Reversible Harmonic Maps between Discrete Surfaces

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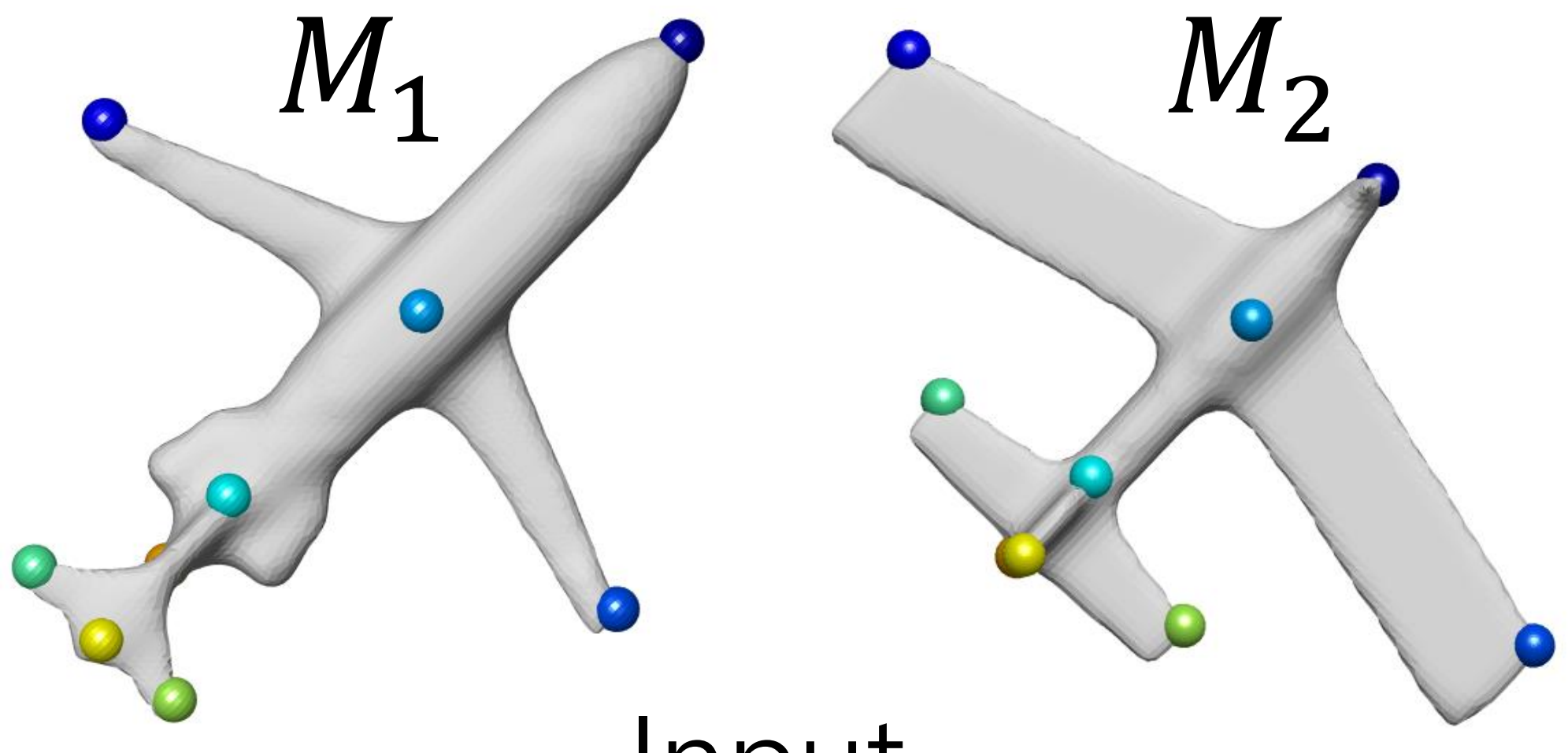
