

# Memory Efficient All-Solutions SAT Solver and its Application for Reachability Analysis

Orna Grumberg, Assaf Schuster, and Avi Yadgar

Computer Science Department, Technion, Haifa, Israel.

**Abstract.** This work presents a *memory-efficient All-SAT engine* which, given a propositional formula over sets of *important* and *non-important* variables, returns the set of all the assignments to the *important* variables, which can be extended to solutions (satisfying assignments) to the formula. The engine is built using elements of modern SAT solvers, including a scheme for learning conflict clauses and non-chronological backtracking. Re-discovering solutions that were already found is avoided by the search algorithm itself, rather than by adding blocking clauses. As a result, the space requirements of a solved instance do not increase when solutions are found. Finding the next solution is as efficient as finding the first one, making it possible to solve instances for which the number of solutions is larger than the size of the main memory.

We show how to exploit our All-SAT engine for performing *image computation* and use it as a basic block in achieving full reachability which is purely SAT-based (no BDDs involved).

We implemented our All-SAT solver and reachability algorithm using the state-of-the-art SAT solver Chaff [19] as a code base. The results show that our new scheme significantly outperforms All-SAT algorithms that use blocking clauses, as measured by the execution time, the memory requirement, and the number of steps performed by the reachability analysis.

## 1 Introduction

This work presents a *memory-efficient All-SAT engine* which, given a propositional formula over sets of *important* and *non-important* variables, returns the set of all the assignments to the *important* variables, which can be extended to solutions (satisfying assignments) to the formula. The All-SAT problem has numerous applications in AI [21] and logic minimization [22]. Moreover, many applications require the ability to instantiate all the solutions of a formula, which differ in the assignment to only a subset of the variables. In [14] such a procedure is used for predicate abstraction. In [7] it is used for re-parameterization in symbolic simulation. In [18, 6] it is used for reachability analysis, and in [13] it is used for pre-image computation. Also, solving QBF is actually solving such a problem, as shown in [15].

Most modern SAT solvers implement the DPLL[9, 8] backtrack search. These solvers add clauses to the formula in order to block searching in subspaces that are known to contain no solution. All-SAT engines that are built on top of modern SAT solvers tend to extend this method by using additional clauses, called *blocking clauses*, to block solutions that were already found [18, 6, 13, 14, 7, 20]. However, while the addition of blocking clauses prevents repetitions in solution creation, it also significantly inflates

the size of the solved formula. Thus, the engine slows down corresponding to the number of solutions that were already found. Eventually, if too many solutions exist, the engine may saturate the available memory and come to a stop.

In [6] an optimization is employed to the above method. The number of blocking clauses and the run time are reduced significantly by inferring from a newly found solution a set of related solutions, and blocking them all with a single clause. This, however, is insufficient when larger instances are considered. Moreover, the optimization is applicable only for formulae which originated from a model.

In this work we propose an efficient All-SAT engine which does not use blocking clauses. Given a propositional formula and sets of important and non-important variables, our engine returns the set of all the assignments to the important variables, which can be extended to solutions to the formula. Setting the non-important variables set to be empty, yields all the solutions to the formula. Similar to previous works, our All-SAT solver is also built on top of a SAT solver. However, in order to block known solutions, it manipulates the backtracking scheme and the representation of the implication graph. As a result, the size of the solved formula does not increase when solutions are found. Moreover, since found solutions are not needed in the solver, they can be stored in external memory (disk or the memory of another computer), processed and even deleted. This saving in memory is a great advantage and enables us to handle very large instances with huge number of solutions. The memory reduction also implies time speedup, since the solver handles much less clauses. In spite of the changes we impose on backtracking and the implication graph, we manage to apply many of the operations that made modern SAT solvers so efficient. We derive conflict clauses based on conflict analysis, apply non-chronological backtracking to skip subspaces which contain no solutions, and apply conflict driven backtracking under some restrictions.

We show how to exploit our All-SAT engine for reachability analysis, which is an important component of model checking. Reachability analysis is often used as a preprocessing step before checking. Moreover, model checking of most safety temporal properties can be reduced to reachability analysis [1]. BDD-based algorithms for reachability are efficient when the BDDs representing the transition relation and the set of model states can be stored in memory [4, 5]. However, BDDs are quite unpredictable and tend to explode on intermediate results of image computation. When using BDDs, a great effort is invested in finding the optimal variables order. SAT-based algorithms, on the other hand, can handle models with larger number of variables. However, they are mainly used for Bounded Model Checking (BMC) [2].

Pure SAT-based methods for reachability [18, 6] and model checking of safety properties [13, 20] are based on All-SAT engines, which return the set of all the solutions to a given formula. The All-SAT engine receives as input a propositional formula describing the application of a transition relation  $T$  to a set of states  $S$ . The resulting set of solutions represents the image of  $S$  (the set of all successors for states in  $S$ ). Repeating this step, starting from the initial states, results in the set of all reachable states.

Similar to [18, 6], we exploit our All-SAT procedure for computing an image for a set of states, and then use it iteratively for obtaining full reachability. Several optimizations are applied at that stage. Their goals are to reduce the number of found solutions

by avoiding repetitions between images; to hold the found solutions compactly; and to keep the solved formula small.

