolates the assertive part of a contrastive proposition. The latter are ordinary logical formulae (propositional in the current context). I then require those assertive parts (of the corresponding components) to be satisfied in the respective structures. The definition of α is by induction on the syntactic structure of its argument.

$$\alpha(\phi) \stackrel{df.}{=} \begin{cases} \phi & if\ \phi\ is\ propositional \\ \alpha(\psi)\wedge\alpha(\chi) & if\ \phi = \psi \lhd\!\rhd \chi,\ or\ \phi = \psi\wedge\chi \\ \alpha(\psi) & if\ \phi = \lhd\ \psi\ ,\ \phi = \rhd\ \psi,\ or\ \phi = !\psi \\ \alpha(\psi)\vee\alpha(\chi) & if\ \phi = \psi\vee\chi \\ \neg(\alpha(\psi)) & if\ \phi = \neg\psi \end{cases}$$

Note that 'α' is *idempotent*, i.e., $\alpha(\alpha(\phi)) = \alpha(\phi)$. I now modify (in Table 3) the definition of satisfaction of contrastive proposition to accommodate nesting, obtaining the previous definition as a special case when the nested operands are propositional.

A simple calculation shows that, for $p,\ q,\ r$ propositional, for example,

$$\langle \mathcal{S},\ \mathcal{A}\rangle \models (p \lhd\!\rhd q) \lhd\!\rhd r\ iff\ \mathcal{S} \models (p\wedge q) \rightarrow \neg r\ and\ \mathcal{A} \models p\wedge q\wedge r$$

An immediate consequence of the above definitions is expressed in the next proposition, stating the associativity of the binary contrastive operator.

Proposition:

$$(\phi \lhd\!\rhd \psi) \lhd\!\rhd \chi \;\Leftrightarrow\; \phi \lhd\!\rhd (\psi \lhd\!\rhd \chi)$$

When interpreted for **but**, we get that p **but** q **but** r means that there is a "triple contrast" between the three propositional arguments: All three hold in the actual world, but in the standard world any two imply the negation of the third. This is a desirable property, which rather conforms with our intuitive understanding of contrast.

This "oblivious" interpretation of the contrastive operators is reminiscent of a phenomenon mentioned in [14], a certain "cancelation of presupposition". In their explication of the particle **even**,

Even Bill likes Mary

presupposes

Someone ($\neq$ Bill) likes Mary.

When embeded, say as in

I just noticed that even Bill likes Mary

they argue that the above presupposition is not present. Thus, contrast may fail to propagate under nesting in a similar way as the failure of presupposition propagation under nesting.

In addition to the previously mentioned 'flat' equivalences, we also get the following by simple truth-value calculation, applying α wherever needed:

- $$\phi \lhd\!\rhd \psi \;\Leftrightarrow\; !(\neg(\phi \wedge \psi)) \wedge (\phi \wedge \psi) \tag{3}$$
- $$!!\phi \;\Leftrightarrow\; !\phi \tag{4}$$
- $$\rhd\rhd\phi \;\Leftrightarrow\; \rhd\phi \tag{5}$$
- $$\lhd\lhd\phi \;\Leftrightarrow\; \lhd\phi \tag{6}$$
- $$\neg(!\phi) \;\Leftrightarrow\; !(\neg\phi) \tag{7}$$

2.3 First-order Contrastive logic

Here it is assumed that the two structures $\mathcal{S}$ and $\mathcal{A}$ have the *same carrier* (or *domain*), and are of the *same signature*. In other words, they interpret the same relational and functional symbols, assigning them possibly different *extents*. For simplicity, avoiding a philosophical debate, it is assumed that (individual) *constants* in the signature have the same denotation in both structures. This assumption does not influence the development of the ideas.

The interpretation of the connectives is inherited from the propositional case. However, in order to deal with quantification, one has to deal with application of contrastive connectives to *open formulae*, i.e., formulae with *free variables*. I would like to make use of the traditional way of handling free variables, by relativizing satisfaction to a *valuation* (known also as an *assignment*), binding free variables to elements of the domain. Here we face the need for a decision, as we have two structures over which satisfaction is defined. Since it was agreed that both have the same domain, only one is used, *joint valuation*, so to speak, which distributes into the two structures when separate satisfaction is involved. I denote valuations generically by $\mathcal{V}$, and the value of variable x in valuation $\mathcal{V}$ by $\mathcal{V}[\![x]\!]$.