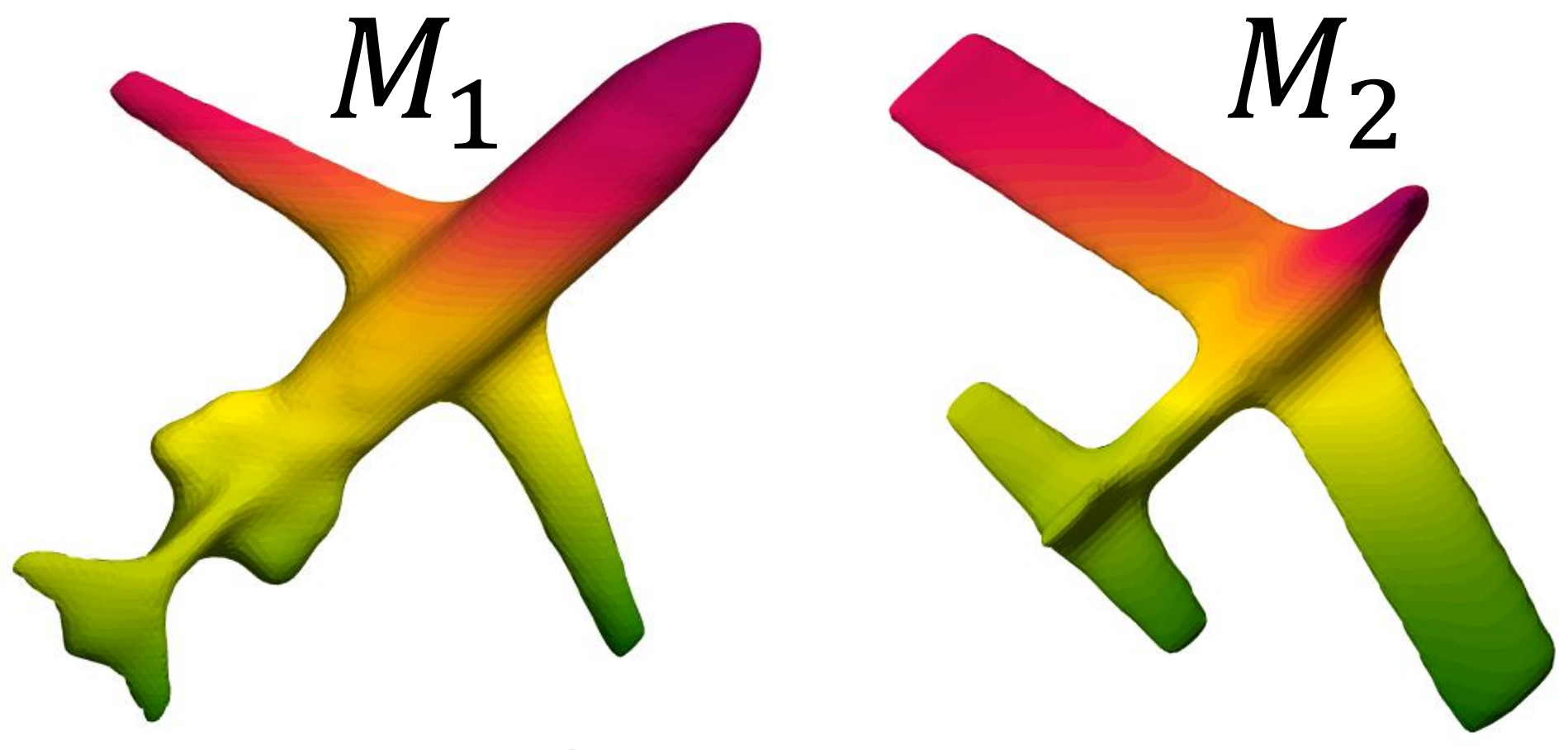
Goal: compute a smooth function ϕ_{12} between two shapes, given a sparse set of landmarks