An important observation is that for image computation, the solved formula is defined over variables describing current states  $\bar{x}$ , inputs  $\bar{I}$ , next states  $\bar{x}'$ , and some auxiliary variables that are added while transforming the formula to CNF. However, many solutions to the formula are not needed: the only useful ones are those which give different values to  $\bar{x}'$ . This set of solutions is efficiently instantiated by our algorithm by defining  $\bar{x}'$  as the important variables. Since the variables in  $\bar{x}'$  typically constitute just 10% of all the variables in the formula [23], the number of solutions we search for, produce, and store, is reduced dramatically. This was also done in [18, 6, 13, 20].

Other works also applied optimizations within a single image [6] and between images [18, 6]. These have similar strength to the optimization we apply between images. However, *within* an image computation we gain significant reductions in memory and time due to our new All-SAT procedure, and the ability to process the solutions outside the engine before the completion of the search. This gain is extended to the reachability computation as well, as demonstrated by our experimental results.

In [11], a hybrid of SAT and BDD methods is proposed for image computation. This implementation of an All-SAT solver does not use blocking clauses. The known solutions are kept in a BDD which is used to restrict the search space of the All-SAT engine. While this representation might be more compact than blocking clauses, the All-SAT engine still depends on the set of known solutions when searching for new ones. Moreover, since our algorithm does not impose restrictions on learning, we believe it can be used to enhance the performance of such hybrid methods as well.

In [3, 12] all the solutions of a given propositional formula are found by repeatedly choosing a value to one variable, and splitting the formula accordingly, until all the clauses are satisfied. However, in this method, all the solutions of the formula are found, while many of them represent the same next state. Therefore, it can not be efficiently applied for quantifier elimination and image computation.

We have built an All-SAT solver based on the state-of-the-art SAT solver Chaff [19]. Experimental results show that our All-SAT algorithm outperforms All-SAT algorithms based on blocking clauses. Even when discovering a huge number of solutions, our solver does not run out of memory, and does not slow down. Similarly, our All-SAT reachability algorithm also achieves significant speedups over blocking clauses-based All-SAT reachability, and succeeds to perform more image steps.

The rest of the paper is organized as follows. Section 2 gives the background needed for this work. Sections 3 and 4 describe our algorithm and its implementation. Section 5 describes the utilization of our algorithm for reachability analysis. Section 6 shows our experimental results, and Section 7 includes conclusions.

## 2 Background

### 2.1 The SAT Problem

The *Boolean satisfiability problem* (SAT) is the problem of finding an assignment  $A$  to a set of Boolean variables  $V$  such that a Boolean formula  $\phi(V)$  will have the value 'true' under this assignment.  $A$  is called a *satisfying assignment*, or a *solution*, for  $\phi$ .

We shall discuss formulae presented in the conjunctive normal form (CNF). That is,  $\phi$  is a conjunction of clauses, while each clause is a disjunction of literals over  $V$ . A literal  $l$  is an instance of a variable or its negation:  $l \in \{v, \neg v \mid v \in V\}$ . We shall consider a clause as a set of literals, and a formula as a set of clauses.

A clause  $cl$  is satisfied under an assignment  $A$  iff  $\exists l \in cl, A(l) = true$ . For a formula  $\phi$  given in CNF, an assignment satisfies  $\phi$  iff it satisfies all of its clauses. Hence, if, under an assignment  $A$  (or a partial assignment), all of the literals of some clause in  $\phi$  are false, then  $A$  does not satisfy  $\phi$ . We call this situation a *conflict*.

## 2.2 Davis-Putnam-Logemann-Loveland Backtrack Search (DPLL)

We begin by describing the *Boolean Constraint Propagation* ( $bcp()$ ) procedure. Given a partial assignment  $A$  and a clause  $cl$ , if there is one literal  $l \in cl$  with no value, while the rest of the literals are all false, then in order to avoid a conflict,  $A$  must be extended such that  $A(l) = true$ .  $cl$  is called a *unit clause* or an *asserting clause*, and the assignment to  $l$  is called an *implication*. The  $bcp()$  procedure finds all the implications at a given moment. This procedure is efficiently implemented in [19, 10, 26, 17, 16].

The DPLL algorithm [9, 8] walks the binary tree that describes the variables space. At each step, a *decision* is made. That is, a value to one of the variables is chosen, thus reaching deeper in the tree. Each decision is assigned with a new *decision level*. After a decision is made, the algorithm uses the  $bcp()$  procedure to compute all its implications. All the implications are assigned with the corresponding decision level. If a conflict is reached, the algorithm backtracks in the tree, and chooses a new value to the most recent variable not yet tried both ways. The algorithm terminates if one of the leaves is reached with no conflict, describing a satisfying assignment, or if the whole tree was searched and no satisfying assignment was found, meaning  $\phi$  is unsatisfiable.

