Geometric Covering

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MSc Thesis

Introduction

Geometric Covering (GC) queries appear in numerous applications:

- Mold design in manufacturing
- Inspection
- Security and surveillance
- Placements of cellular antennas
- Illumination design
- Spraying of paint

Layout of the Rest of the Talk

We are focusing on mold-design and security.

- Related work in mold-design and security.
- A generic unified framework for answering geometric covering.
- Geometric Covering is an *NP*-hard problem.
- Examples of the generic framework as implemented in a 3D mold-design and security.
- Conclusions and future work.

Related Work I
Mold design

- 2-pieces-mold polygonal decomposition in R^3 [Ahn02, Khardekar06, Chen06]
- *n*-pieces-mold polygonal decomposition in R^3 [Liu09, Priyadarshi04, Stoyan10]
- 2-pieces-mold freeform surface decomposition in R^3 [Elber04]
- Algebraic analysis of visibility of freeforms in R^3 [Seong06]
- **Nothing** so far on automatic *n*-pieces-mold freeform decomposition in R^3

Related Work II
Security

- Polygonal 2.5*D* terrain where $z = f(x, y)$.
- Guards on the vertices or above them [Lee91, Goodchild89]
- Edge guards [Bose96, Bose97]
- Different greedy solutions [Goodchild89, Kaucic04]
- Guards limited to strategic locations [Kim04]
- Calculating partial visibility [Franklin94, Rana03]

Set-Cover I

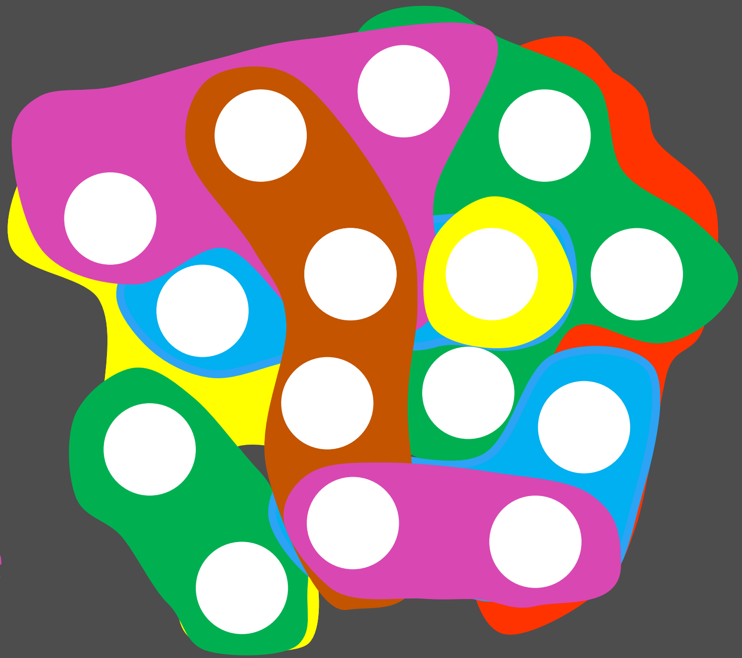
- Set-cover (SC) is a classic computer science query.
- SC is considered a very hard problem to solve (*NP* hard).
- Given some universe ***U*** and a family ***F*** of subsets of ***U*** which their union equals ***U***, a cover of ***U*** is a subfamily of ***F*** whose union still equals ***U***.
- In SC we are seeking a cover with minimal number of subsets.

Set-Cover II

- The universe U is a set of circles.
- A subset of U is a group of circles.
- The family F is all these groups of circles.
- The subfamily F_1 is the brown, yellow, blue and green groups.

  F_1 is a cover of U.
- The subfamily F_2 is the red, purple and yellow group.

  F_1 is a minimal cover of U.

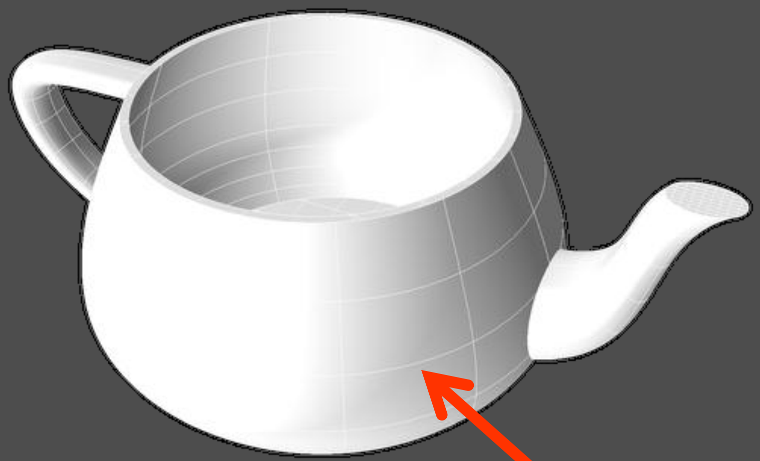


We will now show a reduction from GC problems to SC problems.

Visibility Map I

- We receive a 2 manifold geometry in R^3, C, which has a parameterization x_{uv}, y_{uv}, z_{uv}.
- The domain D_C of C is a 2-dimensional box , a rectangle, possibly trimmed.
- We are creating a discrete representation of D_C as an image, as a visibility map.
- The visibility map can serve as a controlled approximation for the coverage of C.

Visibility Map II



The Utah Teapot with its interior curved in.

The visibility map of the outer **body** of the Utah Teapot

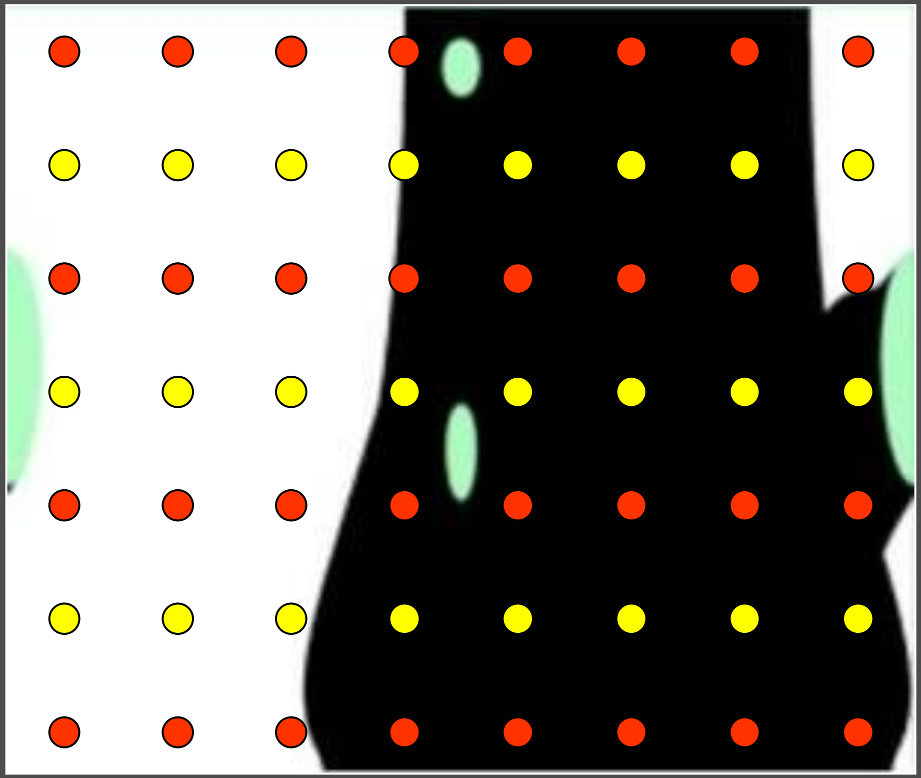
- Visible locations are set to white.
- Hidden locations are set to black.
- Trimmed away bits are set to green - don't care.

Visibility Map III

Linearize the visibility map, as a vector of bits as follow:

- Don’t care locations are simply skipped.
- Each bit is either 1 (visible pixel) or 0 (hidden pixel).
- Sequence the 1/0 bits in some order over the visibility map (for example: left to right, top to bottom).

Visibility Map IV



Visibility map of 8×7

1 1 1 1 0 0 0 1 1 1 1 0 0 0 0 1 1 1 1 0 0 0 0 1 1 1 1 0 0 0 0 0 1 1 1 0 0 0 0 0 1 1 1 0 0 0 0 0 1 1 1 0 0 0 0 0

Vector of 56 bits

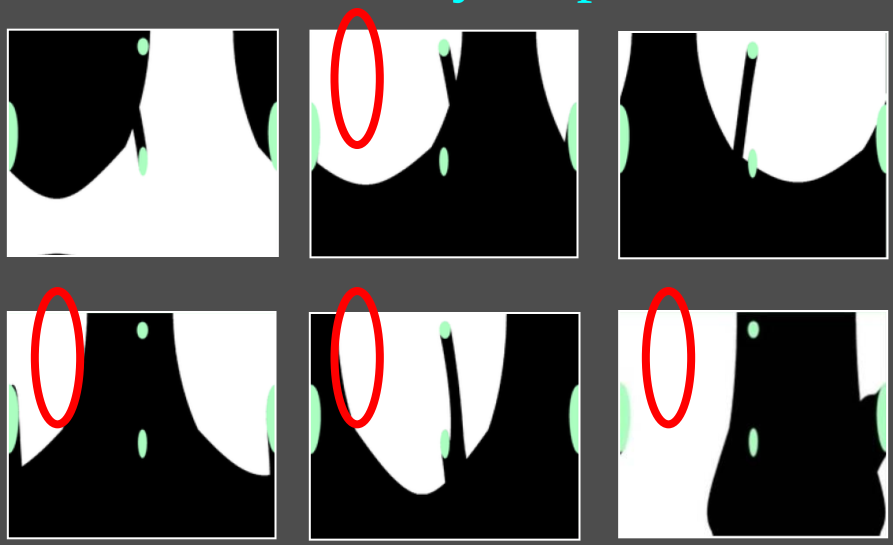
Set-Cover II

Set-cover can be clearly applied to vectors of bits:

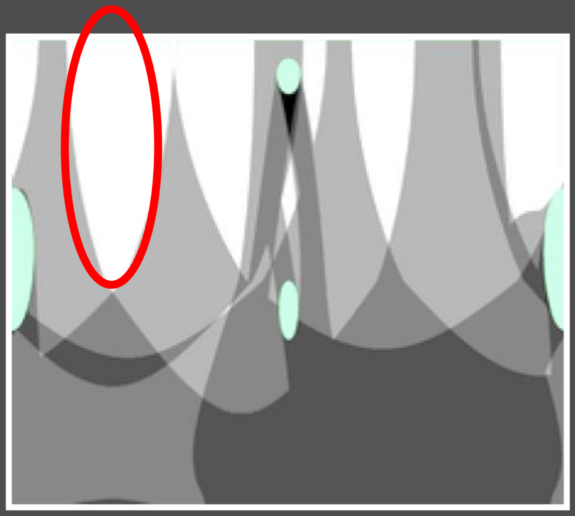
- The universe ***U*** is the domain D_C.
- A subset of ***U*** is a vector of bits.
- A family ***F*** of subsets of ***U*** is a set of vectors of bits from different views around the geometry *C*.
- A cover of ***U*** is a subfamily of ***F***, a set of vectors of bits which their union equals D_C.