Applications:

- **Statistical analysis** and segmentation transfer of medical data
- **Deformation** and **texture transfer**
- Interpolation for **shape synthesis**



Input



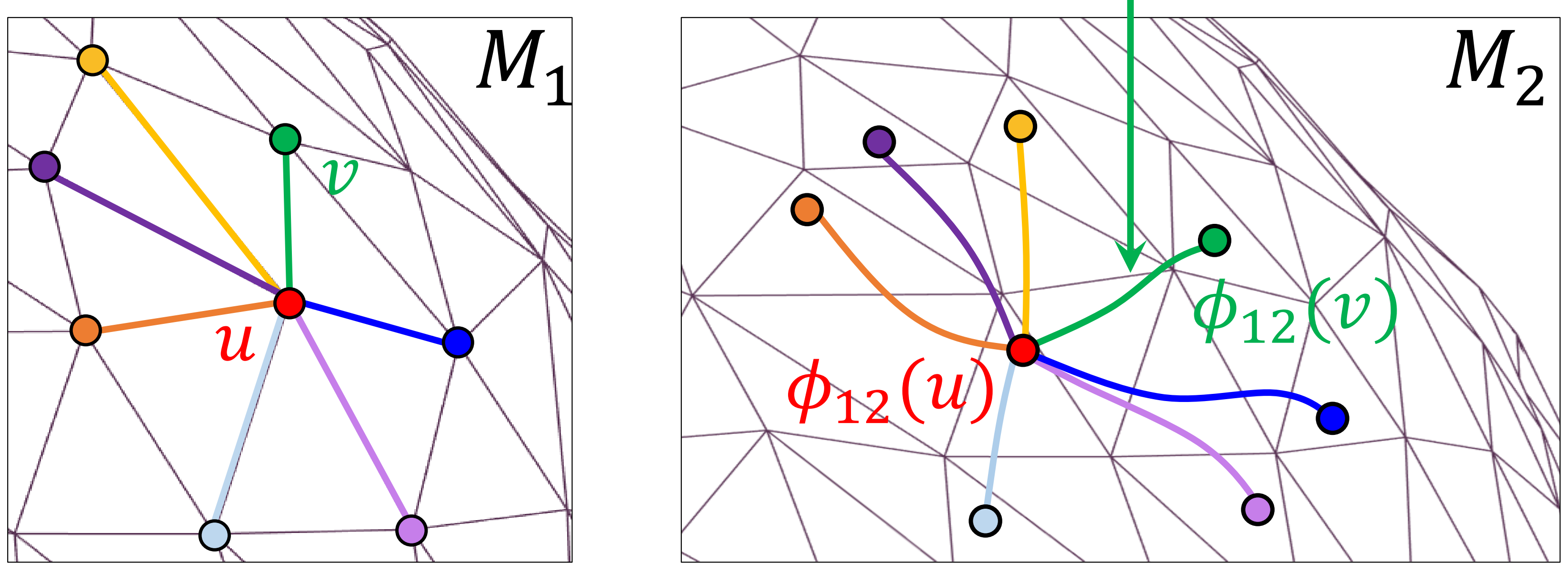
Output

The *Dirichlet* energy measures smoothness:

$$E(\phi) = \int_{M_1} |d\phi_{12}|^2$$

Discretization:

