CONTRIBUTION

We present a patch-based denoising algorithm relying on a sparsity-inspired model (K-SVD) within multi-scale analysis framework, that overcomes some of the disadvantages of the popular algorithms. Our method is competitive with state of the art methods in terms of PSNR while giving superior results with respect to visual quality.

BACKGROUND

Problem Statement: recover $z \in \mathbb{R}^n$ from

\[ Y = Z + \eta \]

$\eta \sim \mathcal{N}(0, \sigma^2)$.

Sparse Model:

$z = Dx, \quad D \in \mathbb{R}^{n \times m}, \quad (n < m), \quad ||x||_0 \leq m$

Sparse Coding:

\[ \min_x ||x||_0 \text{ subject to } ||y - Dx||_2^2 \leq \varepsilon^2. \]

MULTI-SCALE K-SVD DENOISING

- Noisy image $Y$
- $Y_b^W = (W_A Y)_b$: its wavelet transform coefficients
- $\hat{Z}_b^W$: denoised coefficients per band $b$
- $S$ decomposition levels.

Global MAP in the wavelet domain:

\[ \forall b, \{x_{ij,b}, D_b, Z_b^W\} = \arg\min_{x_{ij,b}, D_b, Z_b^W} \frac{1}{\lambda} \sum_{ij} ||Y_b^W - Z_b^W||_2^2 + \sum_{ij} \mu_{ij,b} ||x_{ij,b}||_0 + \sum_{ij} ||D_b x_{ij,b} - R_{ij,b} Z_b^W||_2^2 \]

Numerical Solution

1. Apply wavelet transform $Y_b^W = (W_A Y)_b$
2. Approximate the MAP estimator by K-SVD on patches from the different bands $b \rightarrow Z_b^W$
3. Apply inverse wavelet transform on the denoised coefficients $Z = W_A Z^W$

FUSING SINGLE AND MULTI-SCALE

\[ \hat{Z}_{\text{Single Scale}} = D_1 \hat{x}_1, \quad \hat{Z}_{\text{Multi Scale}} = D_2 \hat{x}_2 \]

\[ \hat{x} = \alpha \]

\[ \hat{\alpha} = \arg\min_{\alpha} ||\alpha||_0 \text{ subject to } ||\hat{y} - A\alpha||_2^2 \leq \varepsilon^c \]

Fused K-SVD denoised patch: $\hat{Z}_f = \frac{1}{\sqrt{3}} D \hat{\alpha}$

REFERENCES