Set-Cover III

Subfamily of the set of visibility maps



The union of the visibility maps

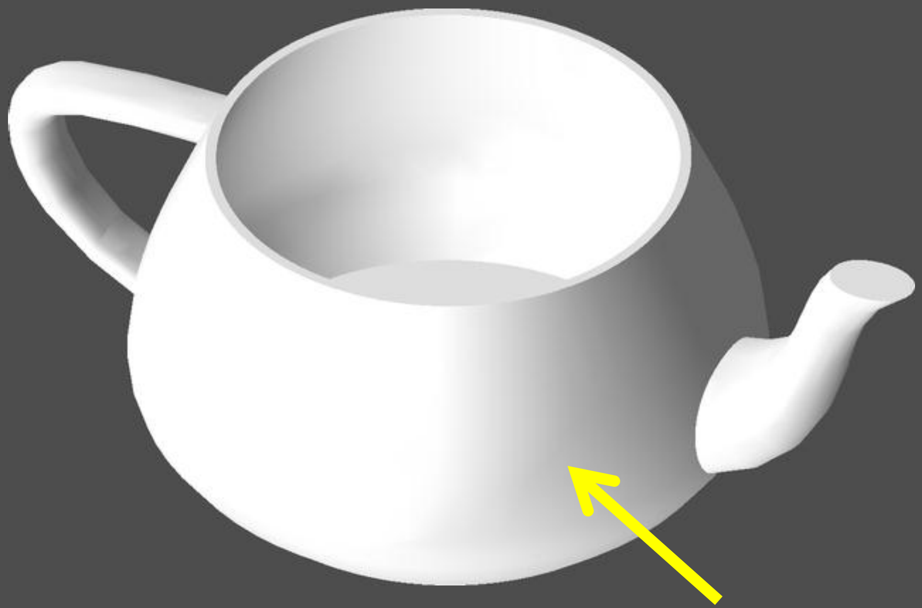


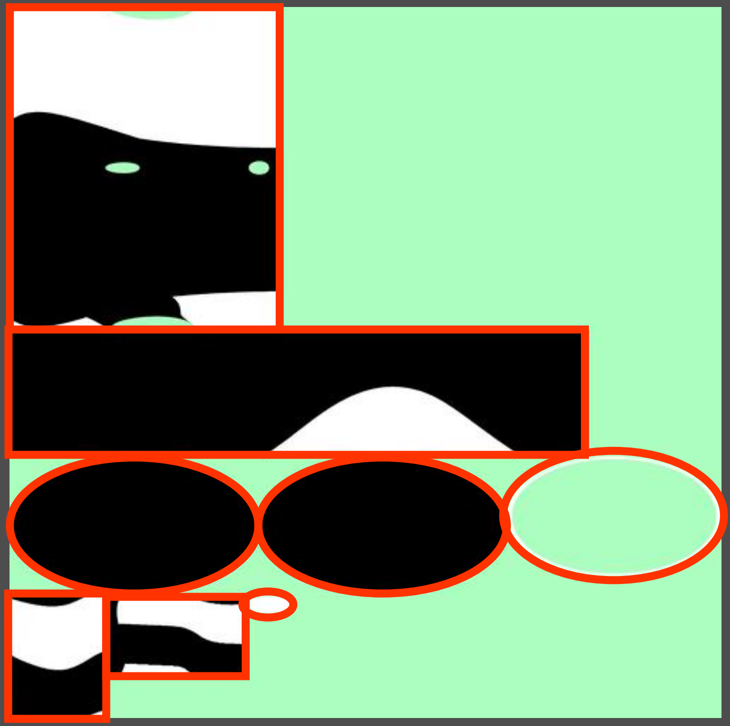
- The set-cover is done in the parametric domain.

Creating Visibility Maps I

- Input geometry C can be a surface or a set of surfaces, possibly trimmed.
- Each surface has its own rectangular domain, created independently of the other surfaces.
- We rearrange the domains of all the surfaces in one large image: The visibility map of C.

Creating Visibility Maps II





Creating Visibility Maps III

- Given C and D_C, the visibility map from direction V_i is computed as follow:
- The surface is tessellated into triangles.
- Two-rendering passes:
  I. A regular (Z-buffer) rendering of C from V_i keeping only the Z-depth information, in `ZBuffer(x, y)`.
  II. Scan conversion of C in the domain, D_C, and deciding visibility by comparing the Z-depths

Creating Visibility Maps IV

Pass II

A tesselation $\boldsymbol{T} = \{T_i\}$ of triangles with UV parametric coordinates is given.

For each triangle T_i in $\boldsymbol{T}$, scan convert T_i by its UV coordinates.

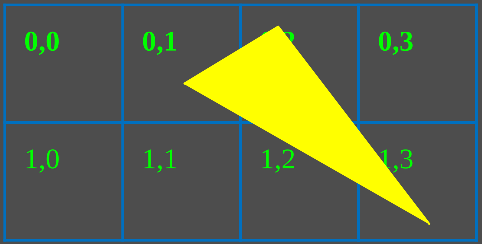
  For each pixel p_{uv} in T_i

    x_{uv}, y_{uv}, $z_{uv} \leftarrow XYZ$ coordinates of p_{uv};

    VisMap$(u, v) \leftarrow z_{uv} \approx$ *ZBuffer*(x_{uv}, y_{uv});

  EndFor

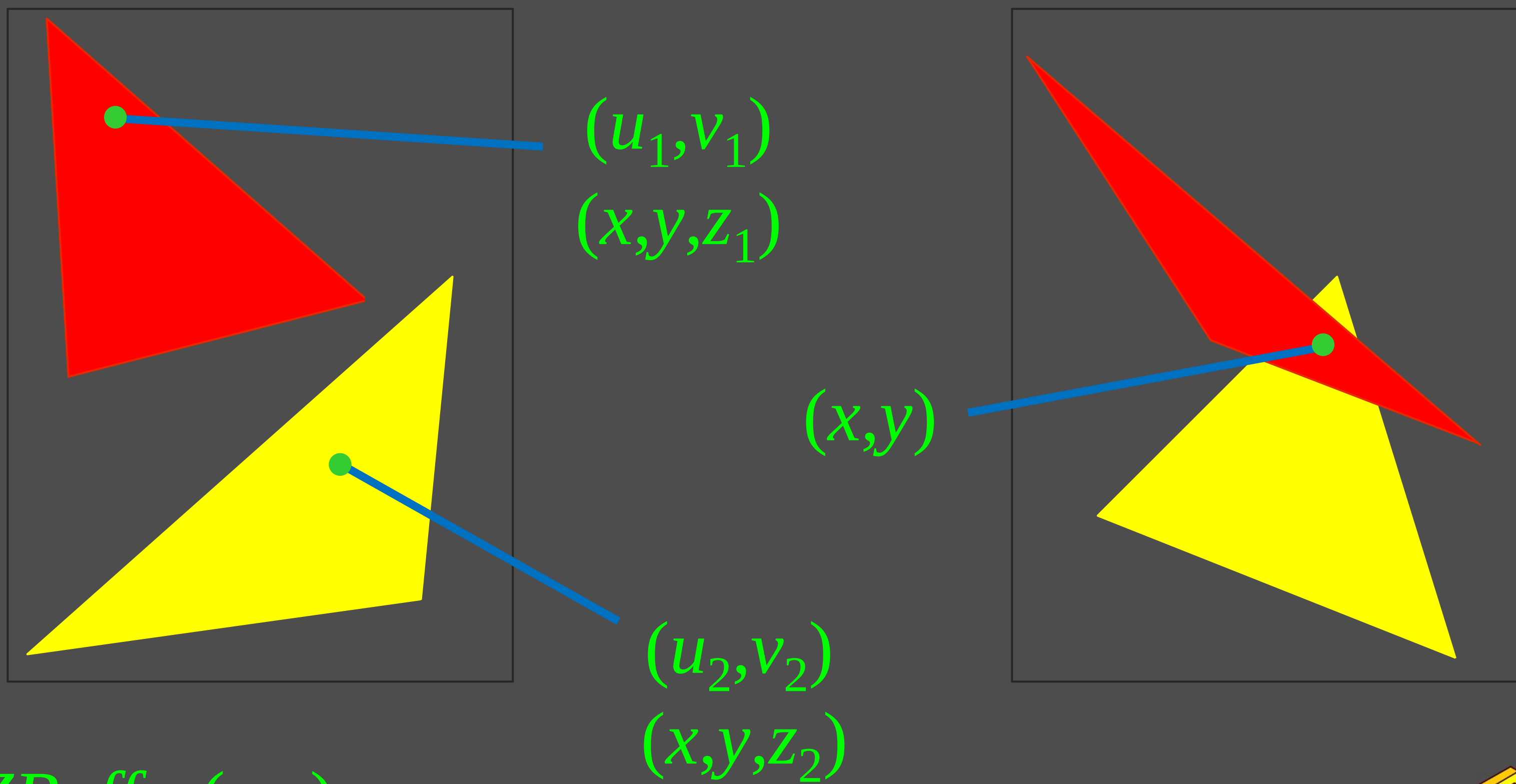
EndFor



UV Domain of 4×2

Creating Visibility Maps V

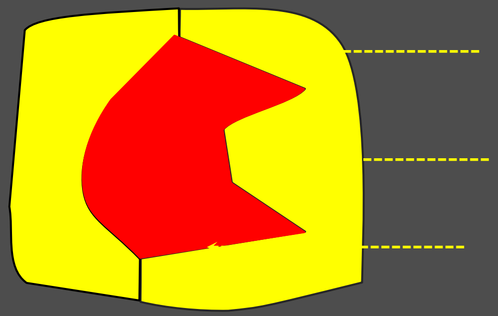
UV domain pass II | Euclidean space pass I