Thus, the following clauses in the semantic definition are obtained:

$$\langle \mathcal{S},\ \mathcal{A},\ \mathcal{V}\rangle \models \phi(x)\ iff\ \langle \mathcal{S},\ \mathcal{A}\rangle \models \phi(\mathcal{V}[\![x]\!])$$

where x may occur free in ϕ, and $\phi(d)$ denotes substitution of d for all free occurrences of x in ϕ.

$$\langle \mathcal{S},\ \mathcal{A}\rangle \models \exists x :\ \phi(x)\ iff\ \langle \mathcal{S},\ \mathcal{A},\ \mathcal{V}\rangle \models \phi(x)$$

for *some* valuation $\mathcal{V}$.

Similarly,

$$\langle \mathcal{S},\ \mathcal{A}\rangle \models \forall x :\ \phi(x)\ iff\ \langle \mathcal{S},\ \mathcal{A},\ \mathcal{V}\rangle \models \phi(x)$$

for *every* valuation $\mathcal{V}$.

Note that these definitions, as usual, have a strong impact on the *Barcan formulae*. We still have

$$\models \exists x : \lhd\, \phi(x)\ \rightarrow\ \lhd\, (\exists x : \phi(x))$$

and

$$\models \exists x : (\ \phi(x) \lhd\!\!\rhd \psi(x)\ ) \rightarrow (\exists x : \phi(x)) \lhd\!\!\rhd (\exists x : \psi(x))$$

(but not the other direction), due to the usage of negation in the definitions of '$\lhd$' and '$\lhd\!\!\rhd$'.

3 Contrastive (Linear-time) Temporal Logic

Let us start again by considering informally the meaning[10] of **already** ϕ. As in previous cases, it has two aspects in its meaning:

- An implication that ϕ holds (at the time of reference - see below) - the *assertive* aspect.

[10] Again, only part of the meaning of this overloaded, ambiguous connective is captured.

- An implication of a *temporal unexpectedness*[11] of ϕ - a *prematureness* aspect.

For example, consider the sentence
When I arrived in the station, the train was already gone.

The first part of the sentence establishes a *time of reference*, namely the arrival time. The second half implies, on the one hand, that at the time of reference the train was not in the station (the *assertive* aspect), and on the other hand that the speaker expected the train to have been in the station at some prior time, to have remained there, and to still being there at the reference time. Note that this expectation lends itself to two explanations:

- The speaker arrived when intended, but the train was supposed to leave later than it actually did.
- The train left on time, but the speaker intended to arrive earlier.

I assume here the first explanation only, assuming that the time of reference is always fixed.

Thus, by employing a Contrastive Temporal Logic one can capture both aspects, in that the assertive aspect constitutes a (temporal) assertion about the actual world, while the prematureness aspect is a (temporal) assertion about the standard world, again interpreted as the expected world. This aspect of prematureness, in a weaker form, is considered to be a presupposition in [12]. Both worlds have to be temporal structures. Here the compatibility among structures implies also that both worlds use, so to speak, the *same* clock. This is made more precise below.

In [16] a comprehensive study of **already** (and related adverbs) is carried out (in the context of their meaning in German). The main aspect of **already** is conceived there as that of a *phase change* from an interval satisfying $\neg\phi$ to an interval satisfying ϕ, where the time of reference falls into the second interval. The *prematureness* aspect is mentioned there as secondary only. In [12], holding in an interval is replaced by a weaker property, merely holding at some past time instant. The relationship of **already** and expected timing is pointed out also in [11] in a similar context of German. Since my purpose here is to illustrate the use of Contrastive Logics in formalizing natural language constructs, the debate about which aspect of meaning is primary and which is secondary is omitted, leaving for linguists to settle. See [16] for further references to linguistic treatments of this subject.

In order to stay within the simplest temporal logic possible, I restrict the discussion to weak intervals (to be made clearer below). The syntax of *PLTL (Propositional (Linear-time) Temporal Logic)* is presented in Table 4.

A *structure* $\mathcal{A}$ for the interpretation of an *PLTL* formula is an infinite sequence σ of *states* s_i, $i \geq 0$, where a state is an assignment to the propositional

[11]That is, occurring earlier than expected.

- Every propositional formula is a formula.