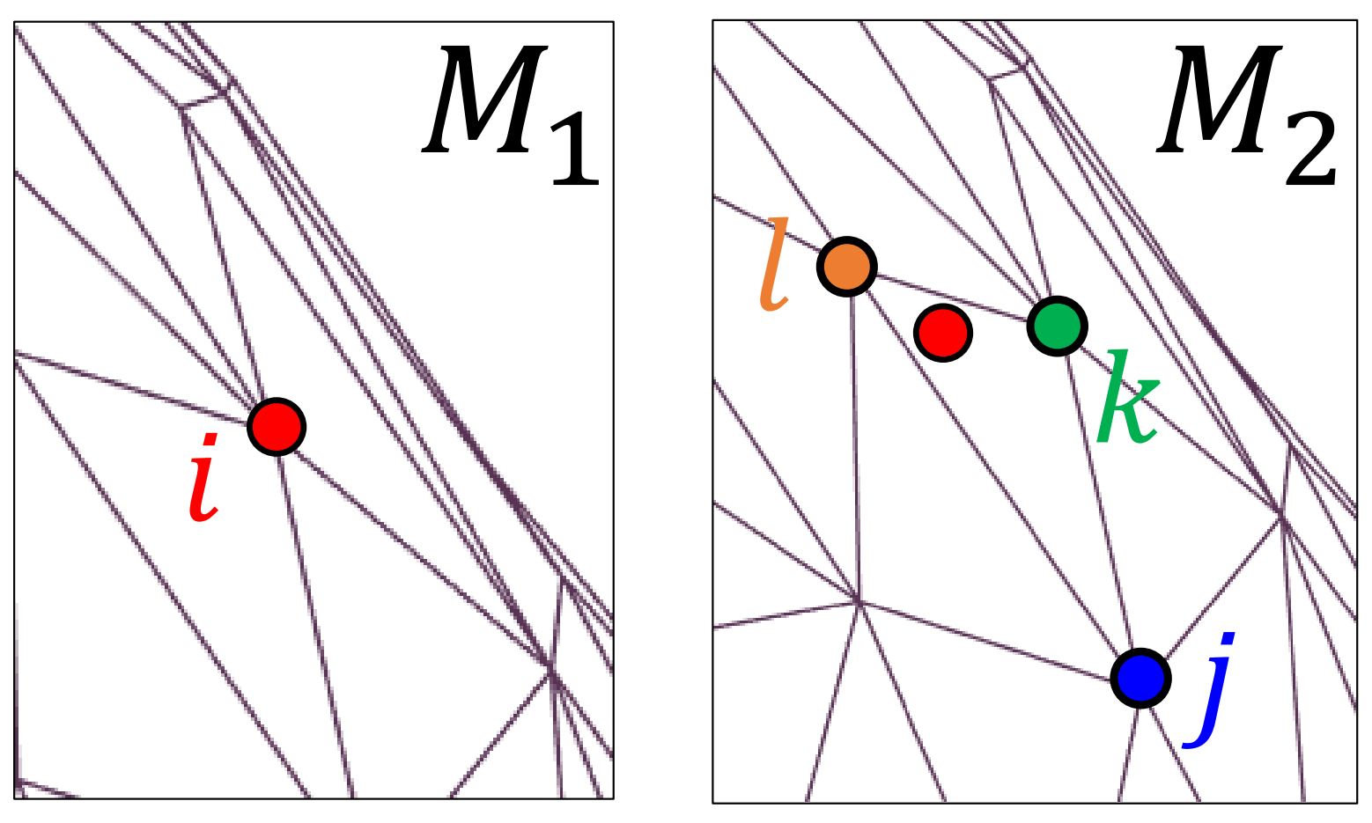
$$E_D(\phi_{12}) = \sum_{(u,v)\in E_1} w_{uv} \underbrace{d^2_{M_2}(\phi_{12}(u), \phi_{12}(v))}_{}$$



We suggest a quadratic approximation using Multi-Dimensional Scaling (MDS):

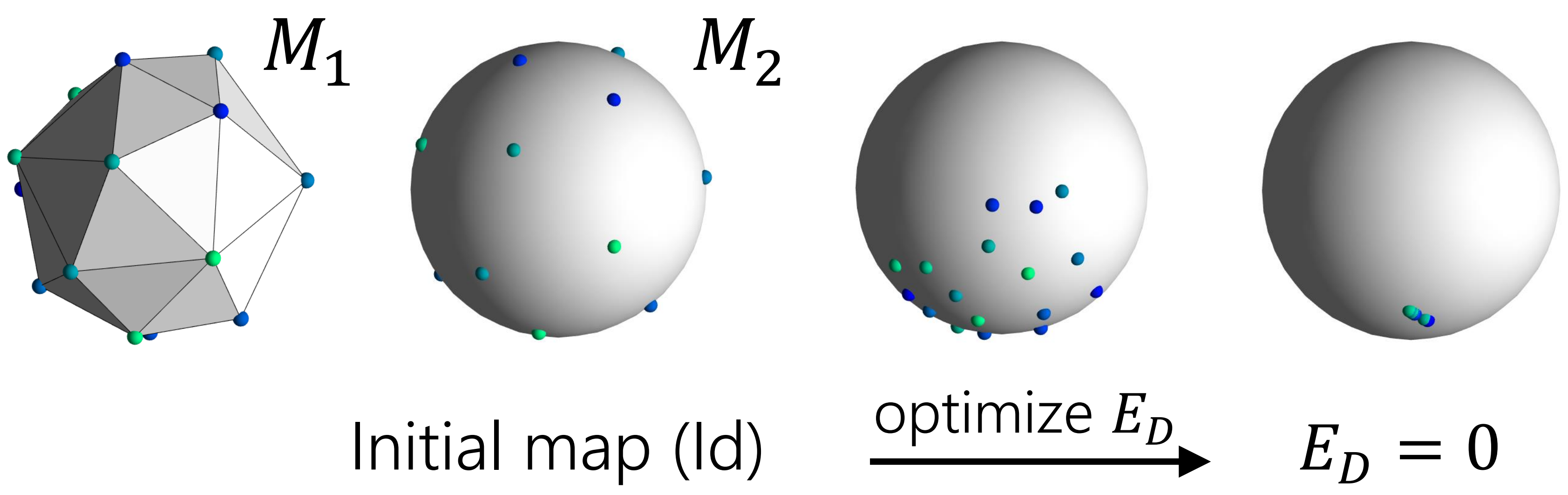
- X_2 is the high dimensional embedding of M_2 (Euclidean distances in the embedding space approximate geodesic distances on M_2)
- P_{12} is a matrix that represents the map ϕ_{12}