$$ZBuffer(x,y) \approx z_1$$

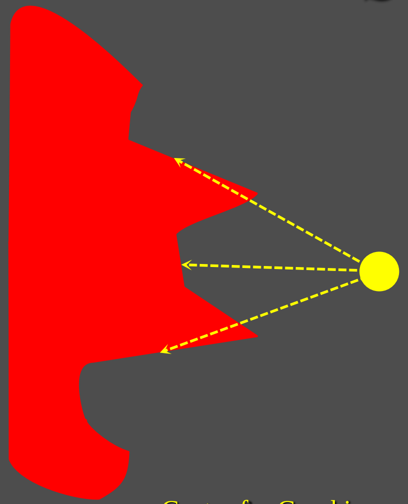
Creating Visibility Maps VII

Mold Design



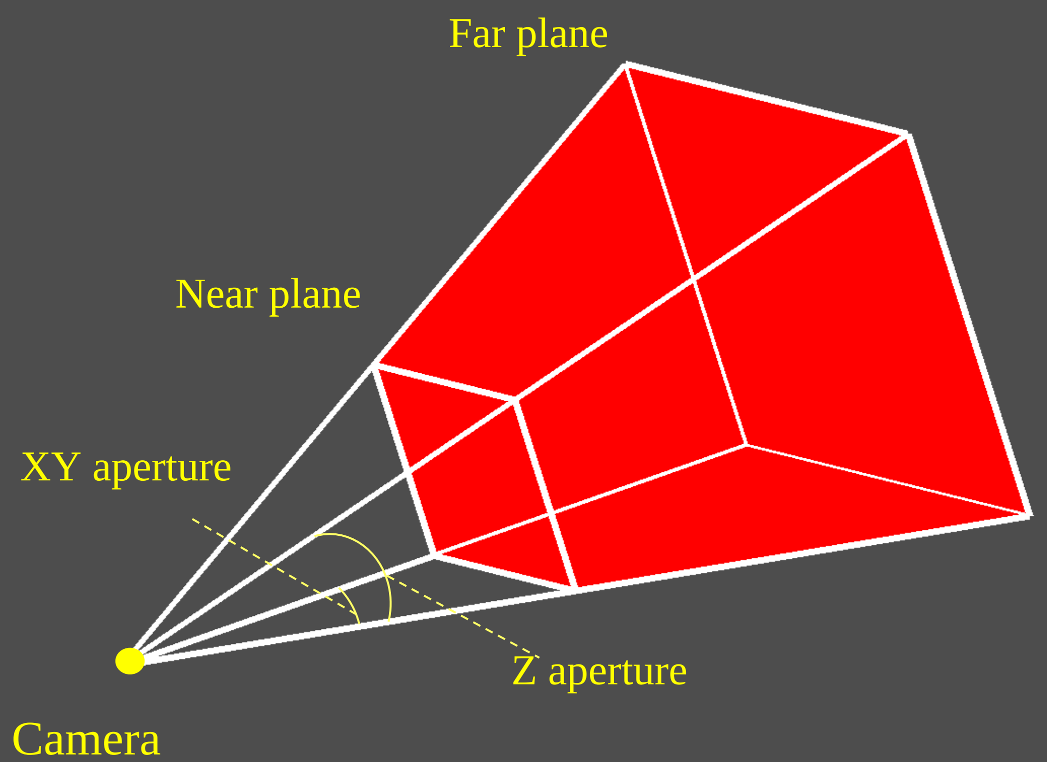
Orthographic projection

Security



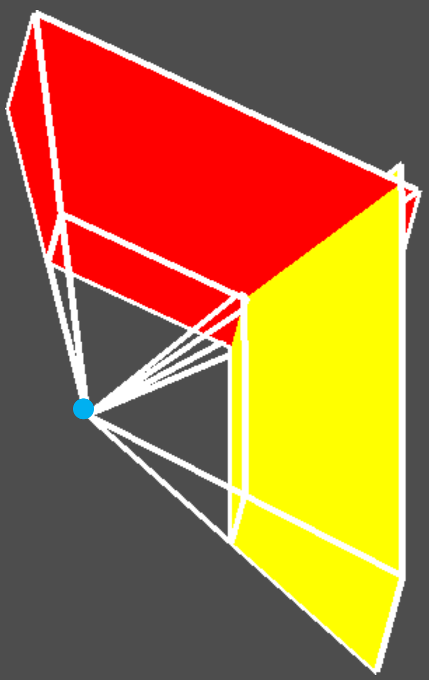
Perspective projection

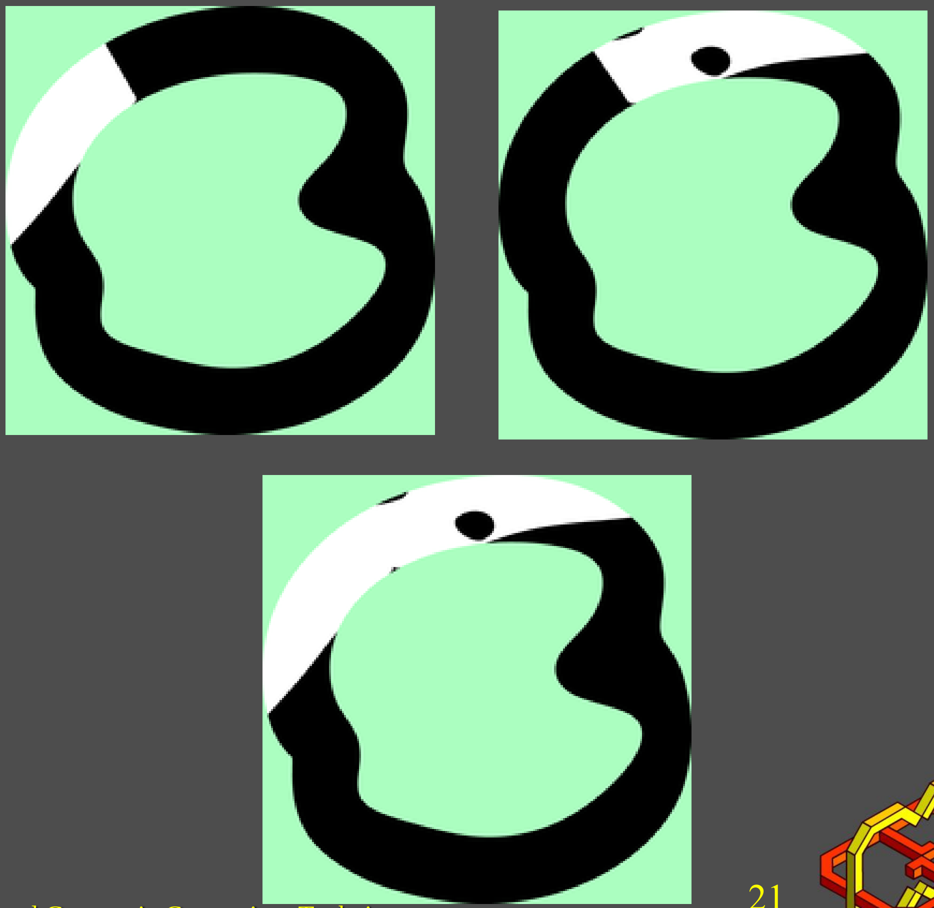
Creating Visibility Maps VIII
Perspective projection I



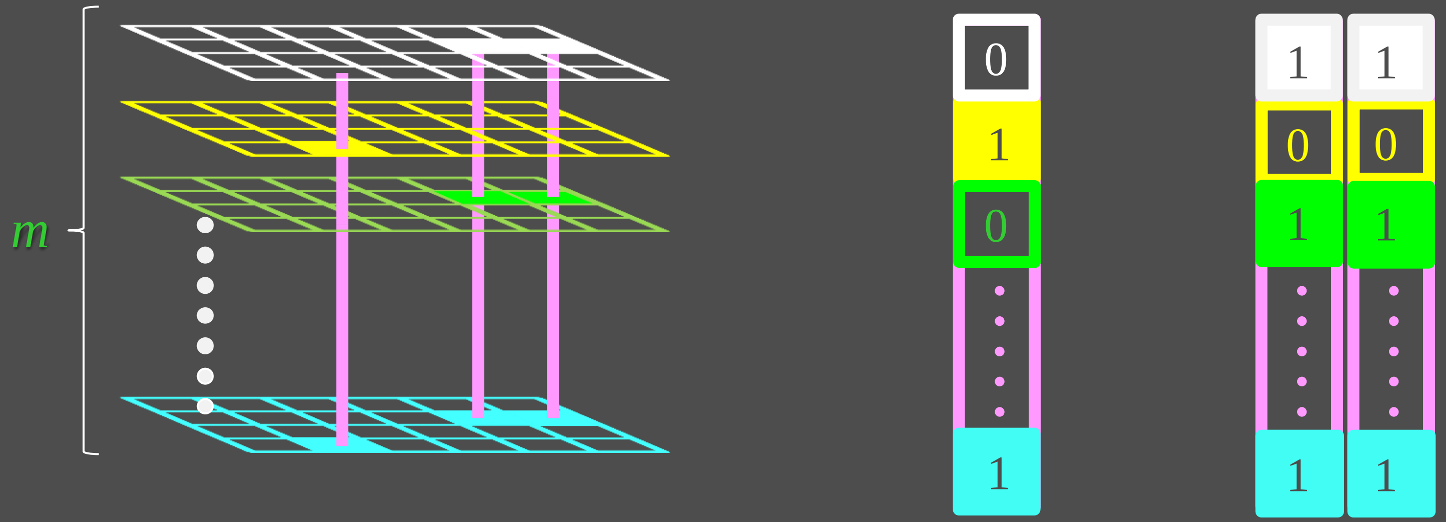
Creating Visibility Maps IX
Perspective projection II