## 2.3 Optimizations

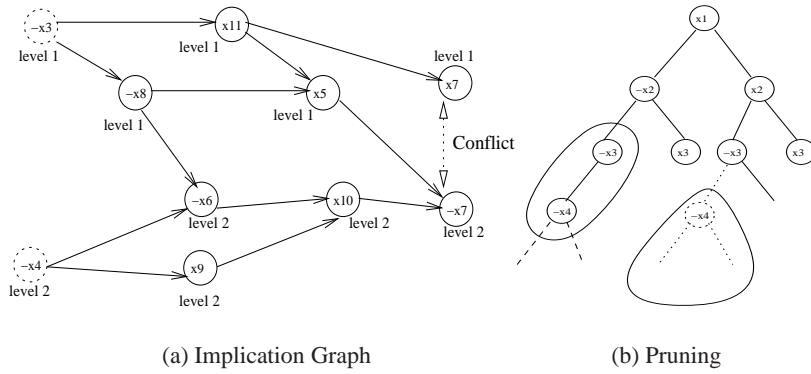
Current state of the art SAT solvers use Conflict Analysis, Learning, and *Conflict Driven Backtracking* to optimize the DPLL algorithm [27, 17]. Upon an occurrence of a conflict, the solver creates a *conflict clause* which implies the reverse assignment to some variable in the highest level. This clause is added to the formula to prune the search tree [27, 17]. In order to emphasize the recently gained knowledge, the solver uses *conflict driven backtracking*: Let  $l$  be the highest level of an assigned variable in the conflict clause. The solver discards some of its work by invalidating all the assignments above  $l$ . The implication of the conflict clause is then added to  $l$  [27]. This implies a new order of the variables in the search tree. Note that the other variables for which the assignments were invalidated, may still assume their original values, and lead to a solution.

Next we describe the *implication graph*, which is used to create conflict clauses. An *Implication Graph* represents the current partial assignment during the solving process, and the reason for the assignment to each variable. For a given assignment, the implication graph is not unique, and depends on the decisions and the order used by the  $bcp()$ . We denote the asserting clause that implied the value of  $l$  by  $ante(l)$ , and refer to it as the antecedent of  $l$ . If  $l$  is a decision,  $ante(l) = NULL$ . Given a partial assignment  $A$ :

- The implication graph is a directed acyclic graph  $G(L, E)$ .

- The vertices are the literals of the current partial assignment:  $\forall l \in L, l \in \{v, \neg v \mid v \in V \wedge A(l) = true\}$ .
- The edges are the reasons for the assignments:  $E = \{(l_i, l_j) \mid l_i, l_j \in L, l_i \in ante(l_j)\}$ . That is, for each vertex  $l$ , the incident edges represent the clause  $ante(l)$ . A decision vertex has no incident edge.
- Each vertex is also assigned with the decision level of the corresponding variable.

When a conflict occurs, there are both true and false vertices of some variable, denoted *conflicting variable*. A *Unique Implication Point* (UIP) in an implication graph is a vertex of the current decision level, through which all the paths from the decision vertex of this level to the conflict pass. There may be more than one UIP, and we order them starting from the conflict. The decision variable of a level is always a UIP. When conflict driven backtracking is used, the conflict analysis creates a conflict clause which is a unit clause. This clause implies the reverse assignment to one of the UIPs. After backtracking, the opposite value of the UIP becomes an implication, and the conflict clause is its antecedent [27]. Figure 1(a,b) shows an implication graph with a conflict and two UIPs, and the pruning of the search tree by a conflict clause.



**Fig. 1.** (a) An implication graph describing a conflict. The roots are decisions.  $\neg x_4$  and  $x_{10}$  are UIPs. (b) Adding the clause  $(x_3, x_4)$  prunes the search tree of the subspace defined by  $\neg x_3 \wedge \neg x_4$ .

## 2.4 The All-SAT Problem

Given a Boolean formula presented in CNF, we would like to find all of its solutions as defined in the SAT problem.

**The Blocking Clauses Method:** A straight forward method to find all of the formula's solutions is to modify the DPLL SAT solving algorithm such that when a solution is found, a blocking clause describing its negation is added to the solver, thus preventing the solver from reaching the same solution again. The last decision is then invalidated, and the search is continued normally. Once all of the solutions are found, there will be no satisfying assignment to the current formula, and the algorithm will terminate.

This algorithm suffers from exponential space growth as it adds a clause at the size of  $V$  for each solution found. Another problem is that the increasing number of

clauses in the system will slow down the *bcp()* procedure which will have to look for implications in an increasing number of clauses.

### 3 Memory efficient All-SAT Algorithm

#### 3.1 Conventions

Assume we are given a partition of the variables to important and non-important variables. Two solutions to the problem which differ in the non-important variables only are considered the same solution. Thus, a solution is defined by a subset of the variables.

A *subspace* of assignments, defined by a partial assignment  $\sigma$ , is the set of all assignments which agree with  $\sigma$  on its assigned variables. A subspace is called *exhausted* if all of the satisfying assignments in it (if any) were already found. At any given moment, the current partial assignment defines the subspace which is now being investigated.

We now present our All-SAT algorithm. A proof of its correctness is given in [25].

#### 3.2 The All-SAT Algorithm

Our algorithm walks the search tree of the important variables. We call this tree the *important space*. Each leaf of this tree represents an assignment to the important variables which does not conflict with the formula. When reaching such a leaf, the algorithm tries to extend the assignment over the non-important variables. Hence, it looks for a solution within the subspace defined by the important variables. Note that the walk over the *important space* should be exhaustive, while only one solution for the non-important variables should be found. This search is illustrated in Figure 2(a).

We incorporate these two behaviors into one procedure by modifying the decision and backtracking procedures of a conflict driven backtrack search, and the representation of the implication graph.

**Important First Decision Procedure:** We create a new decision procedure which looks for an unassigned variable from within the important variables. If no such variable is found, the usual decision procedure is used to choose a variable from the non-important set. This way, at any given time, the decision variables are a sequence of important variables, followed by a sequence of non important variables. At any given time, the *important decision level* is the maximal level in which an important variable is a decision. An example is given in Figure 2(a).

