A HYBRID ACCESS METHOD IN MULTIPLE ACCESS CHANNEL

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Abstract

This paper considers a multiple-access communication channel with an infinite number of users. We show that if a controlled slotted Aloha protocol is used, then messages with variable length will have a negative impact on the average message delay. In order to alleviate this problem, a mixed mode (hybrid) access method is suggested under which the channel bandwidth is split into two sub-channels, managed under different policies. Messages whose length is less than or equal to a critical value are transmitted in one sub-channel under a slotted Aloha policy. The rest of the messages are sent through a separate portion of the channel bandwidth, using a reservation protocol. We show that under this hybrid access method the average delay of a message is greatly improved.

Keywords: Multiple-access channel, Slotted Aloha, Reservation protocol, Mixed-mode protocol, Hybrid channel, Message delay.

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1. INTRODUCTION

Consider a communication network consisting of a large number of users sharing a single packet-switched communication channel. From the large number of studies on multiple-access protocols, it has been found that random-access protocols (slotted Aloha and its variants) are efficient when the data traffic is low and bursty. On the other hand, reservation protocols (explicit or implicit) are efficient when the data traffic is high and steady.

In slotted Aloha (see e.g., [Ab], [Rob1], [Kil]), the time axis is divided into slots of length equal to the transmission time of a single fixed length packet. Users transmit their packets at any time they desire but start transmission at the beginning of a slot. If a conflict occurs (two or more users transmit during the same slot), conflicting users retransmit their packets according to some predetermined conflict resolution algorithm. It is well known that the classical slotted Aloha protocol is unstable for the infinite user model (see [FGL]), unless some auxiliary control on the retransmission probabilities is used. A simple first order recursive retransmission control policy was suggested in [HaVL], which achieves stability for an infinite user model with arrival rate up to \( e^{-1} \). Other control schemes have been also suggested in [Fe], [LaKi] and [KlYe].

In a reservation protocol, every user requests (explicitly or implicitly) the channel for its exclusive use for a period of time. The channel is then assigned to the users (centrally or distributively) according to a scheduling procedure (see e.g., [ToKi], [Bi], [Rob2]).

In the studies above, and others as well, the packets have been assumed to be generated independently according to a Poisson or a Bernoulli process. Under this assumption it is hardly surprising, that the arrival rate is the sole factor on which the trade-off between random access and reservation depends.

In practice, however, messages could be generated independently, but the packet inter-arrivals are usually dependent, e.g., the dependency introduced by long mes-
sages, such as files. Since long messages are more likely to clash under a random access protocol, it seems that they could deteriorate the channel's performance. This gives rise to the idea of using a different access method for long messages.

In this study, we are concerned with three basic questions regarding a network with variable length messages:

(i) **Do variable length messages increase the delay in a slotted Aloha channel?**
(ii) **How does the delay of a message change as a function of its length?**
(iii) **Which access should be used when the messages are of variable length?**

As we shall see below, variable length messages cause a significant increase in the average message delay. Also this effect grows linearly with the message length. Our solution for this problem is based on the usage of a mixed-mode (Hybrid) access protocol, i.e., one which uses two different channel access schemes.

Other mixed mode access protocols have been studied in the literature. In [BoFr], the access protocol switches from slotted Aloha mode to reservation mode whenever a collision occurs. It continues under the later scheme until the queue of reservations is cleared at which time it switches back to slotted Aloha. In [ScKl], a mixed protocol for radio network with small propagation delay is suggested. There, a large carrier sensing user "steals" slots which remain unused by the background of Aloha users. Another scheme which alternates between TDMA and the UBN ([KlYe]) is suggested in [SKP].

Both [BoFr] and [SKP] adapt the access mode to the traffic intensity, based on some backlog information, whereas in [ScKl], long messages are modeled by a single large user which is given a lower priority. In our approach, control over the access of a message to the channel depends on its length. We show that this protocol is substantially superior over slotted Aloha.

In Section 2 we formulate and analyze a controlled slotted Aloha protocol in a variable message length environment. In Section 3 we define and analyze a hybrid access method (mixed mode). Its delay versus throughput graphs are presented in Section 4.
2. SLOTTED ALOHA WITH VARIABLE LENGTH MESSAGES

In the first subsection we shall derive a recursive formula for the stationary distribution of the number of messages in the system, and in the second subsection we shall show that the average message delay increases linearly with its length.