For $\phi,\ \psi$ formulae:

- $\mathbf{P}\ \phi$ is a formula (read *past* ϕ).
- $\phi\ \mathbf{S}\ \psi$ is a formula (read ϕ *since* ψ).
- $\mathbf{F}\ \phi$ is a formula (read *future* ϕ, or more often *eventually* ϕ).
- $\phi\ \mathbf{U}\ \psi$ is a formula (read ϕ *until* ψ).
- $\neg\phi,\ \phi\wedge\psi,\ \phi\vee\psi$ etc. are formulae.

Table 4: Propositional Linear-Time Temporal Logic: Syntax

variables. In addition, satisfaction is defined using an auxiliary index $i \geq 0$, representing the *current* time, or time of reference, as a position in the sequence. The semantics is defined recursively, as shown in Table 5. I use σ_i to denote the ith state in σ.

Now the logic is extended to the corresponding *Propositional Contrastive Temporal Logic* by the addition of operators $\downarrow \phi$ and $\uparrow \phi$, the semantics of which is defined by interpretation on two *temporal structures* $\langle \mathcal{S},\ \mathcal{A}\rangle$.

$$\langle \mathcal{S},\ \mathcal{A}\rangle,\ i \models\ \downarrow \phi\ iff\ \mathcal{A},\ i \models \phi\ \mathbf{S}\ (\neg\phi)\ \textit{and}\ \mathcal{S},\ i \models (\neg\phi)\mathbf{U}\phi \qquad (\mathbf{AL})$$

$$\langle \mathcal{S},\ \mathcal{A}\rangle,\ i \models\ \uparrow \phi\ iff\ \mathcal{A},\ i \models \phi\ \mathbf{U}\ (\neg\phi)\ \textit{and}\ \mathcal{S},\ i \models (\neg\phi)\mathbf{S}\phi \qquad (\mathbf{ST})$$

Note that the current time i distributes to the separate structures. This is what was meant by the "common clock" of both structures. The fact that weak intervals are used is reflected by replacing the phase changes, implying a $\neg\phi$ interval directly followed ϕ interval (or vice versa), by the interval from the point of change to the current time. This seems to suffice for the current purpose.

Again, by interpreting the standard world $\mathcal{S}$ as the world in which events take place at the expected time, one may define:

$$\mathbf{already}\ \phi \overset{df.}{=} \downarrow \phi$$

$$\mathbf{still}\ \phi \overset{df.}{=} \uparrow \phi$$

I leave out the detailed description of the extension to *First-order Temporal Contrastive Logic*, as well as the issue of *nesting* the contrastive temporal

- For ϕ a propositional formula, $\sigma,\ i \models \phi$ iff $\sigma_i \models \phi$.
- $\sigma,\ i \models \mathbf{P}\ \phi$ iff there exists some $j,\ 0 \leq j < i$, such that $\sigma,\ j \models \phi$.
- $\sigma,\ i \models \phi\ \mathbf{S}\ \psi$ iff there exists some $j,\ 0 \leq j < i$, such that $\sigma,\ j \models \psi$, and for every $k,\ j < k \leq i$, it is the case that $\sigma,\ k \models \phi$.
- $\sigma,\ i \models \mathbf{F}\ \phi$ iff there exists some $j,\ j \geq i$, such that $\sigma,\ j \models \phi$.
- $\sigma,\ i \models \phi\ \mathbf{U}\ \psi$ iff there exists some $j,\ j > i$, such that $\sigma,\ j \models \psi$, and for every $k,\ i \leq k < j$, it is the case that $\sigma,\ k \models \phi$.
- Satisfaction of propositional combination is defined by distribution, as usual.

Table 5: Propositional Linear-Time Temporal Logic: Semantics

operators. I only note in passing that the traditional definition of negation by means of $\langle \mathcal{S},\ \mathcal{A} \rangle,\ i \models \neg\phi\ iff\ \langle \mathcal{S},\ \mathcal{A} \rangle,\ i \not\models \phi$ does not satisfy the *duality* relations referred[12] to in [16] for the non-contrastive case (i.e., phase-change only), whereby

$$\mathbf{already}\ \ \phi \ \Leftrightarrow\ \neg\, \mathbf{still}\,(\neg\phi)$$

and

$$\mathbf{still}\ \ \phi \ \Leftrightarrow\ \neg\, \mathbf{already}\,(\neg\phi)$$

This would require what in the terms of [16] is called *strong negation*, which is different then the usual logical negation.

Finally, I also leave out the introduction and study of the temporal version of the *conformal* operators.