**Exhaustive Walk of the Important Space:** An exhaustive walk of the *important space* is performed by extending the original DPLL algorithm with the following procedures: Chronological backtracking after a leaf of the *important space* is handled; Learning a conflict clause and chronological backtracking upon an occurrence of a conflict; Non-chronological backtracking when a subspace of the problem is proved to be *exhausted*.

Chronological backtracking, as in DPLL, is done by flipping the highest important decision variable not yet flipped. This means that under the previous decisions, the last decision variable *must* assume the reverse assignment. Hence, its new value is an implication of the previous decisions. Therefore, in order to perform a chronological backtrack, we flip the highest decision and assign it with the level below. This way, the highest decision is always the highest decision not yet flipped. Higher decisions which were already flipped are regarded as its implications. Note, though, that there is





*lit* and  $\neg lit$  respectively. Therefore, by this non-chronological backtracking, we skip a large *exhausted* subspace.

**Assigning Non-Important Variables:** After reaching a leaf in the *important space*, we have to extend the assignment to the non important variables. At this point, all the important variables are assigned with some value. Note, that since we only need one extension, we actually have to solve the SAT problem for the given formula with the current partial assignment. This is done by allowing the normal work of the optimized SAT algorithm, including decisions, conflict clause learning, and conflict driven backtracking. However, we allow backtracking down to the *important decision level* but not below it, in order not to change the assignment to the important variables. If no solution is found, the current assignment to the important variables can not be extended to a solution for the formula, and should be discarded. On the other hand, if a solution to the formula is found, its projection over the important variables is a valid output. In both cases, we backtrack to the *important decision level* and continue the exhaustive walk of the *important space*.

## 4 Implementation

We implemented our All-SAT engine using zChaff [19] as a base code. This SAT solver uses the VSIDS decision heuristic [19], an efficient *bcp()* procedure, conflict clause learning, and conflict driven backtracking. We modified the decision procedure to match our important-first decision procedure, and added a mechanism for chronological backtracking. We implemented the exhaustive walk over the *important space* using chronological backtracking, and by allowing non-chronological backtracking when subspaces without solutions are detected. We used the original optimized SAT solving procedures above the *important decision level*, where it had to solve a SAT problem. Next, we describe the modifications imposed on the solver in the *important space*.

The original SAT engine represents the current implication graph by means of an *assignment stack*, hereafter referred to as *stack*. The *stack* consists of levels of assignments. The first assignment in each level is the decision, and the rest of the assignments in the level are its implications. Each implication is in the lowest level where it is implied. Thus, an implication is implied by some of the assignments prior to it, out of which at least one is of the same level. For each assigned variable, the *stack* holds its value and its antecedent, where the antecedent of a decision variable is NULL.

In the following discussion, *backtrack to level  $i$*  refers to the following procedure: a) Flipping the decision in level  $i + 1$ . b) Invalidation of all the assignments in levels  $i + 1$  and above, by popping them out of the *stack*. c) Pushing the flipped decision at the end of level  $i$ , with NULL antecedent. d) Executing *bcp()* to calculate the implications of the flipped decision.

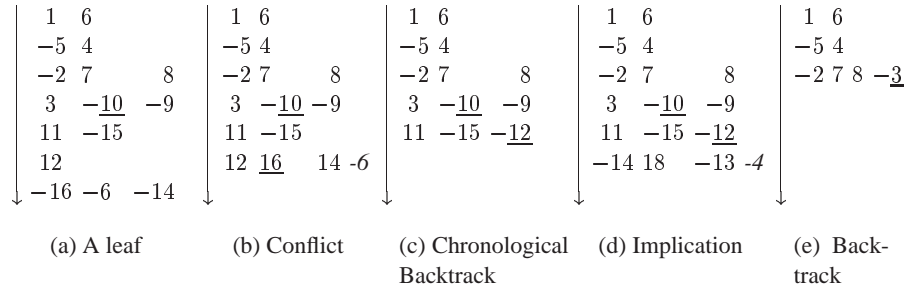
We perform *chronological backtracking* from level  $j$  within the *important space* by backtracking to level  $j - 1$ . This way, the flipped decisions appear as implications of prior ones, and the highest decision not yet flipped is always the highest in the *stack*. The assignments with NULL antecedents, which represent *exhausted* subspaces, remain in the *stack* until a whole subspace which includes them is *exhausted*. An example for this procedure is given in Figure 3(a,b).



Our *stack* now includes decision variables, implications, and flipped decisions, which appear as implications but with no antecedents. Using this *stack* we can construct the implication graph according to the new definition given in section 3.2. Thus we can derive a conflict clause whenever a conflict occurs.

Decisions about *exhausted* subspaces are made during the process of resolving a conflict, as described next. We define a *generated clauses stack*, which is used to temporarily store clauses that are generated during conflict analysis<sup>1</sup>. In the following explanation, refer to Figure 3(b-e). (b) When a conflict occurs, we analyze it and learn a new conflict clause according to the 1UIP scheme [27]. This clause is added to the solver to prune the search tree, and is also pushed into the *generated clauses stack* to be used later. (c) We perform a chronological backtrack. A pseudo code for this procedure is shown in Figure 4('chronological backtrack'). If this caused another conflict, we start the resolving process again. Otherwise, we start calculating the implications of the clauses in the *generated clauses stack*.