2.1. The stationary distribution

Consider a controlled slotted Aloha channel with an infinite number of users and a renewal arrival process of messages. Let \( A(t) \) be the total number of messages arriving during slot \( t, t \geq 1 \). The random variables \( \{ A(t), t \geq 1 \} \) are independent and identically distributed.

Let \( \lambda \) be the expected value of \( A(t) \), and \( \alpha_k = P_k(A(t) = k), k \geq 0 \). We assume that a single user generates an infinitesimal rate of messages, thus at every moment in time its buffer contains at most one message.

Every message consists of a variable number of packets, \( L \), where \( L \) is a geometric random variable. That is,

\[
P_L(L = k) = \alpha(1-\alpha)^{k-1}, \quad k \geq 1, \ 0 < \alpha \leq 1.
\]

We assume that the length of the messages are mutually independent and also independent of the arrival process. The length of a time slot equals the transmission of a single packet, and messages are transmitted packet by packet. Therefore, every user may have an empty buffer, a new message or a partially transmitted message. Moreover, every message may have one packet which has been already involved in a collision.

At the beginning of slot \( t \), every user whose buffer is not empty, transmits the next packet (of its message) with probability \( p_t \), \( 0 < p_t < 1 \). \( p_t \) is a controlled transmission probability. A successful transmission occurs if and only if exactly one user attempts to transmit. If more than one user attempts to transmit in the same slot, a collision occurs, and all collided packets have to be retransmitted later on. Note that the same
transmission probability $p_t$ is used for new and collided packets.

A user whose buffer is not empty will be referred to as an active user. Note that in our model the number of active users equals (with probability one) the number of messages in the system.

Let $N(t)$ be the number of active users at the beginning of slot $t$. Since message lengths are geometrically distributed $(N(t), p_t)$, $t \geq 1$ is a Markov chain.

Further, let $d(t)$ be a 0, 1 valued random variable which assumes the value 1 if and only if a packet is successfully transmitted during slot $t$. From the geometric distribution of the message length, it is clear that $(N(t), p_t)$, $t \geq 1$, has the same stochastic evolution as the corresponding Markov chain in a system in which the message length is one, and messages return to the queue with probability $(1 - \alpha)$ after every successful transmission.

In the following we assume that $p_t = \frac{1}{N(t)}$. This transmission probabilities can not be implemented precisely in a distributed fashion. Nevertheless, they can approximate existing distributed controlled transmission probabilities. For example, the distributed controlled transmission which is suggested in [HaVL] yields $p_t \approx \frac{1}{N(t)}$. An alternative technique is to estimate $N(t)$, as it is shown in [KiYe] and [Se]. In [KiYe], $N(t)$ is estimated with good accuracy by including a single reservation mini-slot at the beginning of each data slot, while in [Se], an exact recursive equation is given for estimating $N(t)$ by observing the output of the channel.

In any event, the deterioration of the delay in a controlled slotted Aloha where $N(t)$ is estimated is more plausible than in our model, which assumes $p_t = \frac{1}{N(t)}$.

Since $p_t = \frac{1}{N(t)}$, $(N(t), t \geq 1)$ is a Markov chain whose evolution is

$$N(t+1) = [N(t) - d(t)]^+ + A(t), \quad (2.1)$$

where
\[ [N(t) - d(t)]^+ = \begin{cases} N(t) - 1 & \text{with prob. } \alpha N(t) p_1 (1 - p_i)^{N(t) - 1} \\ N(t) & \text{with prob. } 1 - \alpha N(t) p_1 (1 - p_i)^{N(t) - 1} \end{cases} \] (2.2)

Hence

\[ [N(t) - d(t)]^+ = \rho(t) - d(t) \text{ with probability one.} \]

Let \( N \) be the number of active users (= messages) under stationary condition at the beginning of a slot, i.e., \( N = \lim_{t \to \infty} N(t) \). Further, let \( \mu_k = P_r(N=k) \). From the stationary balance equations we obtain:

\[ \alpha \sum_{i=1}^{k+1} \frac{1}{i} (1 - \frac{1}{i})^{i-1} \mu_i a_{k+1-i} = \alpha \sum_{i=1}^{k} \frac{1}{i} (1 - \frac{1}{i})^{i-1} \mu_i a_{k+1-i} + \mu_k - \sum_{i=0}^{k} \mu_i a_{k-i}. \] (2.3)

Let

\[ f(k+1) = \alpha \sum_{i=1}^{k+1} \frac{1}{i} (1 - \frac{1}{i})^{i-1} \mu_i a_{k+1-i}. \]

Equation (2.3) provides us with a recursive relation for \( f(k) \):

\[ f(k+1) = \sum_{i=0}^{k} \mu_i P_r(A > k - i), \]

where \( A \) is a random variable with the same distribution as \( A(t) \).