$$\begin{array}{cc} & \begin{array}{ccc} j & k & l \end{array} \\ P_{12} = & \begin{pmatrix} & \vdots & \\ 0.1 & 0.2 & 0.7 \\ & \vdots & \end{pmatrix} i \end{array}$$

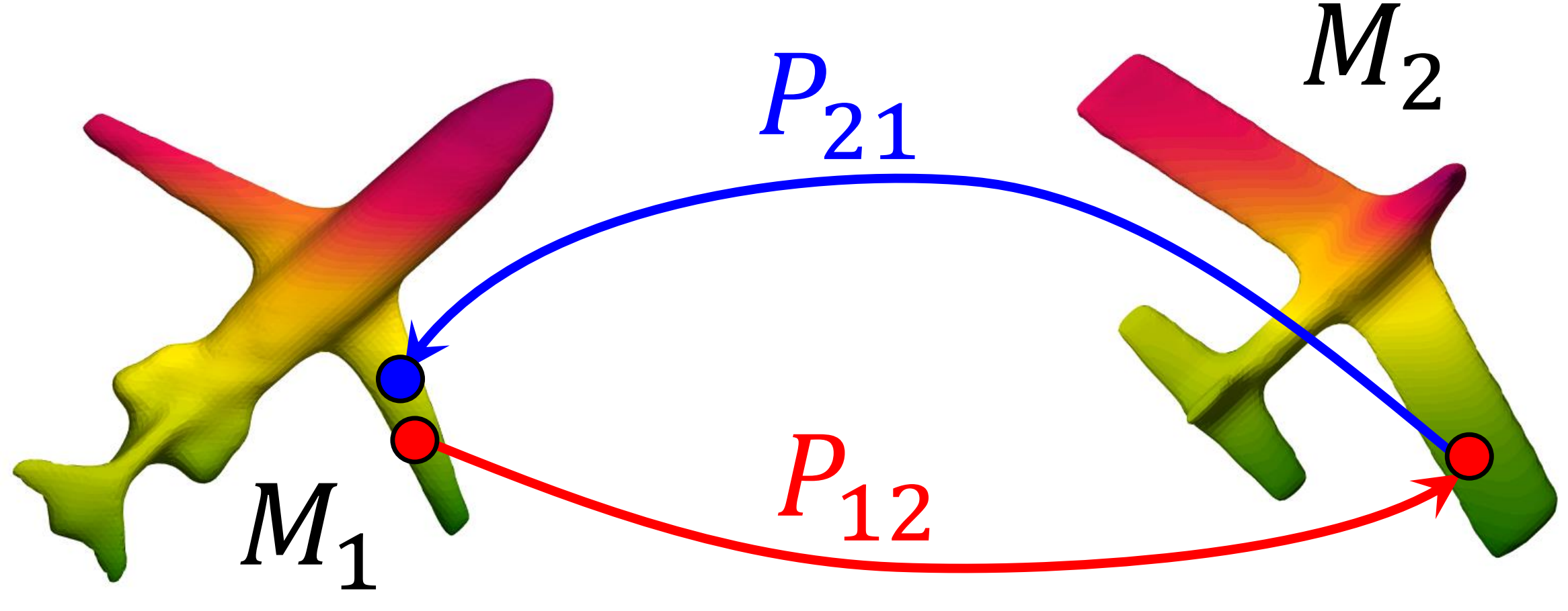


$$\boxed{E_D(P_{12}) = \|P_{12}X_2\|^2_{W_1}}$$

Problem: maps that minimize the Dirichlet energy tend to collapse:



Solution: we add a *reversibility* term that induces **bijectivity**



$$\boxed{E_R(P_{12}, P_{21}) = \|P_{21}P_{12}X_2\|^2_{M_2} + \|P_{12}P_{21}X_1\|^2_{M_1}}$$