Combining visibility maps





Pixel Collapsing I

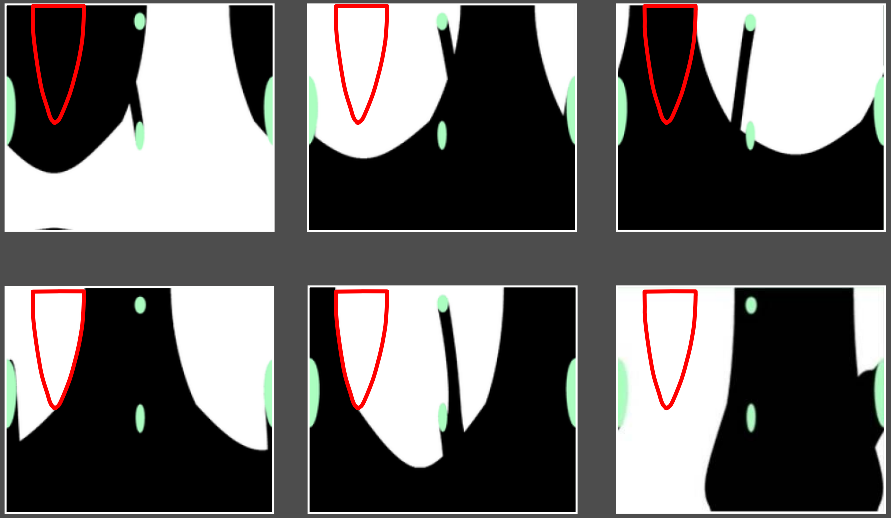


$n \times n \times m$

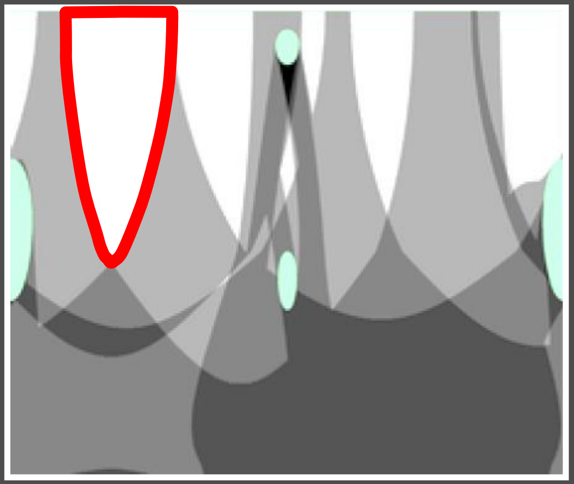
- 2^m possible pixels vector.
- n^2 different pixels vector at most.
- In practice, much less.

Pixel Collapsing II

Subfamily of the set of visibility maps



The union of the visibility maps

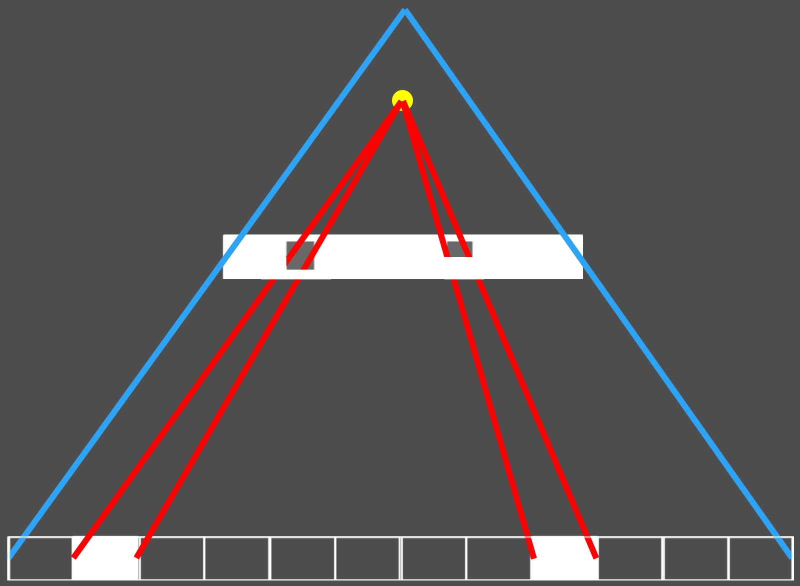


Reduction from SC to GC I

- We have shown a polynomial reduction from GC to SC. For completeness we will also show a polynomial reduction from SC to GC, proving that GC is *NP*-hard as SC is.
- We have a standard SC as described before.
- We will create a geometry corresponding to the universe *U*.
- We will create guards corresponding to the subsets of *U*.
- Solving the GC will solve the SC as well.

Reduction from SC to GC II

Subset of U - a possible guard.

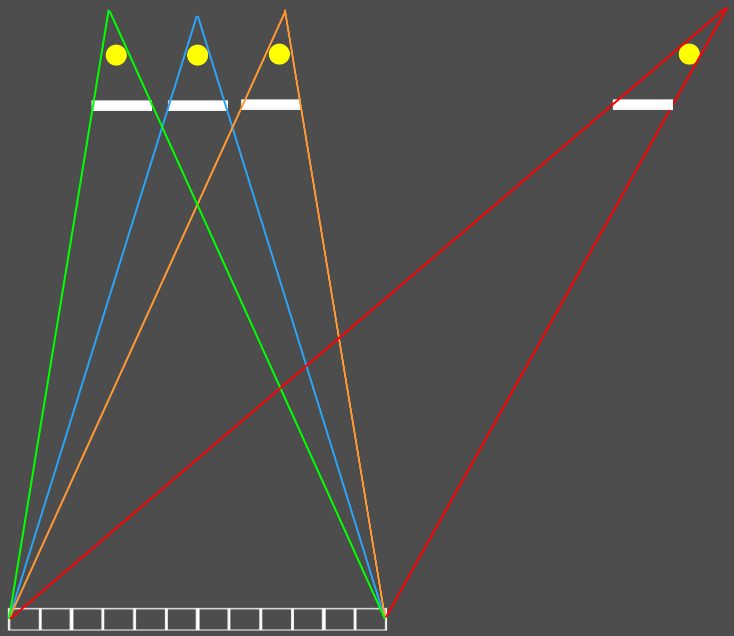


U - a long strip.

Elements of U - regions on the strip.

Reduction from SC to GC III

- *F* - as many guards as are subsets in the problem, spread over the entire plane.
- All the upper strips are entirely covered by each of the guards.



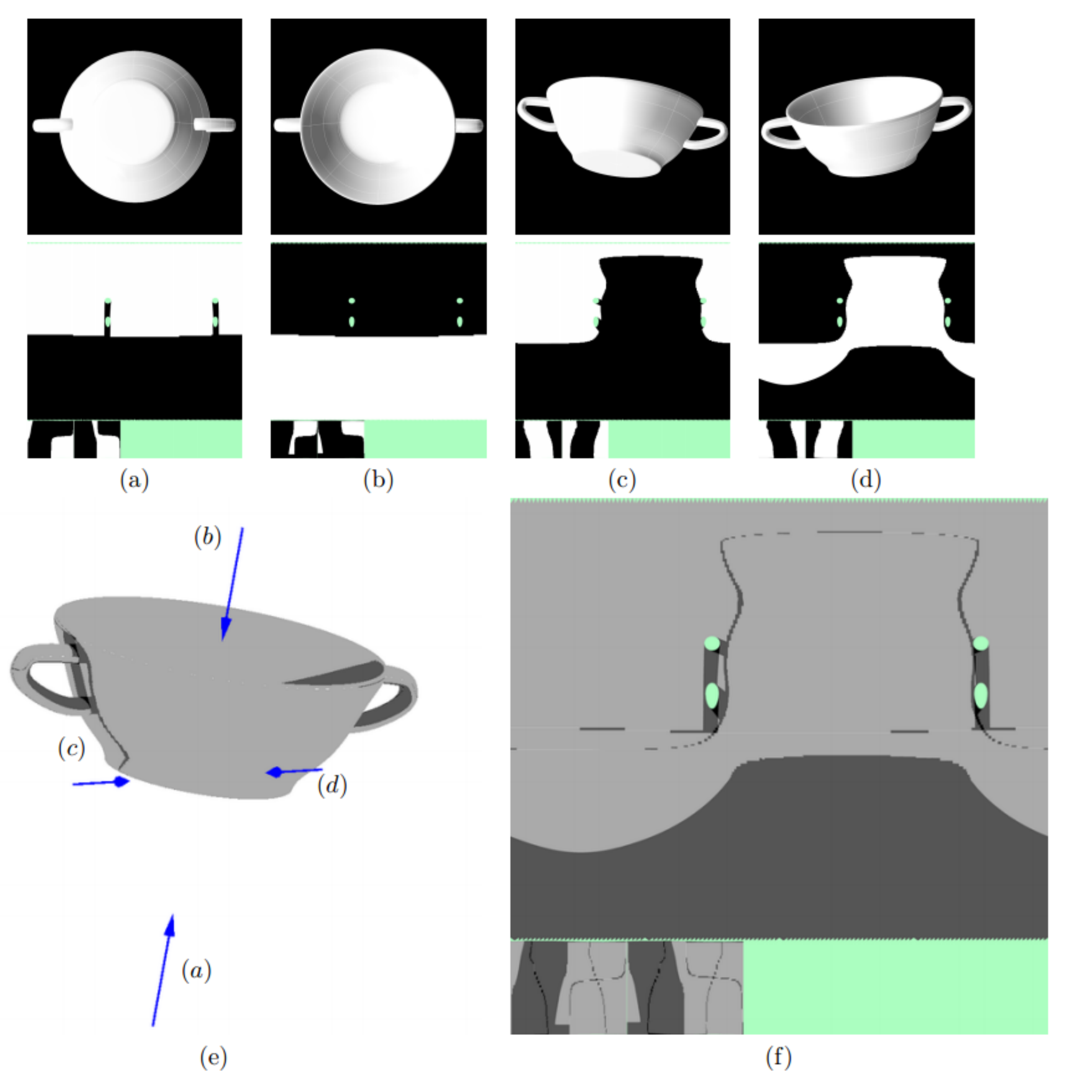
Examples
General Notes

- The following examples were created using Visibility maps of size 4096×4096.
- Both exhaustive (exponential) set cover solution and greedy (non-optimal) solution were sought.
- All implementation is software based and with single thread.
- In the examples we seek high coverage percent rather than a complete coverage.