Hence

\[ \mu_{k+1} = \frac{\sum_{i=0}^{k} \mu_i P_r(A > k - i) - \alpha (1 - \frac{1}{k})^{k-1} a_{k+1-i}}{\alpha a (1 - \frac{1}{k+1})^k} \] (2.4)

Equation (2.4) expresses the stationary distribution of the number of messages in the system in a simple recursive formula. Thus, \( \mu_k, k \geq 0 \), can be computed up to a constant multiplier, and then normalized.

Let \( \bar{N} \) be the expected number of messages in the system under stationary conditions (which is the same as the long-run average number of messages in the system). \( \bar{N} \) is computed by
\[ N = \sum_{k=0}^{\infty} k \mu_k, \]

and from Little's theorem the long-run average delay of a message (including transmission time), \( \bar{D} \) satisfies:

\[ \bar{D} = \frac{N}{\lambda} \quad (2.5) \]

In the sequel we add the system parameters \( A \) and \( \alpha \) to the notation of \( \bar{D} \) and \( N \), that is, \( \bar{D} = \bar{D}(A, \alpha) \) and \( N = N(A, \alpha) \).

In Figure 1 we illustrate the deterioration effect on the message delay. The delay is computed using equations (2.4) and (2.5).

### 2.2. The conditional average delay

In this subsection we show that the long-run average delay of a message consisting of \( k \) packets, \( \bar{D}_k \), is a linear function of \( k \).

Note that \( \bar{D}_{k+1} - \bar{D}_k \) is the expected time (under stationary conditions) that a message with more than \( k \) packets spends in the system from the end of its \( k \)-th successful transmission till the end of its \((k+1)\)-st "successful" transmission. Further, \( \lambda(1-\alpha)^k \) is the arrival rate of messages having more than \( k \) packets.

Let \( N_k \) be the expected number of messages in the system (under stationary conditions), whose transmitted packets are exactly \( k \). From Little's theorem

\[ \lambda(1-\alpha)^k (\bar{D}_{k+1} - \bar{D}_k) = N_k. \quad (2.6) \]

Since every message with \( k \) successfully transmitted packets has another packet with probability \( 1-\alpha \),

\[ N_{k+1} = N_k (1-\alpha). \]

Hence

\[ N_k = (1-\alpha)^k N_0, \quad k \geq 1, \]

and
\[ N = \sum_{k=0}^{\infty} N_k = \frac{\sum_{k=0}^{\infty} N_0(1-\alpha)^k}{\alpha} = \frac{N_0}{\alpha}. \]

Thus,

\[ N_k = \alpha(1-\alpha)^k N, \quad k \geq 0. \tag{2.7} \]

Substituting (2.7) into (2.6) and using (2.5) we obtain

\[ \bar{D}_{k+1} - \bar{D}_k = \alpha \bar{D}. \]

Therefore

\[ \bar{D}_k = \sum_{i=0}^{k+1} (\bar{D}_{i+1} - \bar{D}_i) = \alpha \bar{D} \cdot k. \tag{2.8} \]

As expected, we found that the expected delay of a message increases linearly with its length. This result suggests that messages whose length is greater than a critical value should not use the Aloha access method, but rather use a reservation access method instead. This is the key idea for the hybrid access method analyzed in the next section.