(d) Let  $c_1$  be a clause popped out of the *generated clauses stack*. If  $c_1$  is not asserting, we ignore it. If it implies *lit*, we push *lit* into the *stack* and run *bcp()*. For simplicity of presentation, *lit* is pushed to a new level in the *stack*. If a new conflict is found, we follow the procedure described in Section 3.2 to decide about *exhausted* subspaces. We create a new conflict clause,  $c_2$ , with *lit* as the UIP.  $c_1$  and  $c_2$  imply *lit* and  $\neg lit$  respectively. We calculate  $c_3 \leftarrow resolution(c_1, c_2)$  [9]. (e) We backtrack to the level preceding the highest level of an assigned variable in  $c_3$ . Thus, we backtrack higher, skipping a large *exhausted* subspace. A pseudo code of this procedure is given in Figure 4.



**Fig. 3.** *Important Space* Resolve Conflict (a) Reaching a leaf of the *important space*. '-10' has NULL antecedent (b) Chronological backtrack which causes a conflict. '16' has NULL antecedent, clause  $c_1 = (-1, 2, -14)$  is generated. (c) Chronological backtrack. (d) '-14', the implication of clause  $c_1$ , is pushed into the *stack*, and leads to a conflict. Clause  $c_2 = (-4, 2, -3, 14)$  is generated.  $c_3 \leftarrow resolution(c_1, c_2) = (-1, 2, -3, -4)$ . (e) Backtracking to the highest level in  $c_3$ .

<sup>1</sup> We still refer to the decision stack as *stack*.

```

Important_Space_resolve_conflict() {
  while (conflict ∨ generated_clauses.size > 0) {
    if (conflict) {
      chronological {
      backtrack {
        if (current level == 0)
          return FALSE
        cl ← generate conflict clause with UIP
        generated_clauses.push(cl)
        backtrack one level
      }
    } else {
      Implicate {
      Conflict {
      Clauses {
        cl1 ← generated_clauses.pop()
        if (cl1 is asserting) {
          lit ← unit literal in cl1
          push lit into the stack
          if (bcp() = CONFLICT) {
            cl2 ← generate conflict clause with lit as UIP
            cl3 ← resolution of (cl1, cl2)
            generated_clauses.push(cl3)
            backtrack to the level preceding the highest level in cl3
          }
        }
      }
    }
  }
  return TRUE
}

```

Fig. 4. *Important Space* resolve conflict

## 5 Reachability and Model Checking Using All Solution Solver

### 5.1 Reachability

We now present one application of our All-SAT algorithm. Given the set of initial states of a model, and the transition relation, we would like to find the set of all the states reachable from the initial states. We denote by  $\bar{x}$  the vector of the model state variables, and by  $S_i(\bar{x})$  the set of states at distance  $i$  from the initial states. The transition relation is given by  $T(\bar{x}, \bar{I}, \bar{x}')$  where  $\bar{x}$  represents the current state,  $\bar{I}$  represents the input and  $\bar{x}'$  represents the next state. For a given set of states  $S(\bar{x})$ , the set of reachable states from them at distance 1 (i.e., their successors), denoted  $Image(S(\bar{x}))$ , is

$$Image(S(\bar{x})) = \{\bar{x}' \mid \exists \bar{x}, \exists \bar{I}, S(\bar{x}) \wedge T(\bar{x}, \bar{I}, \bar{x}')\} \quad (1)$$

Given  $S_0$ , the set of initial states, calculating the reachable states is done by iteratively calculating  $S_i$ , and adding them to the reachable set  $S^*$ , until  $S_i$  contains no new states. The reachability algorithm is shown in Figure 5(a).

### 5.2 Image Computation Using All-SAT

We would like to use our All-SAT algorithm in order to implement line 5 in the reachability algorithm (Figure 5(a)). In order to do that, we have to find all of the solutions

for the following formula:

$$S(\bar{x}) \wedge T(\bar{x}, \bar{I}, \bar{x}') \wedge \neg S^*(\bar{x}') \quad (2)$$

Each solution to (2) represents a valid transition from one of the states in the set  $S(\bar{x})$  to a new state  $\bar{x}'$ , which is not in  $S^*(\bar{x}')$ . Including  $\neg S^*(\bar{x}')$  in the image computation was also done in [6, 18].

We now have to construct a CNF representation for Formula 2. We represent each state  $\bar{x}$  as the conjunction of its literals. Therefore,  $S^*(\bar{x}')$  is in DNF, and  $\neg S^*(\bar{x}')$  is in CNF. Creating the CNF representation for  $S$  is done by introducing auxiliary variables. The translations of both sets are linear in the size of the sets. Representing  $T$  as a CNF is also possible by introducing auxiliary variables [2].

Each solution is an assignment to all of the variables in the CNF, and its projection over  $\bar{x}'$  defines the new state. We avoid repetitive finding of the same  $\bar{x}'$  by setting  $\bar{x}'$  to be the important variables in our All-SAT solver. A state is found once, regardless of its predecessors, input, or the assignment to the auxiliary variables added during the CNF construction. Making the decisions from within the model variables proved to be efficient when solving SAT problems for BMC [23].