Mold-Design Examples General Notes

- The following examples were created using 266 views:
  - 130 general views around S^2, duplicated as V and $-V$.
  - 6 views of $\pm X$, $\pm Y$, $\pm Z$.

Example – a Cup Model

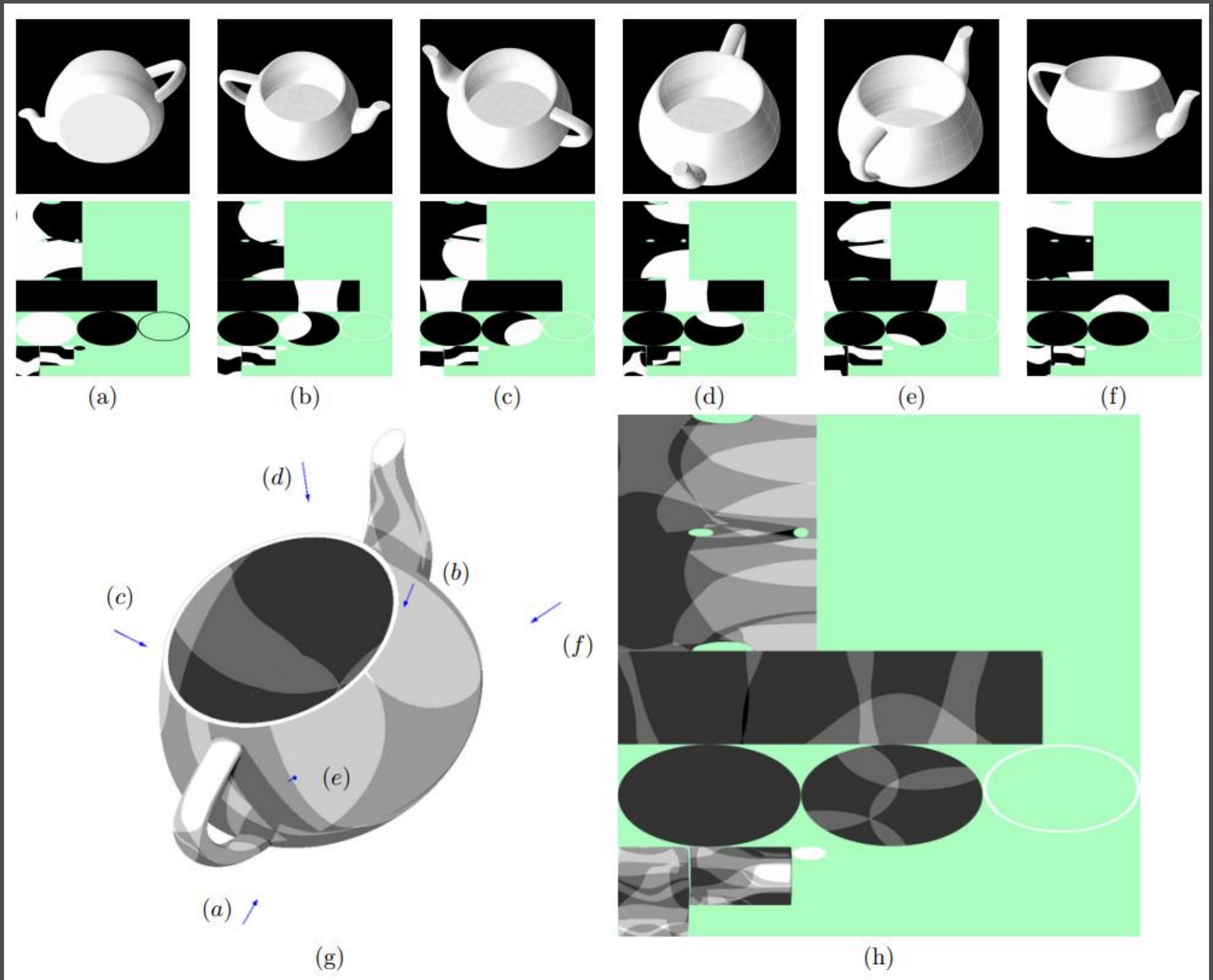


99.827% cover in greedy SC in ~4 seconds.

99.995% cover in exhaustive SC in ~10 hours.

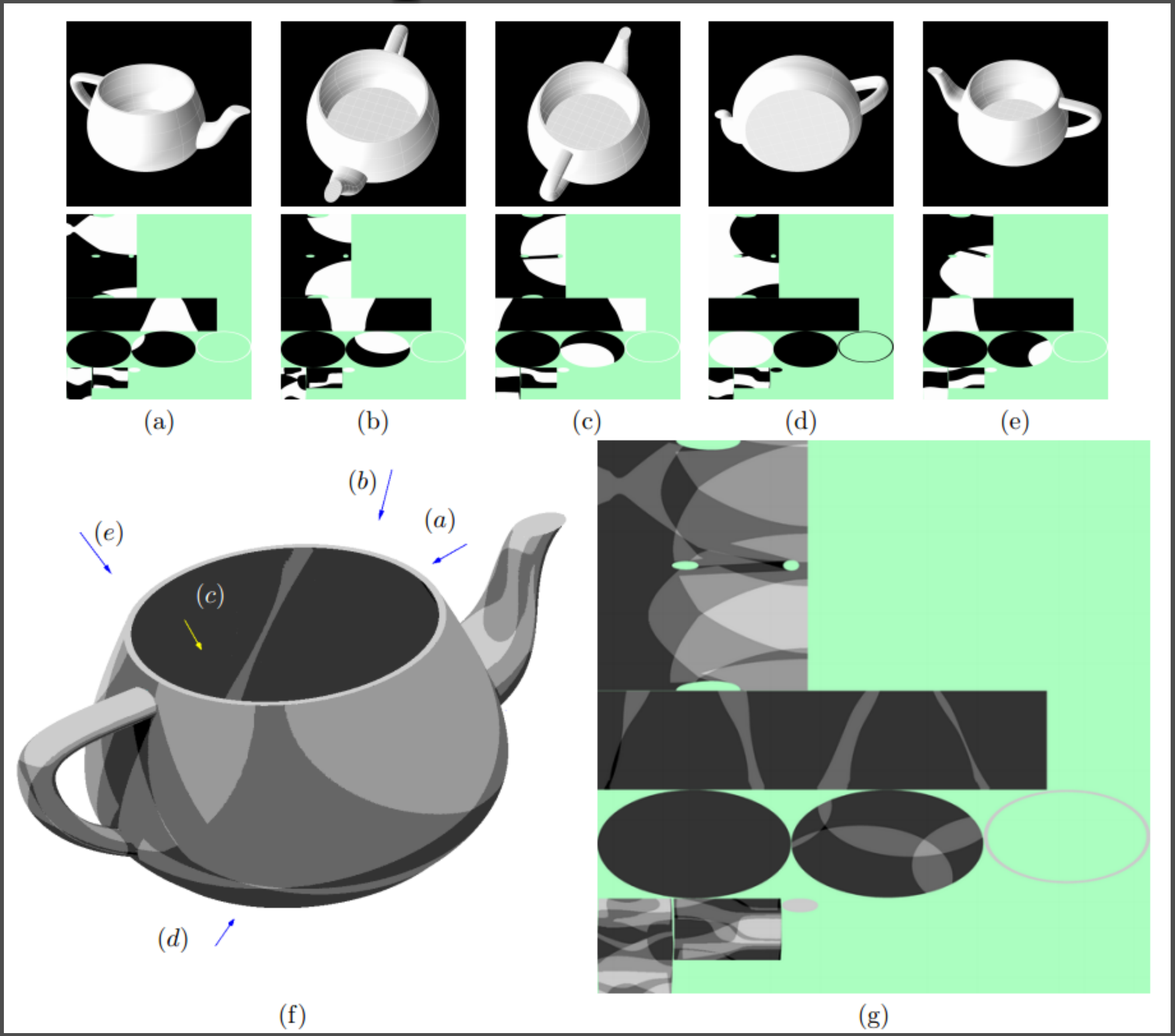
First two view directions 95% cover.

Example – The Utah Teapot I



99.7% cover in greedy SC in ~6 seconds.

Example – The Utah Teapot II

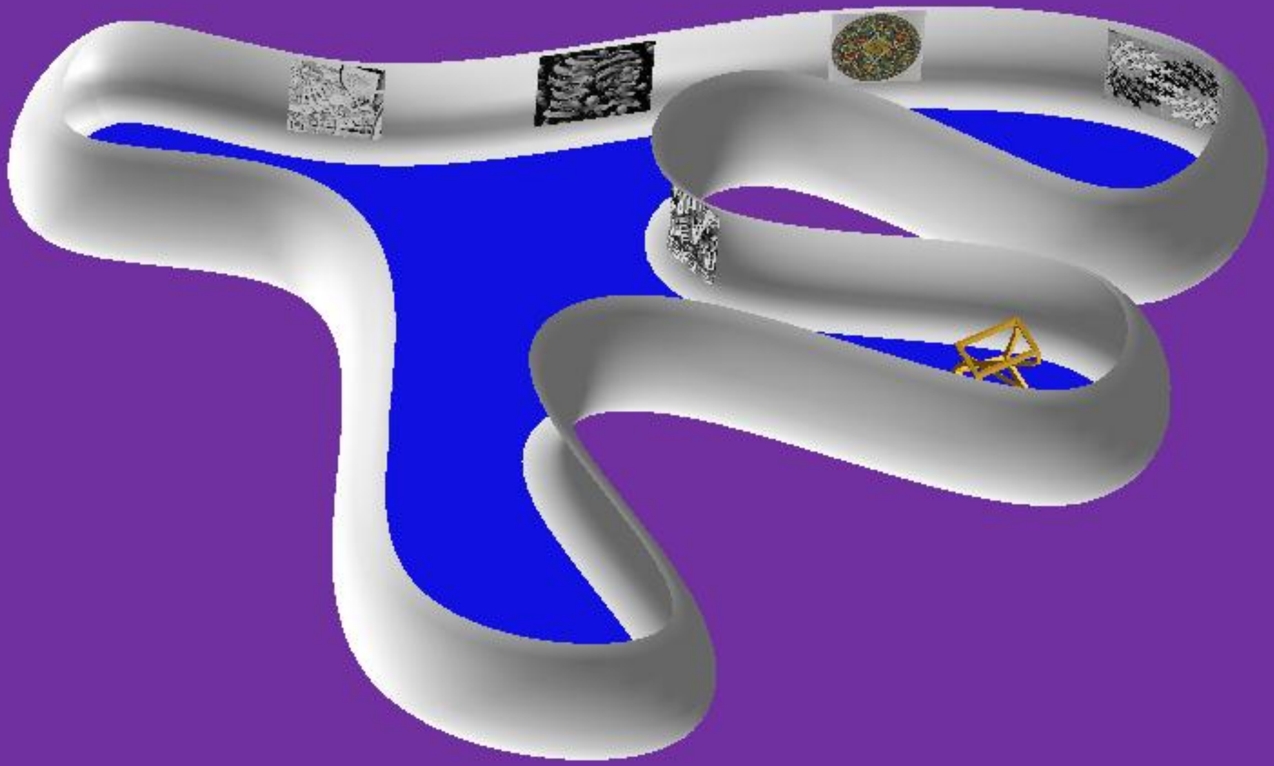


99.7% cover in exhaustive SC in ~433 hours.