3. A HYBRID ACCESS METHOD

A simple method to prevent the deterioration of the average message delay due to long messages, is to introduce the convention that every successful transmission reserves the next slot for its user. Under this reservation protocol, the first packet of every message contends for a slot in the same fashion as in slotted Aloha. The rest of the packets of the message, are transmitted without contention. Note that under this convention, the last packet of every message reserves an additional slot which remains empty. (This can be prevented by adding additional information to every packet.). This idle slot, as well as other idle slots, act as a signal for the users to start contending for the first packet of their messages. It is clear that this reservation scheme can be distributively implemented in the same environment as the slotted Aloha. Moreover, it does not require any changes in the message structure. It seems that for long messages this reservation scheme could be quite efficient since the wasted slots are
negligible. The above properties suggest the following hybrid access method:

The channel bandwidth is split into two sub-channels; a proportion of 0 < φ* < 1 is used as a slotted Aloha sub-channel, whereas the remaining (1−φ*) portion of the total bandwidth acts as a reservation sub-channel. Furthermore, messages whose length L, is less than or equal to a critical value l*, are transmitted through the slotted Aloha sub-channel, while the rest of the messages go through the reservation sub-channel.

It is clear that the two sub-channels created by the constants φ* and l*, are independent. Their delay characteristics are the subject of the following sub-sections.

3.1. The slotted Aloha sub-channel

In this case, a time slot will be able to accommodate a packet, if its length is 1/φ* units. Assuming a stationary arrival process, the arrival rate during one slot for the slotted Aloha sub-channel is:

\[ \frac{\lambda}{\phi^*} P_r(L \leq l^*) = \frac{\lambda}{\phi^*} (1-(1-\alpha)^l^*) \]

Let \( A = A(l^*, \phi^*) \) be the number of messages arriving during one such slot as obtained from the distribution of the original arrival process. For example, if the original arrival process is Poisson with rate \( \lambda \), \( A^* \) has a Poisson distribution with rate \( \frac{\lambda}{\phi^*} (1-(1-\alpha)^l^*) \).

The probability distribution of the message length in the slotted Aloha sub-channel is given by:

\[ P_L(L = k \mid L \leq l^*) = \frac{\alpha (1-\alpha)^{k-1}}{1-(1-\alpha)^l^*}, \quad 1 \leq k \leq l^* \]

and its expected length is

\[ \frac{1}{\alpha} \frac{l^*(1-\alpha)^{l^*}}{1-(1-\alpha)^l^*} \]
Let $\alpha_n$ be the probability that a message which is still in the system and $n$ of its packets have been already transmitted, will leave the system after the $(n+1)$-st transmission.

$$\alpha_n = P_r(L = n+1 | n+1 \leq L \leq t^*) = \frac{\alpha}{1-(1-\alpha)^{t^*-n}}, \quad 0 \leq n \leq t^*-1. \quad (3.1)$$

Let $N^i(t)$ be the number of messages at the beginning of slot $t$, whose transmitted packets equals $i$, $0 \leq i \leq t^*-1$. $N^i(t)$ will be referred to as the number of messages in phase $i$. It is clear that $N(t) = (N^0(t), N^1(t), ..., N^{t^*-1}(t)), t > 1$, forms a Markov chain.

From the balance equations for $N(t)$ under stationary conditions, we can derive a recursive set of linear equations which determines the stationary distribution. An alternative approach, and rather less tedious one is bounding the delay from below and above.

Let

$$\overline{N}^i = \lim_{t \to \infty} E(N^i(t))$$

and

$$\overline{N} = \sum_{i=0}^{t^*-1} \overline{N}^i$$

Since $\overline{N}^{i+1} = \overline{N}^i (1-\alpha_i)$ we have $\overline{N}^{i+1} = \overline{N}^{(1)} \prod_{j=0}^{i} (1-\alpha_j)$ and,

$$\overline{N} = \overline{N}^{(1)} (1 + \sum_{i=0}^{t^*-2} \prod_{j=0}^{i} (1-\alpha_j)).$$

From Little's theorem

$$\overline{D} = \overline{D}^{(0)} \left[ 1 + \sum_{i=0}^{t^*-2} \prod_{j=0}^{i} (1-\alpha_j) \right], \quad (3.2)$$

where $\overline{D}$ ($\overline{D}^{(0)}$) is the expected delay of a packet (in phase 0) under stationary conditions.
3.1.1 A Lower bound to $D$

When the total number of messages in the system is $n$ and the number of messages in phase 0 is $n_0$, the probability that a message from phase 0 will move to the next phase or leave the system after the next slot is:

$$n_0 \frac{1}{n}(1 - \frac{1}{n})^{n-1} \leq n_0 \frac{1}{n}(1 - \frac{1}{n})^{n-1}. \quad (3.3)$$