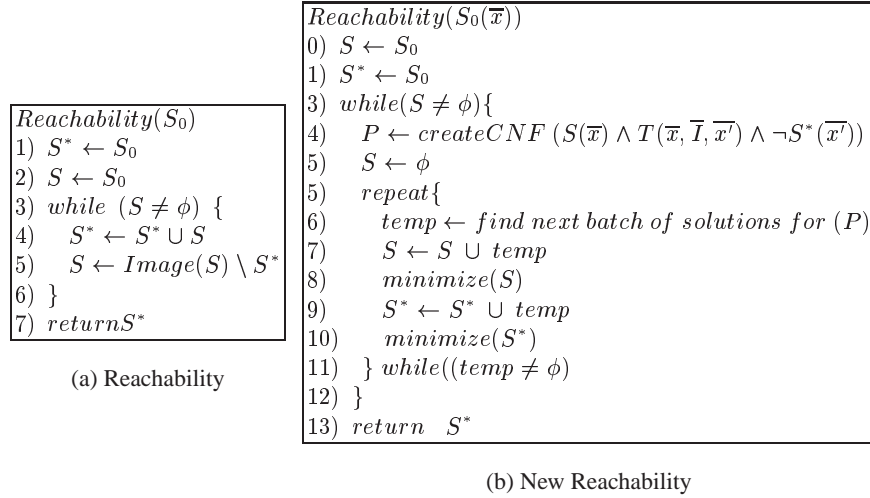
### 5.3 Minimization

**Boolean minimization:** A major drawback of the implementation described above is the growth of  $S^*$  between iterations of the algorithm, and of  $S$  during the image computation. This poses a problem even when the solutions are held outside the solver. Representing each state by a conjunction of literals is pricey when their number increases. Therefore we need a way to minimize this representation. We exploit the fact that a set of states is actually a DNF formula and apply Boolean minimization methods to find a minimal representation for this formula. For example, the solutions represented by  $(x_0 \wedge x_1) \vee (x_0 \wedge \neg x_1)$ , can be represented by  $(x_0)$ .

In our tool, we use Berkeley's 'Espresso'[24], which receives as an input a set of DNF clauses and returns their minimized DNF representation. Our experimental results show a reduction of up to 3 orders of magnitude in the number of clauses required to represent the sets of solutions when using this tool.

**On-The-Fly Minimization:** Finding all the solutions for Formula (2), and then minimizing them, is not feasible for large problems since their number would be too large to store and for the minimizer to handle. Therefore, during the solving process, whenever a preset number of solutions is found, the solver work is suspended, and we use the minimizer to combine them with the current  $S$  and  $S^*$ . The output, then, is stored again in  $S$  and  $S^*$  respectively, to be processed with the next batch of solutions found by the All-SAT solver. This way, we keep  $S^*$ ,  $S$  and the input to the minimizer small. Note, also, that each batch of solutions includes new states only, due to the structure of Formula 2. Employing logical minimization on-the-fly, before finding all the solutions, is possible since previous solutions are not required when searching for the next ones, and the work of the minimizer is independent of the All-SAT solver. Moreover, the minimization and the computation of the next batch of solutions can be performed concurrently. However, this is not implemented in our tool. Note, that this minimization does not effect the performance of the All-SAT solver.

We now have a slightly modified reachability algorithm as shown in Figure 5(b).



**Fig. 5.** Reachability Algorithms. (a) Regular reachability algorithm. (b) Reachability using out All-SAT solver. Lines 5-11 are the implementation of the on-the-fly minimization.

## 6 Experimental Results

As the code base for our work, we use the zChaff SAT solver [19] which is one of the fastest available state of the art solvers. zChaff implements conflict analysis with conflict driven backtrack, and uses efficient data structures. Since zChaff is open source, we were able to modify the code according to our method.

For comparison, we used the same code base to implement the blocking clauses method. In order to avoid repetition of the same assignments to the important variables, we constructed the blocking clauses from the important variables only. We improved the blocking clauses by using the decision method described in Section 3.2. This way, the blocking clauses can be constructed from only the subset of the important variables which are decision variables, since the rest of the assignments are implied by them. This improvement reduced space requirements as well as solution times of the blocking clauses method substantially.

All experiments use dedicated computers with 1.7Ghz Intel Xeon cpu and 1GB RAM, running Linux. The problem instances are from the ISCAS'89 benchmark.

### 6.1 All-SAT Solver

Figure 6 shows the performance of our new All-SAT solver. The problems are CNF representations of the transition relations of the models in the benchmark. In cases where a model name is followed by '\*', the instance consists of multiple transitions and initial conditions of a model. The important variables were arbitrarily chosen.

The table clearly shows a significant speedup for all problems that the blocking clauses method could solve. Smaller number of clauses shortens the time of the *bcp()* procedure, and also allows more work to be performed in higher levels of the memory hierarchy (main memory and cache). The speedup increases with the hardness of the problem and the computation time.