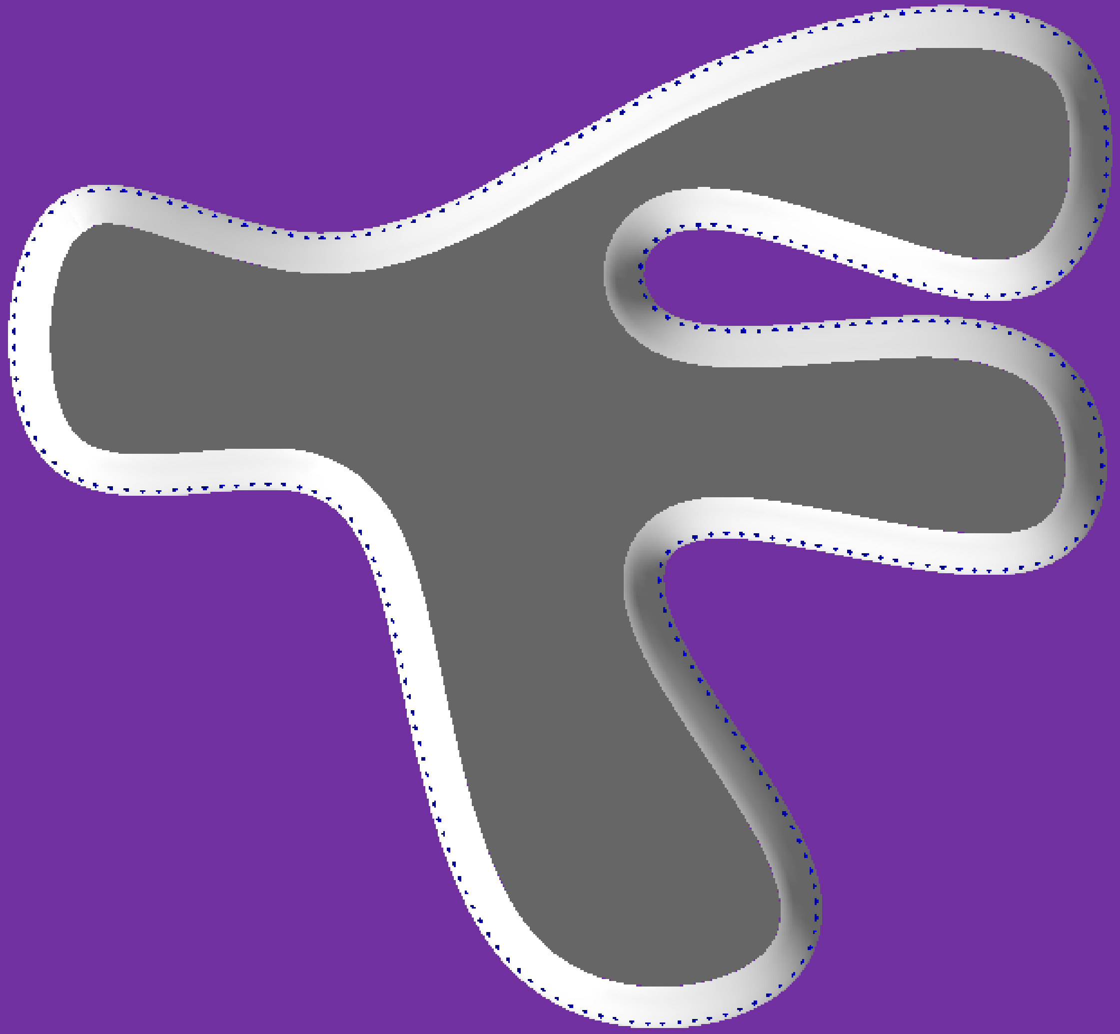
Security Examples General Notes

- The following examples were created using about 300 guards/cameras.
- The guards where evenly spread on a curve or a plane.

