A NEW WAVELET DICTIONARY

How to build an adequate $\Phi$? Multi-scale and border-effects free?

Cropped Wavelets: Let $W_s$ be the wavelet synthesis matrix, $P$ a padding operator. The transform for signal $f$ is defined in terms of a pursuit:

$$g_w = \arg \min_g \|g\|_0 \text{ s.t. } P^T W_s g = f.$$ 

✓ This approach provides optimally sparse wavelet representations by virtually extending the signal borders.

(PARSE) DICTIONARY LEARNING

Dictionary Learning Find a set of atoms, $D$, that enable the sparsest representation, $X$, of signals in $Y$:

$$\min_{D, X} \frac{1}{2} \|Y - DX\|_F^2 \text{ s.t. } \|x_i\|_0 \leq p \forall i$$

Double Sparsity Extension of the above problem enforcing a sparse model on the atoms, i.e. $D = \Phi A$, so

$$\min_{A, X} \frac{1}{2} \|Y - \Phi AX\|_F^2 \text{ s.t. } \|x_i\|_0 \leq p \forall i$$

$\Phi$ is a fast base dictionary (e.g., DCT, Wavelets), while $A$ is sparse.

ONLINE SPARSE DICTIONARY LEARNING

Data: Training samples $\{y_t\}$, base-dictionary $\Phi = P^T W_s$, initial sparse matrix $A^0$. Init.: $G_\Phi = \Phi^T \Phi; U = 0$.

for $t = 1, \ldots, T$

Draw a mini-batch $Y_t$ at random;

$X_t \leftarrow$ Sparse Code ($Y_t, \Phi, A^t, G^t$), and make $S = supp(X_t)$;

$\nabla f(A^{t+1}) = \Phi^T (Y_t - \Phi A^t X_s^t) X_s^{tT}$;

$U_s^{t+1} = \gamma U_s^t + \eta \nabla f(A_s^t)$;

$A_s^{t+1} = P_k [A_s^t - U_s^{t+1}]$;

Update columns and rows of $G$ by $(A_s^{t+1})^T G_\Phi A_s^{t+1}$

end

APPLICATIONS

General Dictionary (32 × 32 patches)

Image Compression (100 × 100 patches/images)

PNSR

Inpainting (100 × 100 patches/images)


References


Contribution

We show how to efficiently handle bigger dimensions and go beyond the small patches in sparsity-based signal and image processing methods. The resulting large trainable atoms – trainlets – not only achieve state of the art performance in dictionary learning when compared to other methods, but they also open the door to new challenges and problems that remained unattainable until now.

 DOUBLE SPARSITY

Extension of the above problem enforcing a sparse model on the atoms, i.e. $D = \Phi A$, so

$$\min_{A, X} \frac{1}{2} \|Y - \Phi AX\|_F^2 \text{ s.t. }\|x_i\|_0 \leq p \forall i$$

$\Phi$ is a fast base dictionary (e.g., DCT, Wavelets), while $A$ is sparse.

Zero Padding

Symmetric Extension

Periodic Extension