Optimization: we use half quadratic splitting and optimize $E_D(P_{12}) + E_D(P_{21}) + E_R(P_{12}, P_{21})$ using alternating optimization

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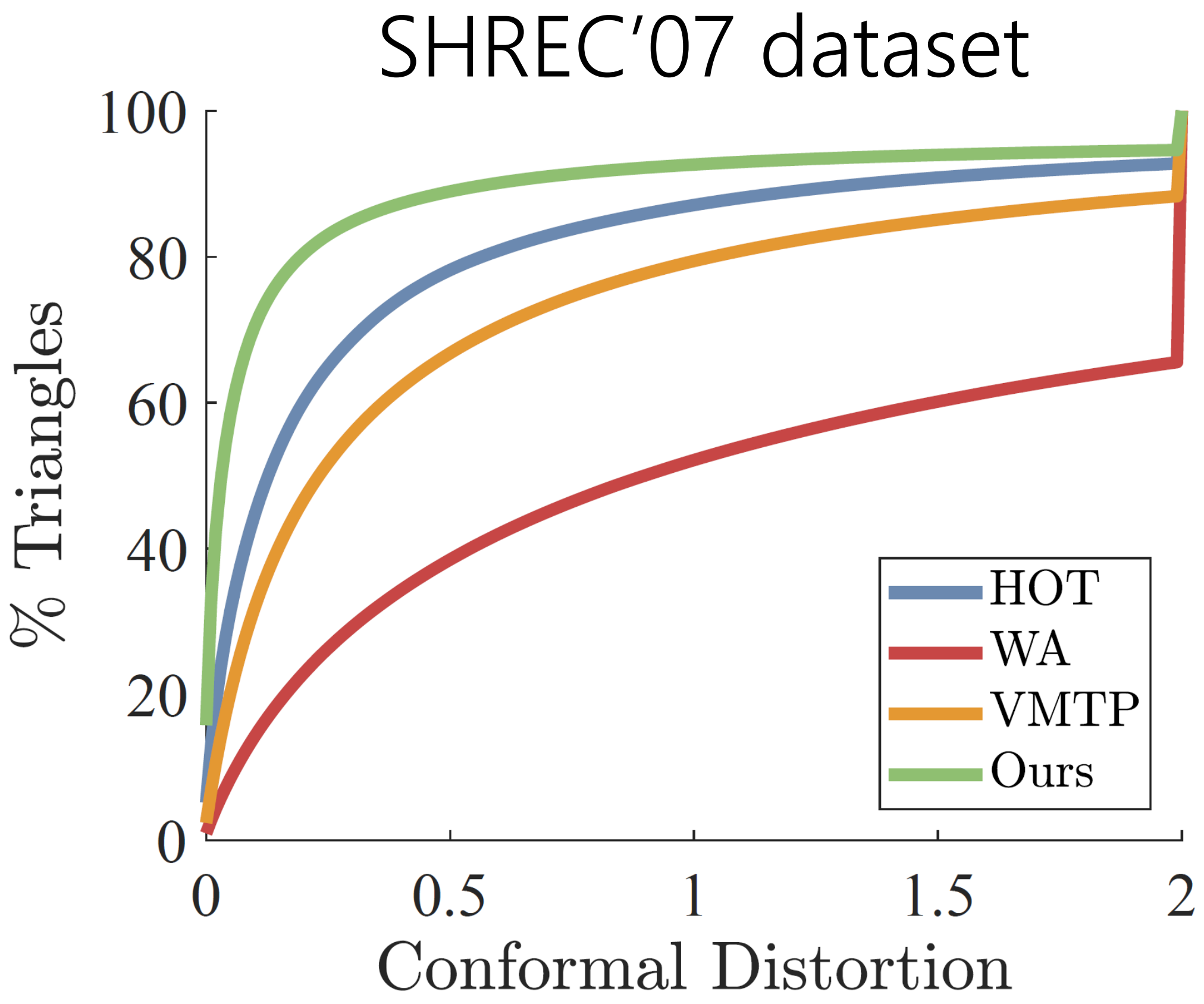
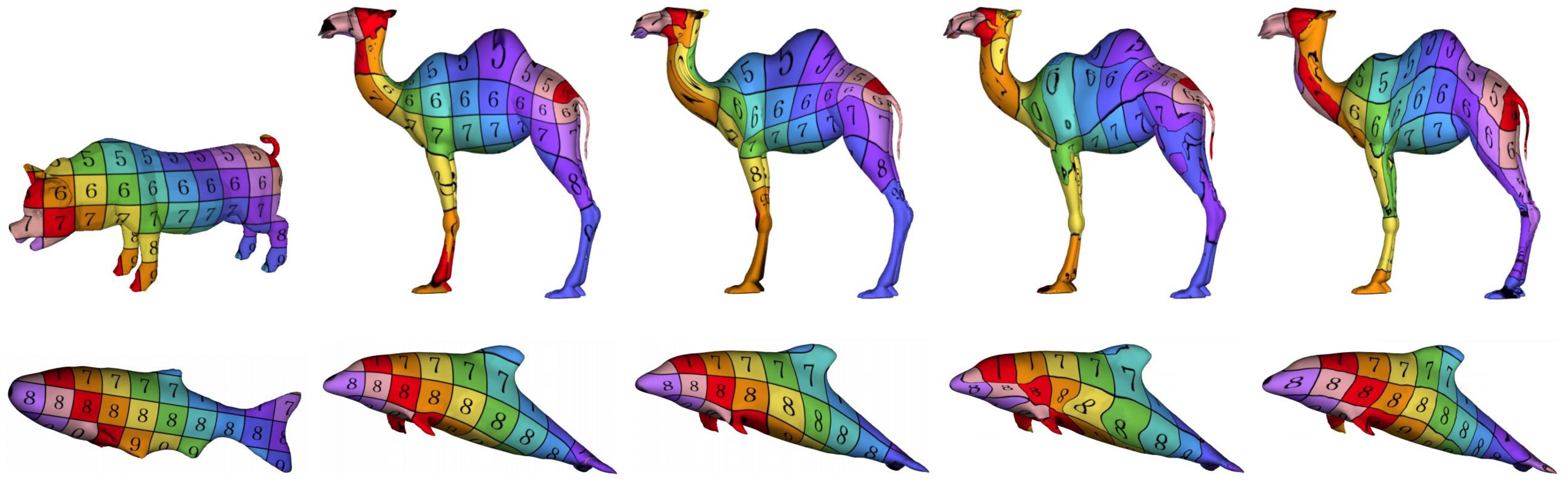
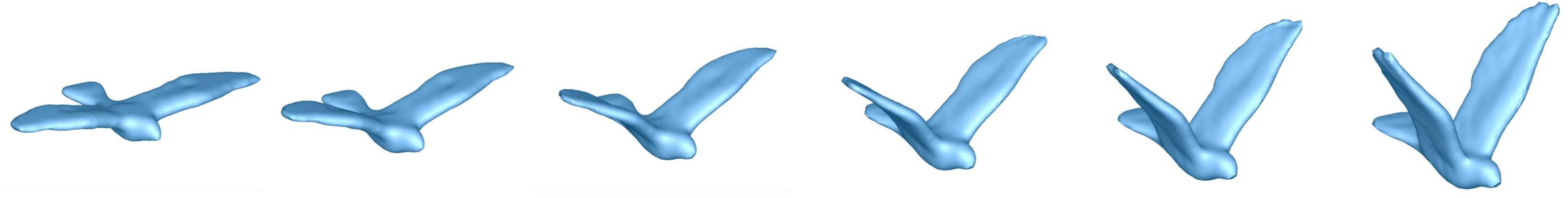
Results





Target Ours HOT [1] WA [2] VMTP [3]



Shape interpolation using [4] and our map

References:

[1] Aigerman, Noam, and Yaron Lipman. "Hyperbolic orbifold tutte embeddings", 2016

[2] Panozzo, Daniele, Ilya Baran, Olga Diamanti and Olga Sorkine-Hornung, "Weighted averages on surfaces", 2013

[3] Mandad, Manish, David Cohen-Steiner, Leif Kobbelt, Pierre Alliez and Mathieu Desbrun, "Variance-minimizing transport plans for inter-surface mapping", 2017

[4] Heeren, Behrend, Martin Rumpf, Peter Schröder, Max Wardetzky and Benedikt Wirth, "Exploring the Geometry of the Space of Shells", 2014

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