name	Clauses	Vars	Sol	T1 (s)	T2 (s)	S.U.	name	Clauses	Vars	Solutions	T1 (s)	T2(s)	S.U.
s510	964	280	64	0.00	0.00	1.00	s641	10064	2338	224	0.03	0.16	5.81
s1488	983	286	64	0.00	0.00	1.00	s953	1279	271	1188	0.07	0.24	3.52
s1494	19153	4919	12	0.00	0.00	1.00	s1238	49699	13476	80	0.06	0.48	8.39
s15850	166	48	2	0.00	0.00	1.00	s9234.1	239	77	640	0.35	0.55	1.57
s208.1	47	22	11	0.00	0.00	1.00	s967	959	277	6504	0.14	0.59	4.35
s23	303	97	5	0.00	0.00	1.00	s38584	1302	299	2272	0.13	0.81	6.31
s298	437	137	8	0.00	0.00	1.00	s1423	1884	560	28590	0.45	29.9	66.44
s382	462	140	8	0.00	0.00	1.00	s1269	2191	679	32768	0.82	50	60.75
s400	462	96	2	0.00	0.00	1.00	s13207	18993	3890	24576	4.40	459	104.39
s420.1	498	153	5	0.00	0.00	1.00	s3271	4426	1349	6.70E+07	1191	M.O.	-
s444	1620	129	2	0.00	0.00	1.00	s9234.1	10007	2317	3.50E+07	3411	M.O.	-
s499	561	161	5	0.00	0.00	1.00	s1512	2320	657	1.30E+08	4601.5	M.O.	-
s526	1528	192	3	0.00	0.00	1.00	s3330	2496	775	1.50E+08	4891	M.O.	-
s635	1438	192	2	0.00	0.00	1.00	s38417	48783	13261	8.30E+06	11379	M.O.	-
s838.1	1406	191	2	0.00	0.00	1.00	s5378	1072	4031	5.00E+08	26493	M.O.	-
s938	558	160	5	0.00	0.00	1.00	s635*	1496	192	> 4E+09	107644	M.O.	-
s526n	479	138	36	0.00	0.00	0.50	s6669	11963	3639	> 4E+09	172800	M.O.	-
s208.1*	160	50	512	0.01	0.01	1.00	s13207.1	18774	3847	> 1E+09	T.O.	M.O.	-
s713	158	48	512	0.01	0.02	2.20	s15850.1	18957	4850	> 1.2E+09	T.O.	M.O.	-
s208.1*	190	61	512	0.01	0.02	2.67	s3384	5073	1784	> 4E+09	T.O.	M.O.	-
s832	454	136	183	0.01	0.02	1.29	s35932	55173	20155	> 7.5E+07	T.O.	M.O.	-
s820	404	129	344	0.01	0.03	3.11	s38584.1	49735	13449	> 2E+08	T.O.	M.O.	-
s208.1*	585	161	480	0.03	0.05	1.77	s4863	10470	3001	> 1.3E+09	T.O.	M.O.	-
s208.1*	610	167	960	0.06	0.10	1.64	s991	1635	560	> 5.5E+08	T.O.	M.O.	-

**Fig. 6.** All-SAT solving time. Vars: The number of variables in the problem. About half the variables are important. Sol: The number of solutions found. T1: The time required for our new All-SAT solver, T2: The time required for the blocking clauses-based algorithm. M.O.: Memory Out. S.U.: Speedup - the ratio T2/T1. The timeout was set to 48 hours.

Our solver is capable of solving larger instances, for which the blocking clauses method runs out of memory. The number of solutions for these instances is simply too high to store in main memory as clauses. In contrast, using our new method, the solutions can be stored on the disk or in the memory of neighboring machines.

The last seven rows in the table show instances for which our solver timed out. In none of these cases, despite the huge number of solutions found (much larger than the size of the main memory in the machine employed), did the solver run out of memory.

## 6.2 Reachability

Figure 7 shows the performance of our reachability analysis tool, calculating reachability for the benchmark models. Since [6] and [18] are the only reachability analysis algorithms that, as far as we know, depend solely on SAT procedures, the table shows a comparison with the performance reported in [6]. Figure 8 shows the instances for which the reachability analysis did not complete. Here, again, a comparison to [6] is shown. The tables show significant speedup for the completed problems, and deeper steps for those not completed.

## 7 Conclusions

In this work we presented an All-SAT engine which efficiently finds all the assignments to a subset of the variables, which can be extended to solutions to a given propositional

Model	# FLOPS	# steps	# states	T1 (sec)	T2 (sec)	Speedup
s386	6	8	13	0.1	0.21	2.10
s298	14	19	218	0.2	0.33	1.65
s832	5	11	25	0.1	0.47	4.70
s510	6	47	47	0.1	0.47	4.70
s820	5	11	25	0.2	0.48	2.40
s208.1	8	256	256	0.1	0.56	5.60
s1488	6	22	48	0.3	0.87	2.90
s1494	6	22	48	0.1	0.87	8.70
s499	22	22	22	0.3	1.74	5.80
s953	29	10	504	0.1	2.01	20.10
s641	19	7	1544	0.2	2.24	11.20
s713	19	7	1544	1	2.53	2.53
s967	29	10	549	0.2	3.12	15.60
s1196	18	3	2615	0.3	6.79	22.63
s1238	18	3	2615	0.2	7.26	36.30
s382	21	151	8865	4	7.7	1.93
s400	21	151	8865	4	7.8	1.95
s444	21	151	8865	4	8	2.00
s526n	21	151	8868	5	9.21	1.84
s526	21	151	8868	5	9.35	1.87
s349	15	7	2625	3	14.8	4.93
s344	15	7	2625	3	15.3	5.10

**Fig. 7.** Reachability Analysis Performance. #FLOPS is the number of flip-flops in the model. #states is the total number of reachable states. T1 is the time required for our tool. T2 is the time as given in [6] using 1.5Ghz dual AMD Athlon cpu with 3GB RAM.