This probability is always smaller than

$$\left(1 - \frac{1}{n_0}\right)^{n_0-1}. \quad (3.4)$$

By replacing the probabilities in (3.3) with the probabilities in (3.4) we obtain a Markov chain $(N^0(t), t \geq 1)$ which is stochastically smaller than $(N^0(t), t \geq 1)$. Moreover, $(N^0(t), t \geq 1)$ is a special case of the model in Section 2 with $A(t) \sim A^*$ and $\alpha = 1$. Thus

$$N^0 \leq \bar{N}^0 = \bar{N}(A^*, 1)$$

and from (3.2)

$$\bar{D} = \bar{D}(A^*, 1)\left[1 + \sum_{i=2}^{\infty} \prod_{j=0}^{i-2} (1 - \alpha_j)\right]. \quad (3.5)$$

where $\bar{D}(A^*, 1)$ is given in Section 2.

3.1.2 An upper bound to $\bar{D}$

From (3.1), $\alpha_j \geq \alpha_0$ for every $0 \leq j \leq t^* - 1$. Therefore, by replacing the probabilities $(\alpha_j, 0 \leq j \leq t^* - 1)$ with $(\alpha_0, \alpha_0, \ldots, \alpha_0)$ we obtain a Markov chain $(N^*(t), t \geq 1)$ which is stochastically greater than the process $\left\{\sum_{t=0}^{t^*-1} N(t), t \geq 1\right\}$. Moreover, $(N^*(t), t \geq 1)$ is a special case of the model in Section 2 with $A(t) \sim A^*$ and $\alpha = \alpha_0$. Thus

$$N^* \leq \bar{N}^* = \bar{N}(A^*, \alpha_0). \quad (3.6)$$

and
Note that for $l^* = 1$ the lower and the upper bounds are equal.

### 3.2. The reservation sub-channel

Here, the length of a time-slot is $\frac{1}{1-\varphi}$ and the arrival rate of messages is

$$\frac{\lambda}{1-\varphi} P_r(L > l^*) = \frac{\lambda(1-\varphi)}{1-\varphi}.$$<ref>

Let $\bar{A}^*$ be the number of messages arriving during one slot at the reservation sub-channel, and

$$\alpha_l^* = P_r(\bar{A}^* = i), \ i \geq 0.$$<ref>

Let $L^*$ be the length of a message arriving at the reservation sub-channel. The random variable $L^*$ is distributed as the conditional distribution of $L$ given $L > L^*$.

It is easy to verify that

$$E(L^*) = l^* + \frac{1}{\alpha}.$$<ref>

Recall that at the end of message transmission, an extra slot is wasted. Therefore, the delay of a message in our sub-channel has the same distribution as the delay of a similar reservation sub-channel, for which the message length is $L^* + 1$ and there are no extra wasted slots at the end of message transmissions. The following uses this analogy.

Let $P_{i,j+1}$ be the probability that a message which is currently present in the reservation sub-channel, and $j$ of its packets have been already transmitted, will leave the system after the $(j+1)$-st transmission. We have

$$P_{i,j+1} = \begin{cases} \alpha & \text{if } j > l^* + 1, \\ 0 & \text{if } j \leq l^* + 1. \end{cases}$$<ref>

Let $N(t)$ be the number of messages in the system at the beginning of slot $t$ and $\bar{S}(t)$ be the number of packets left in a message which is currently reserving the channel. If no user is reserving the channel, $\bar{S}(t) = 0$. The process $\{(N(t), \bar{S}(t)), \ t \geq 1\}$ is a Markov process.
kov chain.

Let \((\mu(k,j), 0 \leq k, l^*+2 \leq j)\) be the stationary probability distribution of 
\(\{(N(t), S(i)), t \geq 1\}\), i.e.,

\[
\mu(k,j) = \lim_{t \to \infty} P_r[(N(t),S(t)) = (k,j)].
\]

From the balance equations of the stationary probabilities we have

\[
\mu(k,0) = \mu(0,0)a^*_0 + \sum_{i=1}^{k} \mu(i,0)a^*_i(1 - (1 - \frac{1}{t})^{i-1}) + \sum_{i=1}^{k+1} \mu(i,0)a^*_{i-1}(1 - \frac{1}{t})^{i-1}p_{0,i} + \sum_{j=1}^{k+1} \sum_{i=1}^{k} \mu(i,j)a^*_{i-1}p_{j,i} + 1,
\]

\[
\mu(k,1) = \sum_{i=1}^{k} \mu(i,0)a^*_{i-1}(1 - p_{j-1,j})(1 - \frac{1}{t})^{j-1}.
\]

