Hele-Shaw Flow Simulation with Interactive Control using Complex Barycentric Coordinates

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The Physics
- Two plates separated by a small gap $h$.
- One/Twofluids trapped between the plates.
- Constant injection/suction of rate $Q$.

The Math
- The flow is described by the following set of equations [1]:
  \[
  (1) \quad W(z) = \frac{Q}{2\pi} \log(z - s) + f(z)
  
  (2) \quad v = -\frac{\partial}{\partial z} W
  
- With the boundary conditions:
  \[
  (3) \quad Re(W) = \gamma \kappa \quad \text{for} \quad z \in \partial \Omega
  
The Simulation
- Uses the Cauchy-Green coordinates [2].
  \[
  f(z) = \sum_{j=0}^{n} C_j(z)f_j
  
  C_j(z) = \frac{1}{2\pi i} \left( \frac{B_{j+1}B_j}{A_{j+1}} \log \left( \frac{B_{j+1}}{B_j} \right) - \frac{B_{j-1}B_j}{A_j} \log \left( \frac{B_{j-1}}{B_j} \right) \right)
  
- At each iteration,
  - Solve equation (3) for $f_j$.
  - Calculate the velocity from (2).
  - Update the vertices positions $z_j$.

References


Real photograph by Antony Hall

Our simulation starting from an interface of a real experiment.

The user interactively moves the singularity during the flow.

Our simulation

Exterior flow with obstacles.

The fluid is pumped from the skeleton of the bunny.