Model	#FLOPS	Depth1 (1000 sec)	Time to reach Depth1 (sec)	Max depth Completed	Actual time for max depth(sec)
S1269	37	1	10	1	10
S1423	74	3	28	4	615
S13207	669	2	8	3	140
S1512	57	4	70	23	31761
S9234	228	8	314	111	251764
S15850	597	5	192	7	8467
S38584	1452	2	1	4	2134

**Fig. 8.** Reachability Analysis Performance. 'Depth 1' is the maximal depth reached in [6] with timeout of 1000 seconds. 'Time to reach Depth 1' is the time required for our tool to complete the same depth. The 'Max depth' and the 'Actual time for max depth' are the maximal steps successfully completed by our tool, and the time required for it. The Timeout is generally 3600 seconds (1 hour), with longer timeouts for s1512, s9234 and s15850.

formula. We achieve this goal by incorporating a backtrack search and a conflict driven search into one complete engine. Our engine's memory requirements are independent of the number of solutions. This implies that, during the computation, the number of solutions already found does not become a parameter of complexity in finding further solutions. It also implies that the size of the instance being solved fits in smaller and faster levels of the memory hierarchy. As a result, our method is faster than blocking clause-based methods, and can solve instances that produce solution sets too large to fit

in memory. We have demonstrated how to use our All-SAT engine for memory-efficient reachability computation.

## References

1. I. Beer, S. Ben-David, and A. Landver. On-the-fly model checking of RCTL formulas. In *10th Computer Aided Verification*, pages 184–194, 1998.
2. A. Biere, A. Cimatti, E. M. Clarke, M. Fujita, and Y. Zhu. Symbolic model checking using SAT procedures instead of BDDs. In *DAC*. IEEE Computer Society Press, June 1999.
3. Elazar Birnbaum and Eliezer L. Lozinskii. The good old davis-putnam procedure helps counting models. *Journal of Artificial Intelligence Research*, 10:457–477, 1999.
4. R. E. Bryant. Graph-based algorithms for boolean function manipulation. *IEEE transactions on Computers*, C-35(8):677–691, 1986.
5. J. R. Burch, E. M. Clarke, K. L. McMillan, D. L. Dill, and L. J. Hwang. Symbolic model checking:  $10^{20}$  states and beyond. *Information and Computation*, 98(2):142–170, June 1992.
6. P. Chauhan, E. M. Clarke, and D. Kroening. Using SAT based image computation for reachability analysis. Technical Report CMU-CS-03-151, Carnegie Mellon University, 2003.
7. P. P. Chauhan, E. M. Clarke, and D. Kroening. A SAT-based algorithm for reparameterization in symbolic simulation. In *DAC*, 2004.
8. M. Davis, G. Logemann, and D. Loveland. A machine program for theorem proving. *CACM*, 5(7), July 1962.
9. M. Davis and H. Putnam. A computing procedure for quantification theory. *JACM*, 7(3):201–215, July 1960.
10. E. Goldberg and Y. Novikov. Berkmin: A fast and robust SAT-solver. In *DATE*, 2002.
11. Aarti Gupta, Zijiang Yang, Pranav Ashar, and Anubhav Gupta. SAT-based image computation with application in reachability analysis. In *FMCAD*, LNCS 1954, 2000.
12. Roberto J. Bayardo Jr. and Joseph Daniel Pehoushek. Counting models using connected components. In *AAAI/IAAI*, pages 157–162, 2000.
13. H. J. Kang and I. C. Park. SAT-based unbounded symbolic model checking. In *DAC*, 2003.
14. S. K. Lahiri, R. E. Bryant, and B. Cook. A symbolic approach to predicate abstraction. In *CAV*. LNCS 2725, July 2003.
15. R. Letz. Advances in decision procedures for quantified boolean formulas. In *IJCAR*, 2001.
16. Chu Min Li and Anbulagan. Heuristics based on unit propagation for satisfiability problems. In *IJCAI (1)*, pages 366–371, 1997.
17. J.P. Marques-Silva and K.A. Sakallah. Conflict analysis in search algorithms for propositional satisfiability. In *IEEE ICTAI*, 1996.
18. Ken L. McMillan. Applying SAT methods in unbounded symbolic model checking. In *Computer Aided Verification*, 2002.
19. M.W. Moskewicz, C.F. Madigan, Y. Zhao, L. Zhang, and S. Malik. Chaff: engineering an efficient SAT solver. In *39th Design Automation Conference (DAC'01)*, 2001.
20. D. Plaisted. Method for design verification of hardware and non-hardware systems. United States Patents, 6,131, 078, October 2000.
21. D. Roth. On the hardness of approximate reasoning. *Artificial Intelligence*, 82(1-2), 1996.
22. S. Sapra, M. Theobald, and E. M. Clarke. SAT-based algorithms for logic minimization. In *ICCD*, 2003.
23. Ofer Shtrichman. Tuning SAT checkers for bounded model checking. In *CAV*, 2000.
24. Berkeley University of California. Espresso, two level boolean minimizer, 1990.
25. A. Yadgar. Solving All-SAT problem for reachability analysis. M.Sc. thesis, Technion, Israel Institute of Technology, Department of Computer Science, 2004.
26. H. Zhang. SATO: An efficient propositional prover. In *CADE*, 1997.
27. L. Zhang, C. F. Madigan, M. W. Moskewicz, and S. Malik. Efficient conflict driven learning in boolean satisfiability solver. In *ICCAD*, 2001.