(3.9i)

(3.9ii)

From (3.7) and (3.9), we get

\[
\mu(k,j) = \begin{cases} 
\sum_{i=1}^{k} \mu(i,0)(1 - \frac{1}{t})^{i-1}a^*_i, & \text{for } j \leq l^*+1, \\
\sum_{i=1}^{k} \mu(i,0)(1 - \frac{1}{t})^{i-1}a^*_i(1 - a)^{j-t^*-1}, & \text{for } j > l^*+1.
\end{cases}
\]

(3.10)

Here, \(a^*_i = P_r(\tilde{A}^*U = k-i)\) is the \(j\)-convolution of \(\tilde{A}^*\).

By substituting (3.10) into (3.8) we obtain the following recursive formula for \(\mu(k+1,0)\):

\[
\mu(k+1,0) \left[ \alpha(1 - \frac{1}{k+1})^k \sum_{j=1}^{l^*} \frac{(1-\alpha)^{j-t^*-2}a^*_j}{a^*_0} \right] = \mu(k,0) - a^*_0\mu(0,0) - \sum_{i=1}^{k} \mu(i,0)a^*_{i-1}(1 - (1 - \frac{1}{t})^{i-1}) + (1 - \frac{1}{t})^{j-1} \sum_{j=1}^{k} \alpha(1-\alpha)^{j-t^*-2} a^*_j\sum_{j=1}^{k} a^*_j.
\]

(3.11)

Equations (3.10) and (3.11) express \(\mu(k,j)\) in a simple recursive formula. The expected number of messages and the expected message delay is readily obtained.
4. A GRAPHICAL PRESENTATION OF THE AVERAGE DELAY

In Figures 2 through 4 we present a comparison between the average message delays under the slotted Aloha and the Hybrid channel. The delays under both access methods are given in the same time units, and presented on a logarithmic scale as a function of the packet arrival rate, $\lambda/\alpha$. Each of the Figures compares the delays for a different value of $\frac{1}{\alpha}$ - the average message length (1.5, 2 and 3 respectively). For all the cases, $t^*=1$ and $\phi^*$ is taken to minimize the average delay in the Hybrid channel.

As can be seen from the Figures, the optimal $\phi^*$ hardly changes when $\alpha$ and $t^*$ are held constant. From calculations which are not presented here, it turns out that a larger $t^*$ does not improve the delay in our examples. Note, however, that for $\alpha = \frac{2}{3}$ the choice of $t^*=0$ (i.e., a slotted Aloha channel) yields smaller delays for values of $\lambda/\alpha$ up to .28. This is easily explained by the short messages and the extra wasted slot per message in the Hybrid channel on the one hand, and the small number of collisions in the slotted Aloha on the other hand.

There are two main trends that can be observed in Figures 2 through 4. The first one is that the rate of the improvement of the Hybrid channel increases with the arrival rate, when the average message length is held constant. The second one is that the rate of the improvement increases with the average message length, when the arrival rate is held constant.
Figure 1: Average delay (without transmission time) vs $\lambda/\alpha$ under slotted Aloha.
Figure 2: Average delay (without transmission time) vs $\lambda/\alpha$ under slotted Aloha and Hybrid.

- $a = 2/3$
- $\xi^* = 1$
- $\phi^* \approx 0.49$

$\phi^* = 0.48 \rightarrow \phi^* = 0.49 \rightarrow \phi^* = 0.50$
Figure 3: Average delay (without transmission time) vs $\lambda/\alpha$ under slotted Aloha and Hybrid.

$\alpha = \frac{1}{2}$

$\phi^* = 1$

$\phi^* \approx 0.34$

$\phi^* = 0.35 \longrightarrow \phi^* = 0.34 \longrightarrow \phi^* = 0.33$
Figure 4: Average delay (without transmission time) vs $\lambda/\alpha$ under slotted Aloha and Hybrid.

$\alpha = 1/3$

$\delta^* = 1$

$\phi^* \approx 0.21$
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