Compressed Learning: A Deep Neural Network Approach

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1. Compressed Sensing

- For a signal \( \mathbf{x} \in \mathbb{R}^N \) and a sensing matrix \( \Phi \in \mathbb{R}^{M \times N} (M \ll N) \) we measure the vector \( \mathbf{y} = \Phi \mathbf{x} \). The sensing rate is defined as \( R = M / N \) and since \( R \ll 1 \) the recovery of \( \mathbf{x} \) is not possible in general.
- Compressed-Sensing (CS) theory [1,2] suggests that for a signal that has a sparse representation in the domain of some linear transform

\[
\mathbf{x} = \Phi \alpha \quad \text{with} \quad \|\alpha\|_0 \ll N
\]

- Recovering the signal \( \mathbf{x} \) is possible (with theoretical guarantees) by solving the following optimization problem

\[
\min_{\alpha} \|\alpha\|_0 \quad \text{st} \quad \mathbf{y} = \Phi \mathbf{D} \alpha
\]

2. Compressed Learning

- Compressed-Learning (CL): applying learning on the projected values \( \mathbf{y} = \Phi \mathbf{x} \) directly, skipping reconstruction
  - [3]: Theoretical foundations
  - [4,5]: Practical results (see later)

This work proposes a holistic way to learn the projection and the recognition jointly via DNN.

3. Our approach

- End-to-end solution to CL
  - The first layer learns the compressed representation towards classification.
  - The second layer expands the output of the sensing layer.
  - DNN is learned jointly with the first two layers.

4. Results

- MNIST classification error (%) (lower is better)

<table>
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</tr>
</tbody>
</table>

- CIFAR10 classification results (%) (higher is better)

<table>
<thead>
<tr>
<th>Sensing Rate</th>
<th>No. of Measurements</th>
<th>Random Sensing + CNN</th>
<th>PCA + CNN</th>
<th>Proposed</th>
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</thead>
<tbody>
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<td>38.22%</td>
<td>52.173%</td>
<td>55.67%</td>
</tr>
</tbody>
</table>

5. Conclusions

- This work presents a novel deep learning approach to Compressed-Learning.
- Jointly optimizing the sensing and inference operators.
- Outperforming state-of-the-art CL methods on MNIST and CIFAR10.
- Extendible to numerous CL applications.

6. References