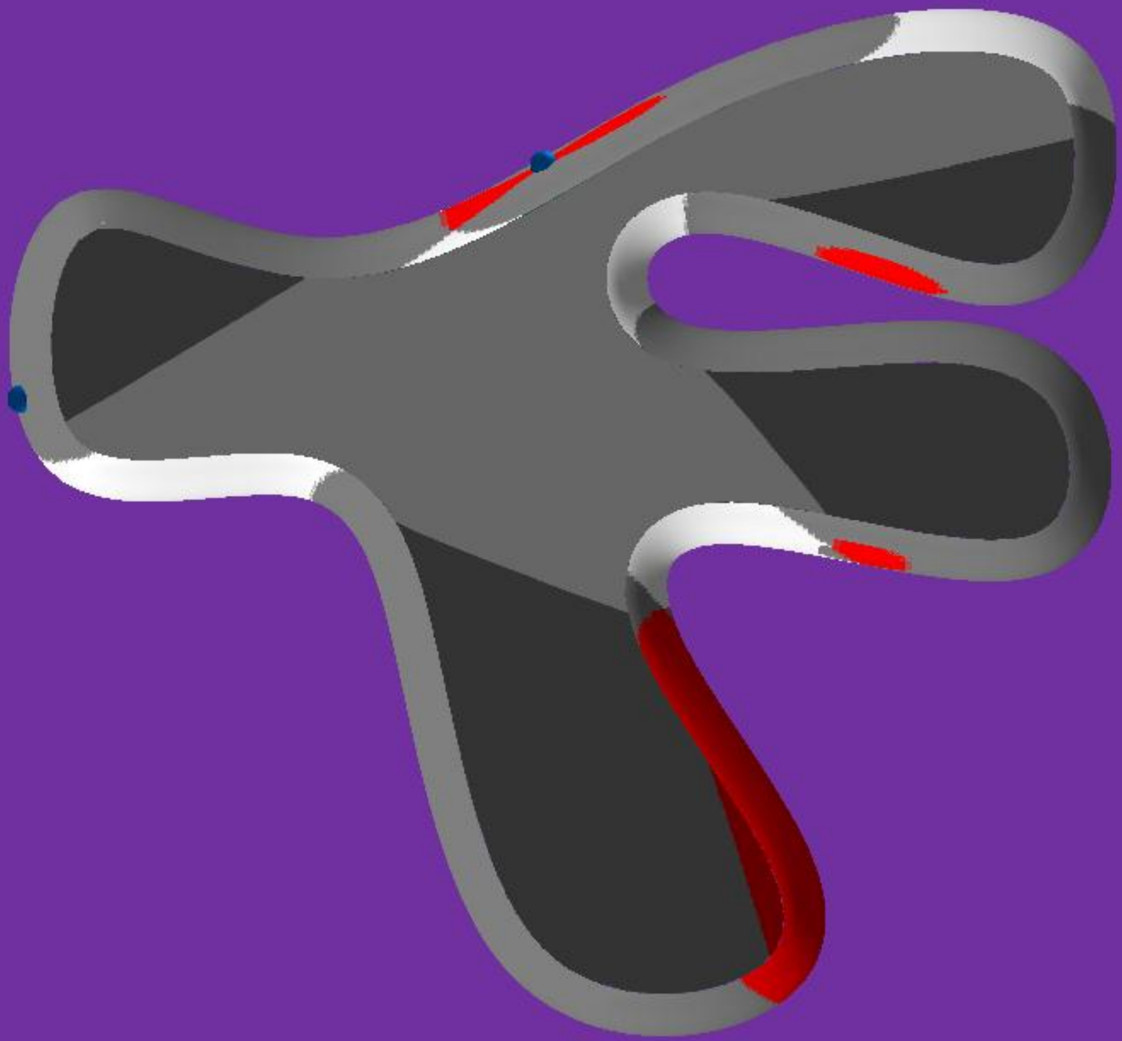
A free form shape gallery



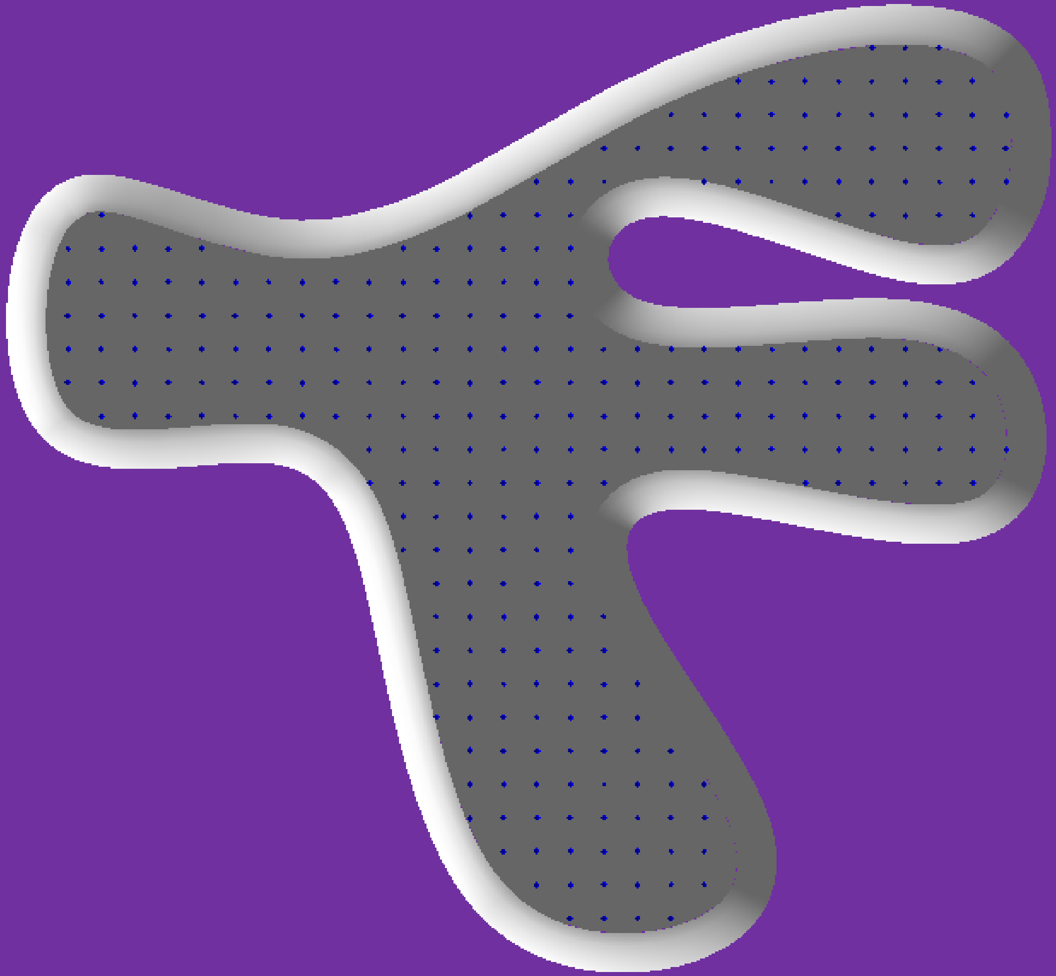
Cameras on the walls



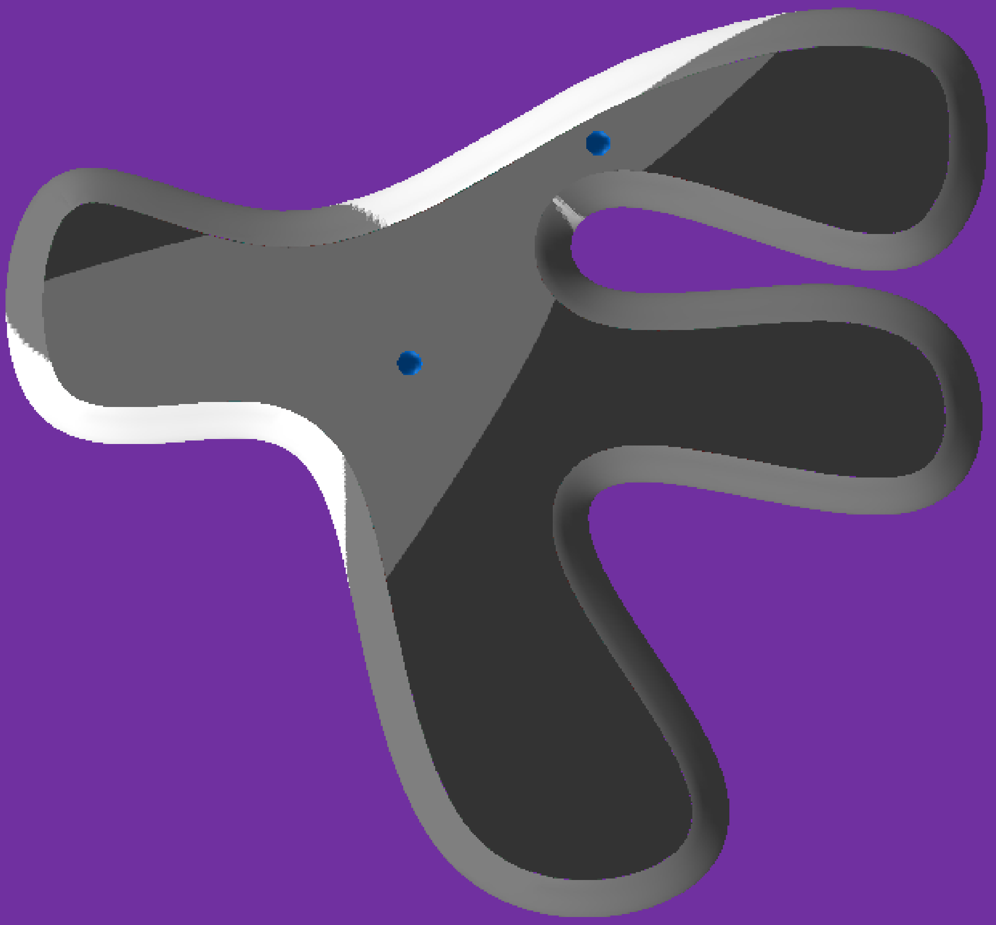
Cameras on the wall - 2 cameras solution