4 Conclusions

In this paper I have introduced the notion of *bilogics*, namely logics interpreted over a *pair* of structures, in contrast to classical logic and many of its variations, the formulae of which are interpreted over one structure. In particular, I have introduced and studied *Contrastive Logic*, suitable for expressing *contrast* and *conformality* between the two structures involved.

A major reason for this study was striving towards an extension of *truth-conditional* semantics to cover several natural-language particles, which have

[12] I do not consider here the connection with **yet**, obtained via what is called there *internal negation* only.

been hitherto considered not to be amenable to such an extensional treatment, and were delegated to the level of non-extensional pragmatics. Precursorsy ideas that hint towards bilogics can be found in [10], [11] and [17].

Other bilogics should be investigated, to see whether more of the semantics of natural languages can be salvaged and treated more rigorously, using the tools that have proved themselves in so many other areas of research and application.

Expectation and actuality

So far, no restriction on the relationship between $\mathcal{S}$, the *standard* structure, and $\mathcal{A}$, the *actual* structure, were imposed. Clearly, to obtain interesting and significant results applicable to situations in which the need for such means arises, such restrictions are needed. In natural-language applications, one way to proceed is to *generate* the standard structure based on contextual information gathered from sentences preceding given contrastive proposition, within some discourse.

In particular, one may be interested in a *computational* interpretation of this generation process. Such an approach is presented in [23], using a so called three-level semantics.

Another source of relevant restrictions may originate from considerations similar to these applied in belief revision theories, where a notion of similarity between structures is introduced, and $\mathcal{S}$ can be taken as the *most similar* structure to $\mathcal{A}$, though non-uniqueness may be problematic here.

While the exact relationship between the expected world and the actual world remains a topic for further research, the approach put forward here can be very helpful in providing a rigorous framework in which the conclusions of such a study *can be expressed.* Once a theory of expectation and its relationship to actuality has been formed, it can be stated as a collection of non-(bi)logical axioms, characterizing the admissible pairs of structures. Thus, instead of using Contrastive logic as depending on a theory of expectations, one can use it to express and impose such a theory.

Contrastive logic and Modal logic

As already mentioned before, there is a certain similarity between Contrastive logic and Modal logic, in that both allude to more than one world (structure) for their semantic definition. However, there is a major difference between the two, due to which the former expresses a relation between the two structures and can be used in making joint statements about the two structures, while the latter does not. The difference originates from the "free" appearence of both structures (interpreting Contrastive logic) as arguments of the satisfaction relation. This contrasts the quantification over worlds in the semantic definition of Modal logic, leaving only one "free" world argument for the satisfaction relation.

It might be worth mentioning, that Contrastive logic might be related to Modal logic in another way. The former can be considered as an utterly degenerate special case of the latter, where the Kripke structures interpretations are restricted to one trivial graph only, having two nodes and one accessability edge between them. It is not clear whether such a representation clarifies anything.

Further work needs to be done also in defining a suitable *proof theory*, hopefully providing complete axiomatizations of Contrastive Logic in all its variants. In this paper, attention was restricted to the semantic aspects only. Maybe the degenerate view mentioned above may help here.

Symmetry of binary contrast

One of the more puzzling consequences of the theory presented is the symmetry of the binary contrast operator, yielding the equivalence

$$\phi \ \mathbf{but}\ \ \psi \ \Leftrightarrow \ \psi \ \mathbf{but}\ \ \phi$$

This equivalence was challenged by several authors that followed the path of this paper. In particular, [18] presents an alternative definition of **but**, based on an extension of the $S5$ Modal logic, which renders **but** asymmetric. However, as the authors there admit, the difference between

$$\phi \ \mathbf{but}\ \ \psi$$

and

$$\psi \ \mathbf{but}\ \ \phi$$

is a matter of focus and emphasis, both being captured by the Modal logic introduced there. Since I am only interested in (extended) truth conditions, I remain happy with the above equivalence. There is no way to distinguish the two orders merely by semantic contents.

Another difference between my presentation and that of [18], which developes contrast in a different way, is in the *dynamicity* of contrast in their approach. In my theory, all expectations are determined "in advance", reflected by the given world $\mathcal{S}$. According to their view, expectations are formed "in the fly". Thus, in ϕ **but** ψ, the expectation of $\neg\psi$ is formed only after "hearing" ϕ. This is an approach more oriented to pragmatics, e.g., utterance situations.

Another attempt to define the meaning of '**but**' is that in [22], in which a more pragmatically oriented approach is pursued, using a three-valued logic set-up. They also get a non-commutative meaning for binary contrast.