Cameras on the ceiling

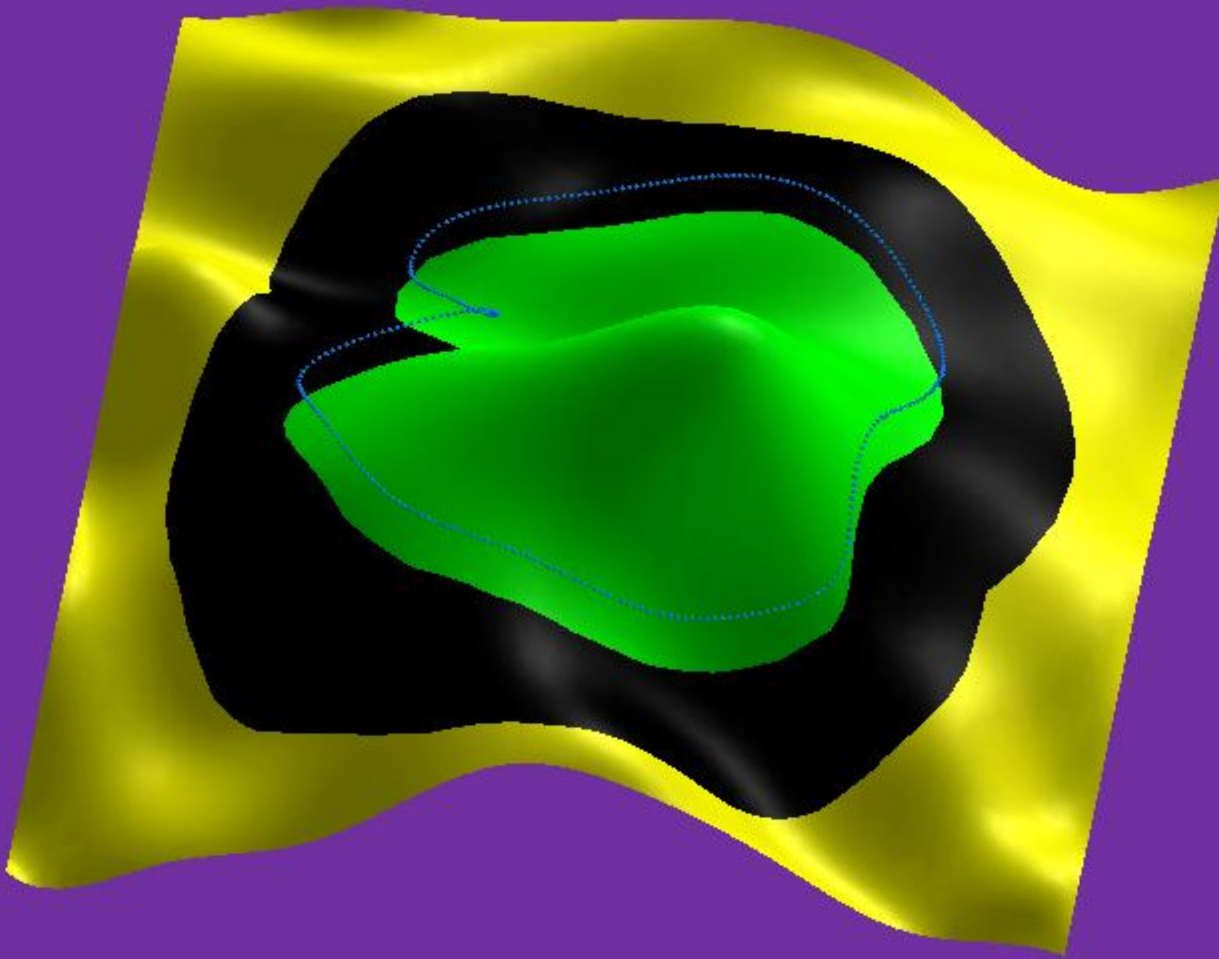


Cameras on the ceiling - 2 cameras solution

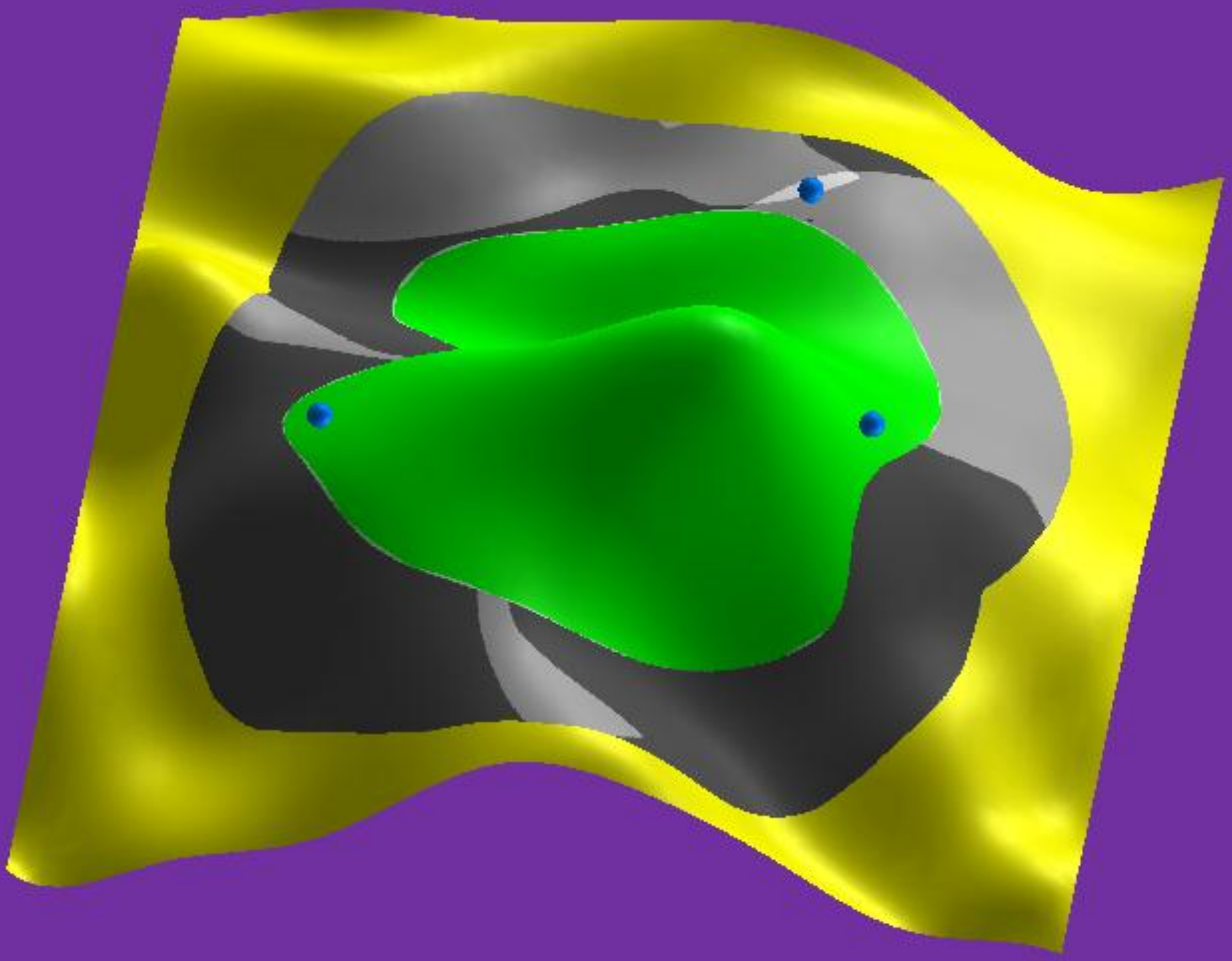


A military compound

A military compound - candidates above the perimeter

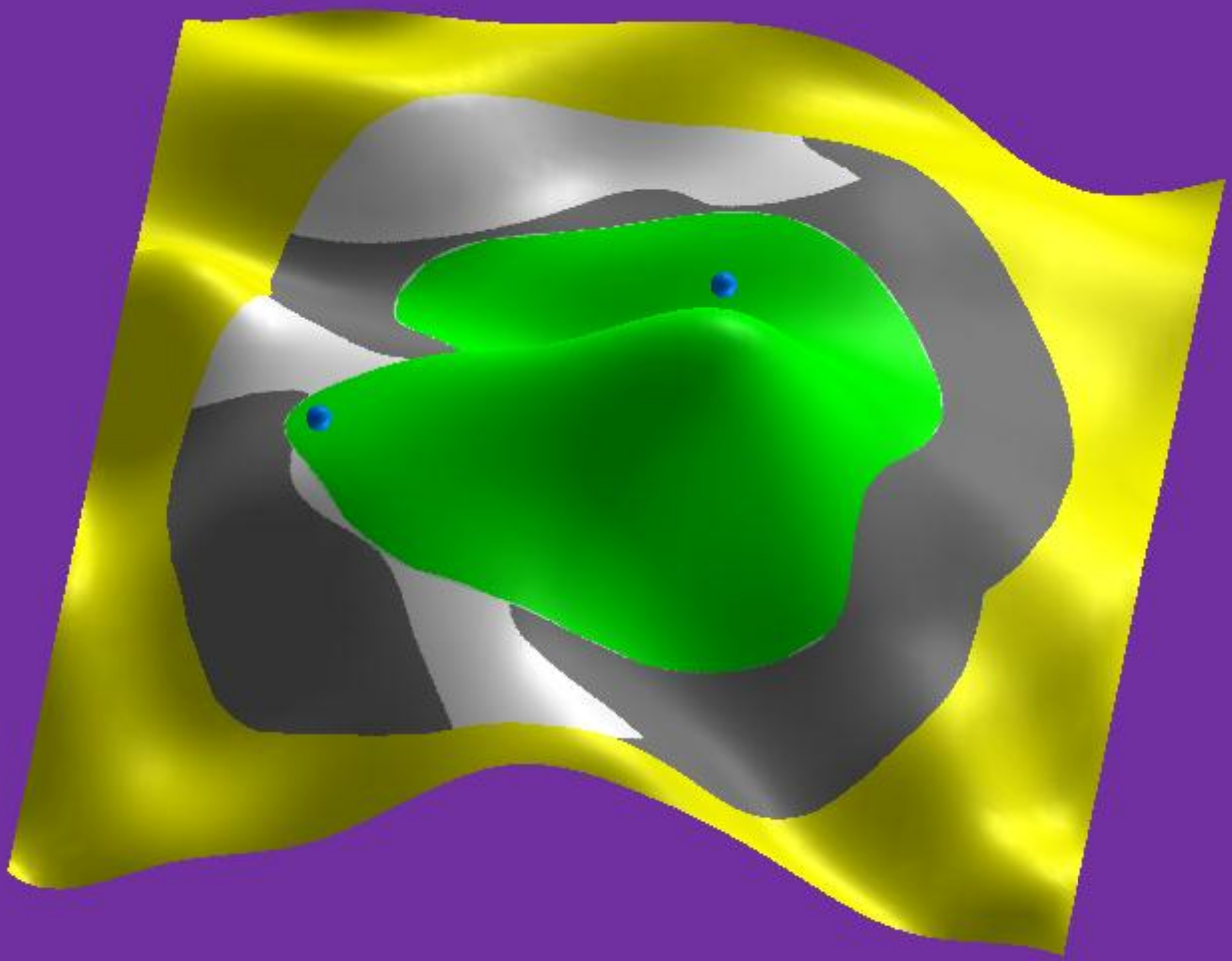


Candidates above the perimeter – 3 guards solution



A military compound - candidates above the compound

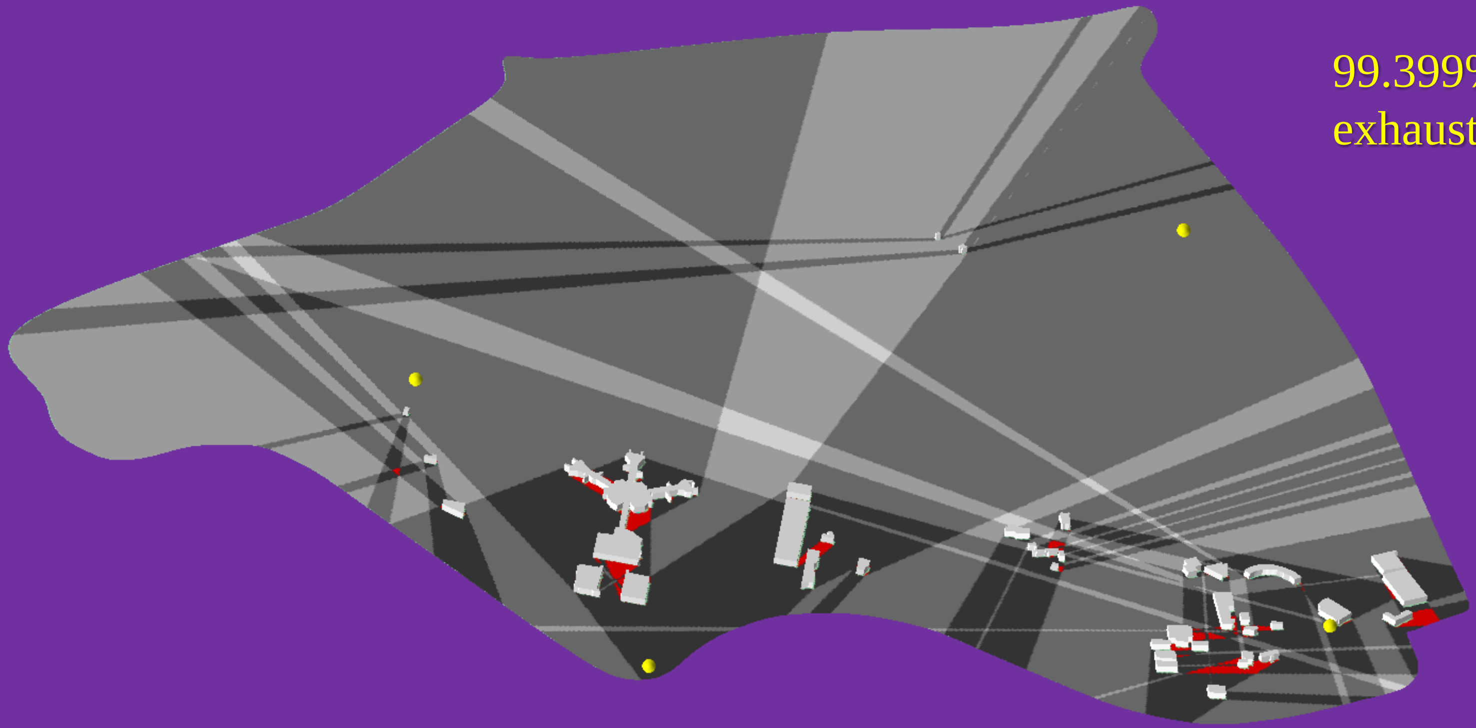
Candidates above the compound– 2 guards solution



Ben Gurion airport

Ben Gurion airport - candidate cameras

Ben Gurion airport - exhaustive 4 views solution



99.399% cover in exhaustive SC.

Conclusions and Future Work I

- We solve the GC problem in the parametric domain and reduce the analysis into the pixel level.
- Though we presented the framework in R^3, nothing prevents the use of this framework in R^n for arbitrary n.
- The reduction to the discrete SC problem allows to optimally solve only discrete GC problems with a few views.
- We are looking for the solution in the continues problem.

Conclusions and Future Work II

- Use of GPU in proposed framework can benefit the computation times (expect ~two orders of magnitudes).
- Viewing angle and location distance limitations can be integrated into the creation of the visibility map.
- Many of the visibility maps are very similar. Can we use this property to reduce set cover calculations?
- The suggested framework can be used in other GC problems beside mold design and security.

End