Finally, computer Science provided a fertile ground for the revival of many non-classical logics, as well as for the formations of new nonstandard logics. The application of Contrastive Logic in Computer Science is also under current investigation. One potential application, not yet investigated, is in line with [20], where related ideas are used for the specification of exception-handling in software systems. I expect Data-Base theory will provide another field of application.

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References

[1] Werner Abraham. But. In *UEA Papers in Linguistics, 3*, pages 1 – 35. University of East Anglia, 1977.

[2] Diane Blakemore. Denial and contrast: A relevance theoretic analysis of *but*. *Linguistics and Philosophy*, 12:15 – 37, 1989.

[3] Lauri Carlson. *Dialogue games: An approach to discourse analysis*. Synthese Language Library 17, D. Reidel publishing co., 1983.

[4] Marcelo Daskal and Tamar Katriel. Between semantics and pragmatics: The two types of *but* - hebrew *'aval'* and *'ela'*. *Theoretical Linguistics*, 4:143 – 172, 1977.

[5] Teun A. Van Dijk. Pragmatic connectives. *Journal of Pragmatics*, 3:447 – 456, 1979.

[6] Simon C. Dik. *Coordination - Its implications for the theory of general linguistics*. North-Holland, 1968.

[7] Thorstein Frentheim. Conjunction and pragmatics. In *Proc. of the Scandinavian seminar on philosophy of language*, volume 2, pages 19 – 46, Upsala, 1974.

[8] L.T.E. Gamut. *Intensional logic and logical grammar (Vol. 2)*. The university of Chicago press, 1991.

[9] Gerald Gazdar and Geofrrey K. Pullum. Truth-functional connectives in natural language. *CLS*, 12:220 – 234, 1976. Chicago Linguistic Society.

[10] Hans G. Herzberger. Dimensions of truth. *Journal of philosophical logic*, 2:535 – 556, 1973.

[11] J. Hoepelman and C. Rohrer. Remarks on *noch* and *schon* in German. In Philip J. Tedeschi and Annie Zaenen, editors, *Syntax and Semantics (Vol. 14,* Presupposition*)*, pages 103 – 126. Academic Press, 1981.

[12] Laurence R. Horn. Ain't it hard (anymore). In *Proc. of the 6th regional meeting, Chicago linguistic society*, pages 318 – 327, Chicago, IL, April 16 - 18 1970.

[13] Hans Kamp. A theory of truth and semantic representation. In Jeroen Groenendijk and Martin Stokhof, editors, *Formal methods in the study of language*, pages 277 – 322. Mathematical Center, 1981.

[14] Lauri Kartunen and Stanley Peters. Conventional implicature. In Choon-Kyu Oh and David A. Dinneen, editors, *Syntax and Semantics (Vol. 11,* Presupposition*)*, pages 1 – 56. Academic Press, 1979.

[15] Robin Lakoff. IF's, aNDs, and bUTs about conjunction. In C. J. fillmore and D. T. Langendoes, editors, *Studies in Linguistic Semantics.* Holt, Reinhart and Winston, New York, 1971.

[16] Sebastian Löbner. German *schon - erst - noch*: An integrated analysis. *Linguistics and Philosophy*, 12:167 – 212, 1989.

[17] Ingolf Max. A new formal tool: Functional variables representing asserions and presuppositions. In *International conference on computational linguistics (COLLING)*, volume 1, Budapest, Hungary, August 1988. John Von Neuman society for computing sciences.

[18] J.-J. Ch. Meyer and W. van der Hoek. A modal contrastive logic. In *Proc. of the 2nd Bar-Ilan symposium on foundations of artificial inteligence*, Bar-Ilan university, Tel-Aviv, 1991.

[19] Marga Reis. Further *and*'s and *but*'s about conjunction. *CLS*, 10:539 – 550, 1974. Chicago Linguistic Society.

[20] Pierre-Yves Schobbens. Inheritance with exceptions for software specification: on the meaning of 'but'. Technical report, Dept. of Computer science, Catholic university of Louvaine, 1989.

[21] Deirdre Wilson. *Presuppositions and non-truth-conditional semantics.* Academic Press, 1975.

[22] Yoad Winter and Mori Rimon. Contrast and implication in natural language. *Journal of semantics*, 11(4):365–406, 1994.

[23] Wlodek Zadrozny and Karen Jensen. Semantics of paragraphs. *Computational Linguistics*, 17(2):171–209, June 1991.